

July 2013

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### Recommended Citation

Kaur, Tajinder; Kumar, Dinesh; Walia, Ekta; and Sandhu, Manjit (2013) "Curvelet Denoising with Improved Thresholds for Application on Ultrasound Images," *International Journal of Computer and Communication Technology*. Vol. 4 : Iss. 3 , Article 2.

Available at: <https://www.interscience.in/ijcct/vol4/iss3/2>

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# Curvelet Denoising with Improved Thresholds for Application on Ultrasound Images



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**Abstract** - In medical image processing, image denoising has become a very essential exercise all through the diagnose. Negotiation between the preservation of useful diagnostic information and noise suppression must be treasured in medical images. In case of ultrasonic images a special type of acoustic noise, technically known as speckle noise, is the major factor of image quality degradation. Many denoising techniques have been proposed for effective suppression of speckle noise. Removing noise from the original image or signal is still a challenging problem for researchers. In this paper, a Curvelet transform based denoising with improved thresholds is proposed for ultrasound images.

**Keywords** - Image Denoising; Curvelet Transform; Ultrasound Images.

## I. INTRODUCTION

### A. Motivation

Noise elimination is a main concern in computer vision and image processing. For example, in many applications where operators based on computing image derivatives are applied, any noise in the image can result in serious errors. Noise presence is manifested by undesirable information, not related to the scene under study, which perturbs the information relative to the form observable in the image. It is translated into more or less severe values, which are added or subtracted to the true gray level values on a number of pixels. Noise can appear in images from a variety of sources: during the acquisition process, due to cameras' quality, and resolution, but also due to the acquisition conditions, such as the illumination level, calibration and positioning. Moreover, it can be a function of the scene environment: background, shape composition, material nature, etc.

Noise distribution in the image varies considerably inter- and intrainages. Noise can be inserted into the image signal in different ways. It can be random and coherent with this signal. In this case, it is included inside the spatial frequency domain of the image and cannot be suppressed other than by using a priori knowledge about the image. This often produces a loss in spatial resolution. If the noise is periodic, it is out of the useful information, and its suppression is often easy without information lost. Finally, when the captor provides a repetitive and redundant signal, which is normally the case for moving scenes, the noise present is said to be incoherent with the information being sought and can be easily eliminated.

Jong-Sen Lee [1] has proposed the computational

techniques involving contrast enhancement and noise filtering on two-dimensional image arrays are developed based on their local mean and variance. These algorithms are nonrecursive and do not require the use of any kind of transform. V. V. Frost et al. have proposed the standard image processing techniques which are used to enhance noncoherent optically produced images are not applicable to radar images due to the coherent nature of the radar imaging process [2]. Piotr S. Windyga has proposed a generic -dimensional filter with the primary purpose of eliminating impulsive-like noise is presented. This recursive nonlinear filter is composed of two conditional rules, which are applied independently, in any order, one after the other [3]. Yongjian Yu and Scott T. Acton have proposed speckle reducing anisotropic diffusion (SRAD), a diffusion method tailored to ultrasonic and radar imaging applications. SRAD is the edge-sensitive diffusion for speckled images, in the same way that conventional anisotropic diffusion is the edge-sensitive diffusion for images corrupted with additive noise [4]. Jean-Luc Starck et al. have proposed the radon, ridgelet and curvelet transforms for image denoising. They apply these digital transforms to the denoising of some standard images embedded in white noise [5]. S.Sudha. et al. have proposed the wavelet based image denoising using adaptive thresholding which describes a new method for suppression of noise in image by fusing the wavelet Denoising technique with optimized thresholding function, improving the denoised results significantly [6]. M.Singh et al. described a comparative study of various spatial domain filters for speckle suppression in Ultrasound images [7]. K.R.Joshi et al proposed the quality metrics for speckle in coherent imaging and their limitations. It also describes a new metric-SDI, its uniqueness in quantifying the speckle and comparison of performance with existing metrics [8]. Li

Hongqiao and W.Shengqian have proposed a new method of wavelet based image denoising with soft-thresholds [9]. M.N. Nobil et al. proposed an efficient and simple method for noise reduction from medical images. They modified median filter by adding more features [10]. Rayudu et al. have proposed the curvelet transform for ultrasound image denoising. Speckle reduction/filtering i.e. visual enhancement techniques are used for enhancing the visual quality of the images [11].

**B. Main Contribution**

To overcome the limitations of SRAD Filter and wavelet transform, the curvelet transform is proposed for denoising of ultrasound images with improved thresholds for preserving and enhance the edges. The performance of each filter will be compared using parameter PSNR (Peak Signal to Noise Ratio).

The organization of the paper as follows: In section I, a brief review of image denoising and related work is given. Section II, presents a concise review of curvelet transform. Section III, presents the thresholding methods for image denoising and proposed system framework. Experimental results and discussions are given in section IV. Based on above work conclusions are derived in section V.

**II. CURVELET TRANSFORM (CT)**

**A. Radon Transform**

The Radon transform of an object  $f$  is the collection of line integrals indexed by  $(\theta, t) \in [0, 2\pi) \times R$  given by

$$Rf(\theta, t) = \int f(x_1, x_2) \delta(x_1 \cos \theta + x_2 \sin \theta - t) dx_1 dx_2 \quad (1)$$

where  $\delta$  is the Dirac distribution. The ridgelet coefficients  $CRT_f(a, b, \theta)$  of an object  $f$  are given by analysis of the Radon transform via

$$CRT_f(a, b, \theta) = \int Rf(\theta, t) a^{-1/2} \psi((t-b)/a) dt \quad (2)$$

Basic algorithm for discrete radon transform is as follows

Compute the two-dimensional Fast Fourier Transform (FFT) of function  $f$ .

Using an interpolation scheme, substitute the sampled values of the Fourier transform obtained on the square lattice with sampled values of  $\hat{f}$  on a polar lattice: that is, on a lattice where the points fall on lines through the origin.

Compute the one-dimensional Inverse Fast Fourier Transform (IFFT) on each line; i.e., for each value of the angular parameter.

**B. Discrete Ridgelet Transform (DRT)**

A continuous ridgelet transform is calculated by applying 1D wavelet transform to the slices of radon transform  $R_f(\theta, \cdot)$ . In radon transform a famous projection-slice theorem is used

$$\hat{f}(\omega \cos \theta, \omega \sin \theta) = \int R_f(\theta, t) e^{-2\pi i \omega x} dt \quad (3)$$

This theorem says that the Radon transform can be obtained by applying the one-dimensional inverse Fourier transform to the two-dimensional Fourier transform of function restricted to radial lines through the origin. The relation among the Fourier, radon and ridgelet domain is depicted in Fig. 1.

To complete the ridgelet transform, apply a one-dimensional wavelet transform along the radial variable in Radon space. The sum up of above procedure is shown in Fig. 2 in the form of flow chart. The DRT of an image of size  $n \times n$  is an image of size  $2n \times 2n$ , introducing a redundancy factor equal to 4 [5].

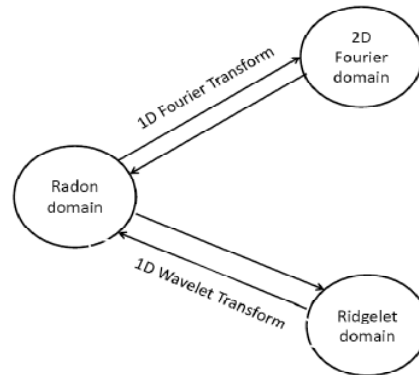


Fig. 1: Relations between transforms.

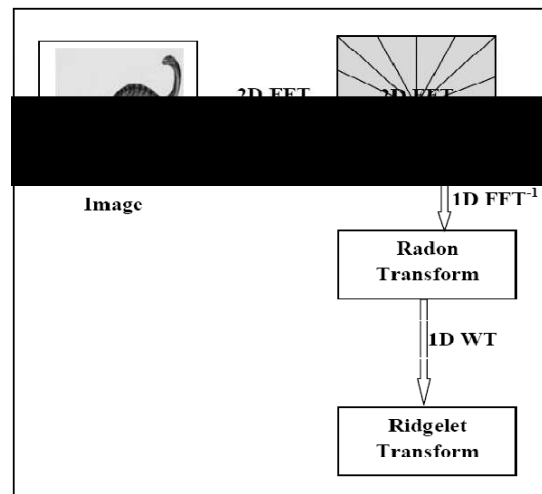


Fig. 2: Flowchart of Discrete ridgelet transform.

C. Curvelet Transform

The idea of Curvelet (Starck *et al.*, [5]) is to represent a curve as a superposition of functions of various lengths and widths obeying the scaling law  $width \approx length^2$ . This can be done by first decomposing the image into sub-bands, i.e., separating the object into a series of disjoint scales. Each scale is then analysed by means of a local ridgelet transform.

Recently, Starck *et al.* [5] showed that “`a trous” subband filtering algorithm [12] is especially well-adapted to the needs of the digital Curvelet transform. The algorithm decomposes an  $n$  by  $n$  image  $I$  as a superposition of the form

$$I(x, y) = c_j(x, y) + \sum_{j=1}^J \omega_j(x, y) \tag{4}$$

Where  $\omega_j$  represents “the details of  $I$ ” at scale  $2^{-j}$ . Thus, the algorithm outputs  $J + 1$  sub band arrays of size  $n \times n$ . [The indexing is such that, here,  $j = 1$  corresponds to the finest scale (high frequencies).]

Curvelet Transform Algorithm

Starck *et al.* [5] presented a sketch of the discrete Curvelet transform algorithm:

1. apply the `a trous algorithm with  $J$  scales [12];
2. set  $B_1 = B_{min}$ ;
3. for  $j = 1, \dots, J$  do
  - i. partition the subband  $\omega_j$  with a block size  $B_j$  and apply the digital ridgelet transform to each block;
  - ii. if  $j \text{ modulo } 2 = 1$  then  $B_{j+1} = 2B_j$ ;
  - iii. else  $B_{j+1} = B_j$ ;

Note that the coarse description of the image  $c_j$  is not processed. Fig. 3 gives an overview of the organization of the algorithm. Extensive literature of Curvelet Transform theory can be found in the reference [5].

III. IMAGE DENOISING

A. Denoising by Hard Thresholding

Suppose that one is given noisy data of the form:

$$\bar{I}(x, y) = I(x, y) + \sigma Z(x, y) \tag{5}$$

Where  $Z(x,y)$  is unit-variance and zero-mean Gaussian noise. Denoising a way to recover  $I(x,y)$  from the noisy image  $\bar{I}(x, y)$  as proper as possible. Rayudu *et al.* [11]

have proposed the hard thresholds for Ultrasound image denoising as shown below:

Let  $y_\lambda$  be the noisy Curvelet Coefficients ( $y = C*I$ ). They used the following hard-thresholding rule for estimating the unknown Curvelet coefficients:

$$\hat{y}_\lambda = y_\lambda; \quad \text{if } |y_\lambda|/\sigma \geq k \bar{\sigma}_\lambda \tag{6}$$

$$\hat{y}_\lambda = 0; \quad \text{else}$$

In their experiments, they have chosen a scale dependent value for  $k$ ;  $k = 4$  for the first scale ( $j = 1$ ) while  $k = 3$  for the others ( $j > 1$ ).

Algorithm:

1. Apply Curvelet transform to the noisy image and get the scaling coefficients and Curvelet coefficients.
2. Chose the threshold by Eq. (6) and apply thresholding to the Curvelet coefficients (leave the scaling coefficients alone).
3. Reconstruct the scaling coefficients and the Curvelet coefficients thresholded and get the denoised image.

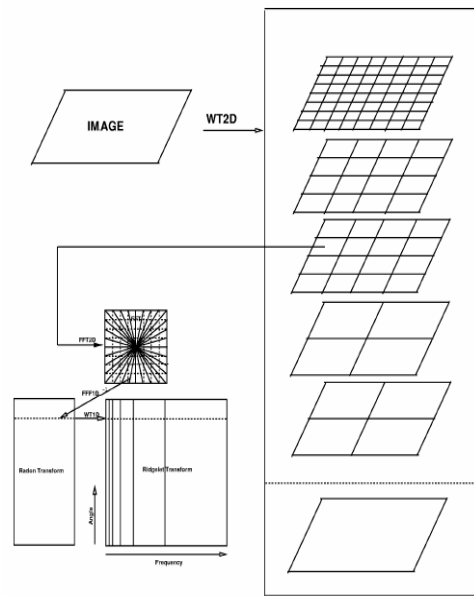


Fig. 3 Curvelet transform flowgraph.

B. Proposed Thresholding algorithm

The hard thresholding is ineffective in many examples. Though the NeighCoeff [13] scheme which considers neighboring Curvelet coefficients to be proposed in this work. In this scheme, the size of neighbor varies with the dependence of the coefficients.

$$S_{j,k}^2 = \sum_{n=-N}^N C_{j,k+n}^2; \quad N = N_0 - j \quad (7)$$

Here  $j$  is the level in curvelet decomposition and  $(2N+1)$  is the size of neighbor.  $N_0$  can be selected according to the size of image and the support of the Curvelet coefficients:

$$C_{j,k} = \begin{cases} C_{j,k} \left( 1 - \frac{\alpha \lambda^2}{S_{j,k}^2} \right) & \text{if } S_{j,k}^2 \geq \alpha \lambda^2 \\ 0 & \text{else} \end{cases} \quad (8)$$

where  $\lambda$  is given by  $2 \log n$  and  $\alpha$  is a parameter that adjusts the threshold.

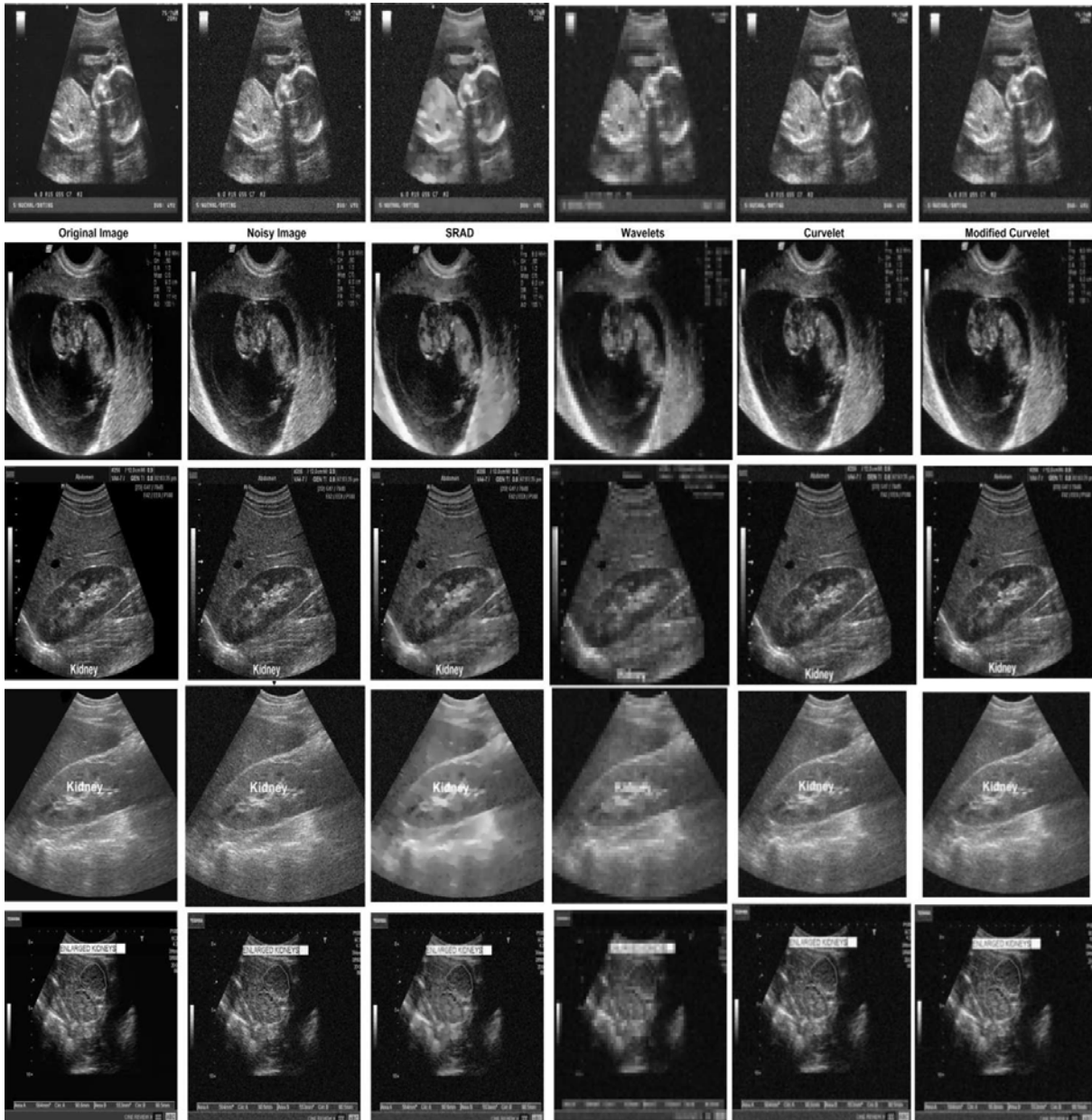


Fig. 4: (first column) sample images, (second column) noisy images, (third column) denoising results of SRAD technique, (fourth column) denoising results of wavelet transform technique, (fifth column) and denoising results of curvelet transform with hard thresholding (sixth column) denoising results of the proposed method.

C. Proposed Denoising Algorithm

Algorithm:

1. Apply Curvelet transform to the noisy image and get the scaling coefficients and Curvelet coefficients.
2. Chose the threshold by Eq. (8) and apply thresholding to the Curvelet coefficients (leave the scaling coefficients alone).
3. Reconstruct the scaling coefficients and the Curvelet coefficients thresholded and get the denoised image.

IV. EXPERIMENTAL RESULTS AND DISCUSSIONS

Removal of noises from the images is a critical issue in the field of digital image processing. The phrase Peak Signal to Noise Ratio (PSNR), often abbreviated PSNR, is an engineering term for the ratio between the maximum possible power of a signal and the power of corrupted noise that affects the fidelity of its representation. As many signals have wide dynamic. The MSE and PSNR is defined as:

$$PSNR = 20 \log_{10} \left( \frac{255}{MSE} \right) \tag{8}$$

$$MSE = \frac{1}{mn} \sum_{i=0}^{m-1} \sum_{j=0}^{n-1} \|I(i, j) - k(i, j)\|^2 \tag{9}$$

Fig. 4 shows the denoising results of different methods on five sample images. In Fig. 4, the first column presents the five sample ultrasound images; second column shows the sample images with Gaussian noise of zero mean and 0.01 variance; third, fourth fifth, and sixth columns denote the denoising results of the SRAD, wavelet, Curvelet with hard thresholding and proposed method respectively. From Fig. 4, it is clear that the proposed method outperforming the SRAD, wavelet and Curvelet transform with hard thresholding based denoising techniques.

TABLE I DENOISING RESULTS OF VARIOUS METHODS IN TERMS OF PSNR UNDER GAUSSIAN NOISE OF ZERO MEAN AND 0.01 VARIANCE.

Image No.	SRAD	Wavelets	Curvelet with Hard Thresholding	Proposed Method
1	27.79	27.5	34.74	35.61
2	26.28	24.88	32.55	33.62
3	23.99	18.57	24.56	23.99
4	27.49	33.73	39.89	41.14
5	20.48	15.42	20.25	21.1

Table I & II show the results of different methods on five sample images. From Table I & II, it is clear that the proposed method outperforming the SRAD and wavelet based denoising techniques.

V. CONCLUSIONS

A new thresholding algorithm is proposed in this paper for curvelet based image denoising on ultrasound images. The performance of the proposed thresholding algorithm is compared with the SRAD, wavelet transform and Curvelet transform with hard thresholding. The results after investigation show that the proposed method is outperforming the other existing methods in terms PSNR.

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