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CMA Channel Equalization Through An Adaptive MMSE Equalizer Based RLS Algorithm

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Abstract - The adaptive algorithm has been widely used in the digital signal processing like channel estimation, channel equalization, echo cancellation, and so on. One of the most important adaptive algorithms is the RLS algorithm. We present in this paper a multiple objective optimization approach to fast blind channel equalization. By investigating first the performance (mean-square error) of the standard fractionally spaced CMA (constant modulus algorithm) equalizer in the presence of noise, we show that CMA local minima exist near the minimum mean-square error (MMSE) equalizers. Consequently, CMA may converge to a local minimum corresponding to a poorly designed MMSE receiver with considerable large mean-square error. The step size in the RLS algorithm decides both the convergence speed and the residual error level, the highest speed of convergence and residual error level.

Key Words - CMA, RLS, Adaptive MMSE.

I. INTRODUCTION

Blind equalization has the potential to improve the efficiency of communication systems by eliminating training signals. Difficulties of its application in wireless communications, however, are due largely to the characteristics of the propagation media - multipath delays and fast fading. The challenge is achieving blind equalization using only a limited amount of data.

A widely tested algorithm is the constant modulus algorithm (CMA). In the absence of noise, under the condition of the channel invertibility, the CMA converges globally for symbol-rate IIR equalizers and fractionally spaced FIR equalizers. It is shown in [9] that CMA is less affected by the ill-conditioning of the channel. However, Ding *et. al.* [2] showed that CMA may converge to some local minimum for the symbolrate FIR equalizer. In the presence of noise, the analysis of convergence of CMA is difficult and little conclusive results are available. Another drawback of CMA is that its convergence rate may not be sufficient for fast fading channels. However the asymptotic performance of these eigen structure-based schemes is limited by the condition of the channel [12, 13]. Specifically, the asymptotic normalized mean square error (ANMSE) is lower bounded by the condition

number of the channel matrix.. The main feature of the proposed approach is the improved convergence property over the standard CMA equalization and the improved robustness for ill-conditioned channels.

II. BLIND CHANNEL EQUALIZATION AND TYPES

The field of blind channel equalization has been existence for a little over twenty years. Research during this time has centered on developing new algorithms and formulating a theoretical justification for these algorithms. Blind channel equalization is also known as a self-recovering equalization.. The objective of blind equalization is to recover the unknown input sequence to the unknown channel based solely on the probabilistic and statistical properties of the input sequence. The receiver can synchronize to the received signal and to adjust the equalizer without the training sequence. The term blind is used in this equalizer because it performs the equalization on the data without a reference signal. Instead, the blind equalizer relies on knowledge of the signal structure and its statistic to perform the equalization. 1. Blind signal is the unknown signal which would be identified in output signal with accommodated noise signal at receiver. 2. Channel equalization uses the idea & knowledge of training

sequences for channel estimation where as Blind channel equalization doesn't utilizes the characteristics of training sequences for frequency and impulse response analysis of channel. 3. Blind Channel Equalization differs from channel equalization and without knowing the channel characteristics like transfer function & SNR it efficiently estimate the channel and reduces the ISI by blind signal separation at receiver side by suppressing noise in the received signal.

III. CMA - CONSTANT MODULUS ALGORITHM

In digital communication, equalizer was designed to compensate the channel distortions, through a process known as equalization. There are two types of equalization which are: 1) Trained equalization, 2) Blind (self-recovering) Equalization.

Blind equalization finds important application in data communication system. In data communications, digital signals are generated and transmitted by the sender through an analog channel to the receiver. Linear channel distortion as a result of limited channel bandwidth, multipath and fading is often the most serious distortion in digital communication system. Blind equalization improves system bandwidth efficient by avoiding the use of training sequence. The linear channel distortion, known as the Inter-symbol interference (ISI), can severely corrupt the transmitted signal and make it difficult for the receiver to directly recover the transmitted data. Channel equalization and identification has proven to be an effective means to compensate the linear distortion by removing much of the ISI.

Channel Equalization:

A typical communication system design involves first passing the signal to be transmitted through a whitening filter to reduce redundancy or correlation and then transmitting the resultant whitened signal. At the receiver, the recorded signal is passed through the inverse whitening filter and the original signal is thus restored. However, the channel will affect the transmitted signal because of a) Channel noise b) Channel dispersion leading to inter symbol interference. For example, by reflection of the transmitted signal from various objects such as buildings in the transmission path, leading to echoes of the transmitted signal appearing in the receiver. Therefore, it is necessary to pass the received signal through a so called equalizing filter to undo the dispersion effect as shown in figure 2 below.

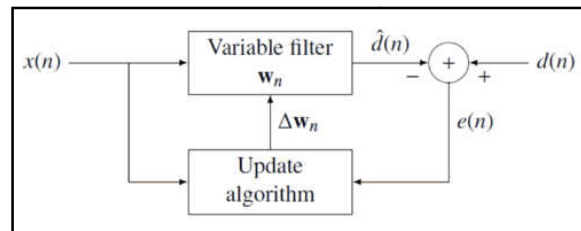
Blind Channel Equalization:

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the input sequence. The receiver can synchronize to the received signal and to adjust the equalizer without the training sequence. The term blind is used in this equalizer because it performs the equalization on the data without a reference signal. Instead, the blind equalizer relies on knowledge of the signal structure and its statistic to perform the equalization. Figure 2 shows such a system. There has been extensive research on this subject for single user applications as well as multi-user applications. The Constant Modulus Algorithm is one such algorithm employed for the blind adaptation problem.

IV. RLS ALGORITHM

The idea behind RLS filters is to minimize a cost function C by appropriately selecting the filter coefficients \mathbf{W}_n , updating the filter as new data arrives. The error signal $e(n)$ and desired signal $d(n)$ are defined in the negative feedback diagram below:



The error implicitly depends on the filter coefficients through the estimate $\hat{d}(n)$:

$$e(n) = d(n) - \hat{d}(n) \tag{1}$$

The weighted least squares error function C —the cost function we desire to minimize—being a function of $e(n)$ is therefore also dependent on the filter coefficients:

$$C(w_n) = \sum_{i=0}^n \lambda^{n-i} e^2(i)$$

where $0 \leq \lambda \leq 1$ is the "forgetting factor" which gives exponentially less weight to older error samples.

The cost function is minimized by taking the partial derivatives for all entries k of the coefficient vector W_n and setting the results to zero

$$\frac{\partial C(W_n)}{\partial w_n(k)} = \sum_{i=0}^n 2\lambda^{n-i} e(i) \frac{\partial e(i)}{\partial w_n(k)} = \sum_{i=0}^n 2\lambda^{n-i} e(i)x(i-k) = 0$$

Next, replace $e(n)$ with the definition of the error signal

$$\sum_{i=0}^n \lambda^{n-i} \left[d(i) - \sum_{l=0}^n w_n(l)x(i-l) \right] x(i-k) = 0$$

Rearranging the equation yields

$$\sum_{l=0}^n w_n(l) \left[\sum_{l=0}^n \lambda^{n-i} x(i-l)x(i-k) \right]$$

$$= \sum_{l=0}^n \lambda^{n-i} d(i)x(i-k)$$

This form can be expressed in terms of matrices

$$R_x(n)w_n = r_{dx}(n)$$

where $R_x(n)$ is the weighted sample correlation matrix for $x(n)$, and $r_{dx}(n)$ is the equivalent estimate for the cross correlation between $d(n)$ and $x(n)$. Based on this expression we find the coefficients which minimize the cost function as

$$w_n = R_x^{-1}(n)r_{dx}(n) \quad (2)$$

This is the main result of the discussion.

Choosing λ :

The smaller λ is, the smaller contribution of previous samples. This makes the filter *more* sensitive to recent samples, which means more fluctuations in the filter coefficients. The $\lambda = 1$ case is referred to as the *growing window RLS algorithm*.

The RLS algorithm for a p -th order RLS filter can be summarized as

Parameters: p =filter order

$$\lambda = \text{forgetting factor,}$$

$$\delta = \text{value to initialize, } P(0)$$

$$w_n = 0$$

In itialization: $x(k) = 0, k = -p, \dots, -1$

$$P(0) = \delta^{-1}I$$

where I is the identity matrix of rank $p + 1$

Computation: For $n = 0, 1, 2, \dots$

$$X(n) = \begin{bmatrix} x(n) \\ x(n-1) \\ \dots \\ \dots \\ x(n-p) \end{bmatrix}$$

$$\alpha(n) = d(n) - X^T(n)w(n-1)$$

$$g(n) = P(n-1)X(n) \left\{ \lambda + X^T(n)P(n-1)X(n) \right\}^{-1}$$

$$P(n) = \lambda^{-1}P(n-1) - \left\{ g(n)X^T(n)\lambda^{-1}P(n-1) \right\}$$

$$w(n) = w(n-1) + \alpha(n)g(n)$$

V. ADAPTIVE MMSE EQUALIZER

The Sampled signal after MMSE Equalizer can be expressed in matrix form as

$$\hat{s}(i) = w^H y(i) \quad (3)$$

$$\text{Where } y(i) = H^T(i)s(i) + n(i), \quad (4)$$

M is the length of the MMSE Equalizer: $w = [w_1, w_2, w_3, w_4, w_5, \dots, w_M]^T$ is the equalizer coefficients vector; Then the error signal $e(i)$ is given by

$$e(i) = d(i) - \hat{s}(i) \quad (5)$$

where $d(i)$ is the desired response. For MMSE equalizer, $d(i) = s(i + D)$, D is a time delay parameter which is $L + 1$ usually. The MMSE criterion is used to derive the optimal equalizer coefficients vector w :

$$w = \text{minimize } E \left\{ |e|^2 \right\} \quad (6)$$

We make the assumption that signal $s(i)$ and noise $n(i)$ are independent identity distribution stochastic

Variable and uncorrelated each other, then the equalizer coefficients vector w can be expressed as[2]:

$$w = (H^H H + \frac{1}{SNR} I)^{-1} H^H \delta_D \quad (7)$$

Where $\delta_D = [0, \dots, 1_D; 0, \dots, 0]_{1 \times (L+M-1)}^T$

$SNR = \frac{\sigma_s^2}{\sigma_n^2}$ denotes the signal noise ratio I is

$M \times M$ identity matrix.

To reduce the complexity caused by matrix inversion of ideal MMSE equalizer, we propose an adaptive MMSE equalizer algorithm. In code-multiplexed pilot CDMA systems, conventional adaptive equalizer is difficult to implement for lack of reference signal. In this paper, the steepest descent method [4] is used to derive adaptive equalizer algorithm in code-multiplexed pilot CDMA systems.

According to Equation.3 and Equationn.5, the mean square error (MSE) J can be expressed as $J(w) = E[e(i)e(i)^*] = \sigma_s^2 - w^H p - p^H w + w^H R w$ (8)

where autocorrelation matrix $R = E[y(i)y^H(i)]$; cross-correlation vector $p = E[y(i)d^*(i)]$, σ_s^2 denotes the signal power; $(\cdot)^*$ represents conjugate operation. Because the wireless channel is time-varying, the equalizer coefficients vector w must be updated real time. Conventional adaptive algorithm requires reference signal $d(i)$, while in the downlink of code-multiplexed pilot CDMA systems, $d(i)$ is difficult to distill. To resolve this problem, the steepest decent method is used. From Eqn.8, the gradient vector is

$$\frac{\partial J(w)}{\partial w} = -2p + 2Rw \quad (9)$$

then the equalizer coefficients updating equation is $w(i+1) = w(i) + 2\mu[p - Rw(i)]$ (10)

where parameter μ is a positive real-valued constant which controls the size of the incremental correction applied to the equalizer coefficients vector.

For the autocorrelation matrix:

$$\begin{aligned} R &= E[y(i)y^H(i)] \\ R &= E[s(i)s^H(i)]\{H^H(i)H(i)\}^T + E[n(i)n^H(i)] \\ R &= \sigma_s^2 \{H^H(i)H(i)\}^T + \sigma_n^2 I \end{aligned} \quad (11)$$

the cross-correlation vector

$$\begin{aligned} p &= E[y(i)d^*(i)] = E[(H^T(i)s(i) + n(s))s^*(i-D)] \\ p &= \sigma_s^2 H^T(i)\delta_D \end{aligned} \quad (12)$$

From Equation.7,8,9, we can obtain the time recursive equation of MMSE equalizer by:

$$w(i+1) = w(i) + 2\mu\sigma_s^2[H^T(i)\delta_D - (\{H^H(i)H(i)\}^T + \frac{1}{SNR}I)w(i)] \quad (13)$$

As can be seen from Equation.13, the updating process avoids the matrix inversion operation. On the other hand, the updating process abstains the requirement to store the autocorrelation matrix $\mathbf{R}(i)$ and only the equalizer coefficients vector of last time is needed. From Eqn.13 we know, the channel convolution

matrix $\mathbf{H}(i)$ is required to update the equalizer coefficients vector.

For CMA, channel response can be estimated through code-multiplexed pilot. In this paper, the low complexity sliding-window method is used to estimate the channel coefficients, which can be expressed as

$$\hat{\beta}_l(i) = \frac{1}{2\sqrt{\alpha}pw(i+1)T_s} \int_{\tau_i+(i-\frac{w}{2})T_s}^{\tau_i+(i+\frac{w}{2})T_s} y(t)c_p^*(t-\tau_l)dt \quad (14)$$

where $\hat{\beta}_l(i)$ is estimation of the complex gain of l -th path; w is the length of sliding-window in symbols and should be selected properly according to the varying speed of the channel.

VI. EXPERIMENTAL RESULTS

The performances for the Adaptive MMSE and adaptive CMA algorithm through and RLS algorithm is experimental performed with accurate figures from 1-8.

The performances of the channel estimation is an analyze in such a way that transmitted bits and receiver bits, Equalizers and very important task that is Convergences.

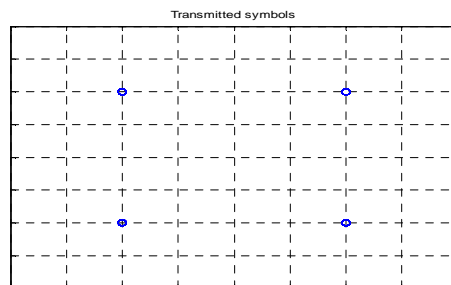


Fig. 1: Transmitter side of Adaptive CMA

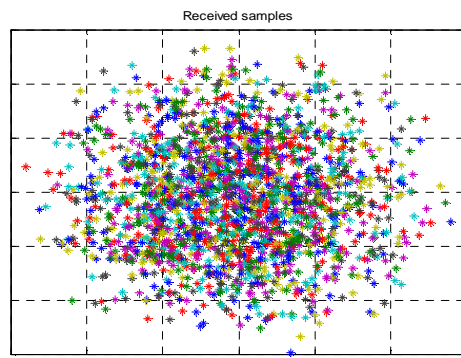


Fig. 2 : Receiver side of Adaptive CMA

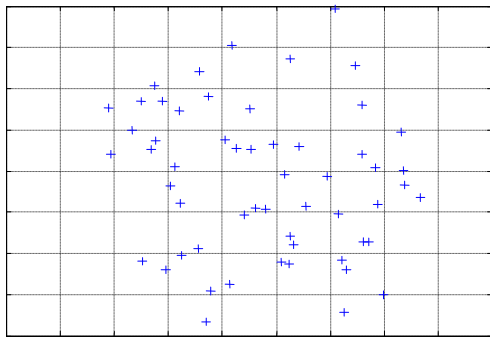


Fig. 3: Adaptive CMA Equalizer

The first figures from 1-4 are obtained for Adaptive CMA Equalizer, with more efficient for equalization and convergences. Secondly from figure 5-8 are obtained for an Adaptive MMSE equalizer through RLS algorithm. The efficient of equalization and convergences is too good. The time complexity is very less and more efficient for advance communication systems.

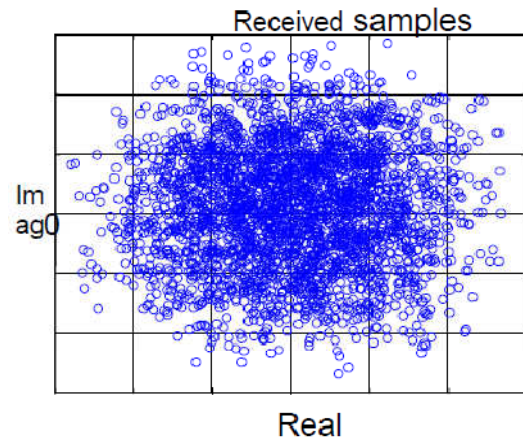


Fig. 6 : Receiver side of Adaptive MMSE Equalizer

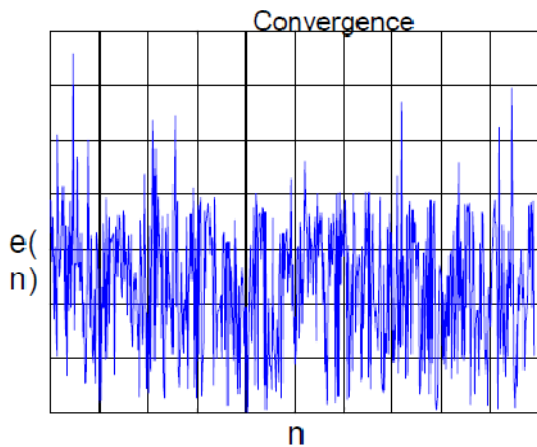


Fig. 4 : Convergence of Adaptive CMA equalizer

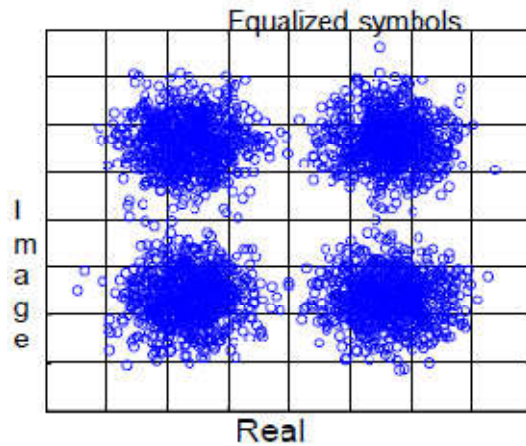


Fig. 7 : Adaptive MMSE Equalizer through RLS

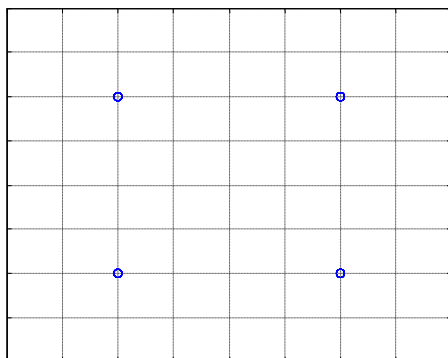


Fig. 5: Transmitter side of Adaptive MMSE Equalizer

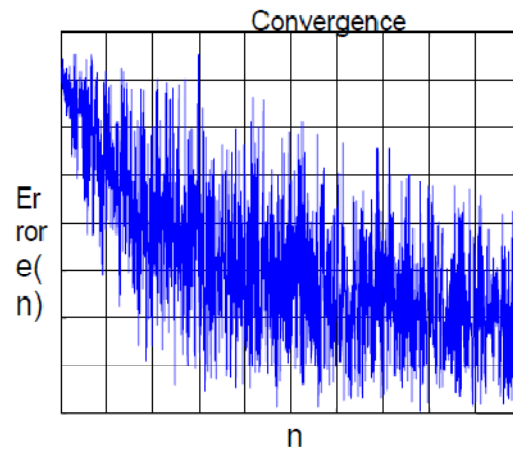


Fig. 8: Convergence of Adaptive MMSE Via RLS algorithm

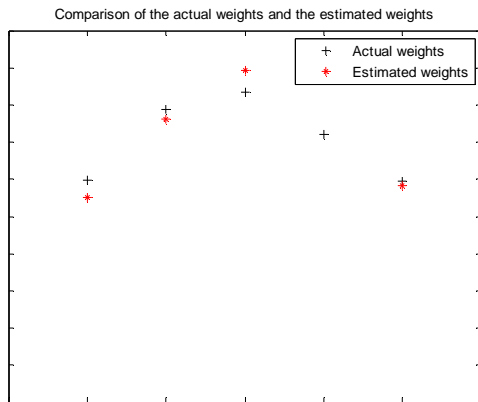


Fig. 9: Comparison of the actual weight and estimated weight

VII. CONCLUSION

In this paper, Aim at conventional Rake receiver can't satisfy the performance requirement in high data rate transmission, while ideal MMSE equalizer is difficult to real-time implement because its large computational complexity, a low complexity adaptive MMSE equalizer algorithm is proposed. In future conclusion, the proposed low complexity adaptive MMSE equalizer in code-multiplexed CDMA system can be proposed and this system has better practical application value.

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