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STRESS CONCENTRATION IN ISOTROPIC & ORTHOTROPIC COMPOSITE PLATES WITH CENTER CIRCULAR HOLE SUBJECTED TO TRANSVERSE STATIC LOADING

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Abstract- The present study brings out the thorough analysis of isotropic and orthotropic fixed rectangular plate with center circular hole under transverse static loading condition. In this paper influence of stress concentration and deflection due to singularity for isotropic and orthotropic composite materials under different parametric conditions is obtained. The effect of thickness -to- width of plate (T/A) and diameter-to-width (D/A) ratio upon stress concentration factor (SCF) for different stresses were studied. An isotropic and one composite material were considered for analysis to determine the variation of SCF with elastic constants. Deflection in transverse direction were calculated and analyzed. Results are presented in graphical form and discussed. Three-dimensional finite element models were created using ANSYS software. Results showed that maximum stress appear near the vicinity of the hole at the upper and lower portions of the plate. The effect of material properties, (E1/E2) on SCF for stresses along x, y and z axis is established through this analysis.

Keywords- Composite, deflection, finite element analysis, stress concentration factor, , transverse loading.

I. INTRODUCTION

The usage of composites is increasing in aerospace and other engineering industrial applications, because of their high strength to weight ratios, high stiffness, low density and long fatigue life. As the application of composites to commercial product has increased, so has the need for design aspects for structural components increase. Accurate knowledge of deflections, stresses and stress concentration factors are required for design of such plates with singularities such as circular hole. Any abrupt change in geometry of plate under loading gives rise to stress concentration; as a result, stress distribution is not uniform throughout the cross section.

Rao et al. [1] evaluated the stress around square and rectangular cut-outs in symmetric laminates. It has been analyzed that the maximum stress and its location is mainly influenced by the type of loading. Kumar et al. [2] has studied the post buckling strengths of composite laminate with various shaped cut-outs under in plane shear. Ozen et al. [3] presented the failure loads of mechanical fastened pinned and bolted composite joints with two serial holes. Tsai–Wu failure criterion was used to predict first failure loads by finite element analysis for the geometrical parameters. Ozben et al. [4] compiled FEM analysis of laminated composite plate with rectangular hole and various elastic modulus under transverse loads. Ghezzi et al. [5] performed a numerical and experimental analysis of the interaction between two notches in carbon fibre laminates. Jain and Mittal [6] analyzed the effect of fibre orientation on stress concentration factor in fibrous plate with central circular hole under transverse static loading by using two dimension finite element methods. The numerical analysis of the stress distribution in-plane -

stress assumption and within the fibrous plate theory framework has been conducted on two symmetric laminates. Mittal and Jain [7] have analyzed the stress concentration and deflection in isotropic, orthotropic and fibrous composite plates with central circular hole subjected to transverse static loading by using two dimensional finite element methods. She and Guo [8] have analyzed the variation of three dimensional stress concentration factors along the wall of elliptic holes in finite thickness plates of isotropic materials subjected to remote tensile stress using finite element method. Gruber et al. [9] developed analytical solution methods for the analysis of stress concentration in fibre reinforced multilayered composites with pin loaded holes. Ukadgaonker and Kakhandki [10] analyzed the stress around an irregular shaped hole for different in-plane loading conditions for an orthotropic fibrous plate. Toubal et al. [11] studied stress concentration in a circular hole in composite plate. Kotousov and Wang [12] have presented analytical solutions for the three dimensional stress distributions around typical stress concentrators in an isotropic plate of arbitrary thickness based on the assumption of a generalized plane strain theory. Troyani et al. [13] have determined the in-plane theoretical stress concentration factors for short rectangular plates with centered circular holes subjected to uniform tension using finite element method. Ukadgaonker and Rao [14] proposed a general solution for stresses around hole in symmetric laminates under in-plane loading by introducing a general form of mapping function and an arbitrary biaxial loading condition to the boundary conditions, and the basic formulation is extended for multilayered plates. Ting et al. [15-16] presented the alternative method to study the stress distributions of

the multiple circular or multiple elliptical holes with the rhombic pattern in the infinite domain. Xiwu et al. [17-18] studied a finite composite plate weakened by elliptical holes under different in-plane loading, treated as an anisotropic multiple connected plates, based on the classical plate theory. Using the complex potential method in the plane theory of elasticity of an isotropic body, an analytical solution concerned with stress concentration around an elliptical hole or holes in finite composite fibrous plate was obtained.

In view of the above review, it can be concluded that a detailed analysis of more cases for stress concentration in composite plates with hole subjected to transverse loadings needs to be carried out. The present work aims to study the effect of T/A and D/A on SCF in isotropic and orthotropic composite (e-glass/epoxy) plates with central circular hole subjected to transverse static loading with all edges fixed and, also the deflection. The effect of T/A and D/A ratio, where T is the plate thickness and A is the plate width and D is the hole diameter on SCF for normal stresses in X, Y, Z directions ($\sigma_x, \sigma_y, \sigma_z$), shear stress (τ_{xy}) and deflection in transverse direction (U_z) is investigated by using three dimensional finite element analysis. The deflection in z-direction for plate with hole (U_z) of different materials under transverse loading is compared with deflection in transverse direction in plate without hole (U_z).

Results are obtained for one isotropic and composite material to find out the variation of SCF for different material parameters.

II. FORMULATION OF PROBLEM

The model of fixed plate of dimension (0.2 m X 0.1 m) having thickness (T) with a central circular hole of diameter (D) under uniformly distributed loading of P (N) in transverse direction (Fig.1) is taken for analysis. The plate is fixed at all edges.

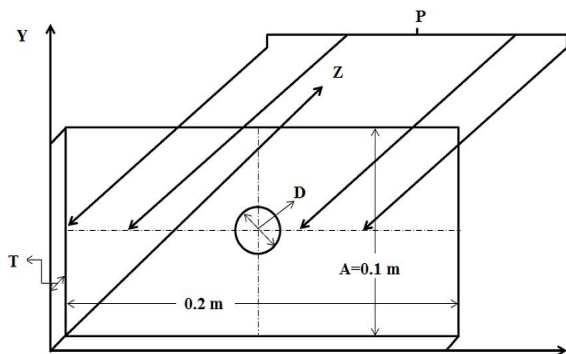


Figure 1. Plate with hole with all edges fixed under transverse load

The material properties [19] of isotropic plate are as: $[E, \mu]: [39 \text{ GPa}, 0.3]$ and, of orthotropic composite plate are as: $[(E_1, E_2, E_3, G_{12}, G_{23}, G_{31}, \mu_{12}, \mu_{23}, \mu_{31}): [(39, 8.6, 8.6, 3.8, 3.8, 3.8 \text{ GPa}, 0.28, 0.28, 0.28)]$. Here, E, G and μ represent modulus of elasticity, modulus of rigidity and Poisson's ratio

III. ANALYSIS

Finite element method was chosen for analysis of the model. The model was meshed using a 3-D solid element, Solid 186 with three degrees of freedom and 60 nodes per element in ANSYS. Typical mesh of the plate using the above element has been shown in Figure. 2. Mapped meshing is used so that more elements employed near the hole boundary. Due to the symmetric nature of different models investigated, it was necessary to discretize the quadrant plate for finite element analysis. Main task in finite element analysis is selection of suitable elements. Numbers of checks and convergence test are made for selection of suitable elements from different available elements and to decide the element length. Element length is selected as 0.002 m after running the convergence tests. The example of the discretized three dimensional finite element model, used in study is shown in Fig. 2.

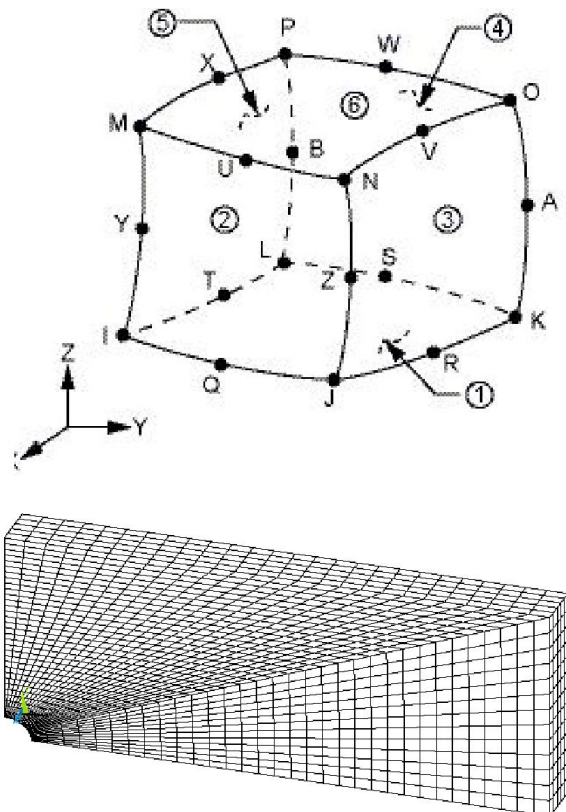


Figure 2. Element used and generated mesh of the plate for D/A = 0.2 and T/A = 0.05.

IV. RESULT AND DISCUSSION

Models generated are analyzed and results thus obtained for material combinations are presented in graphs. Variation of SCF (for $\sigma_x, \sigma_y, \sigma_z, \sigma_{xy}$), deflection versus D/A ratios for all edges fixed rectangular plate with central circular hole loaded transversely for two different materials were presented in below figures. Results are discussed case by case further. Fig. 3 shows variation of SCF (σ_x) versus D/A ratio for isotropic and e-glass/epoxy material. Following observations can be made from

the analysis represented in Fig.3. Both materials considered are following a similar behavior with continuous decrease in SCF (σ_x) with corresponding increase in D/A and T/A ratio respectively. Isotropic and e-glass/epoxy material shows their maximum SCF value of 1.02, 1.35 at T/A=0.10 and D/A=0.1 respectively, whereas minimum SCF value of 0.89, 0.87, were obtained at T/A=0.01 and D/A=0.5 ratio respectively.

Fig. 4 shows variation of SCF (σ_y) versus D/A ratio for isotropic and e-glass/epoxy material. Isotropic material show increase in SCF (σ_y) with corresponding increase in D/A ratio and decrease in SCF with continuous increase in T/A ratio. Isotropic material show maximum SCF (σ_y) of 1.07 at T/A=0.01 and D/A=0.1 ratio, whereas minimum SCF of 0.98 is attained at T/A=0.10 and D/A=0.5 ratios.

E-glass/epoxy follows the opposite behavior for increase in SCF (σ_y) with corresponding increase in T/A and D/A ratios respectively. Maximum SCF (σ_y) of 0.82 is obtained for T/A=0.10 and D/A=0.5 parameters, whereas minimum SCF of 0.70 is obtained at T/A=0.01 and D/A=0.1 respectively.

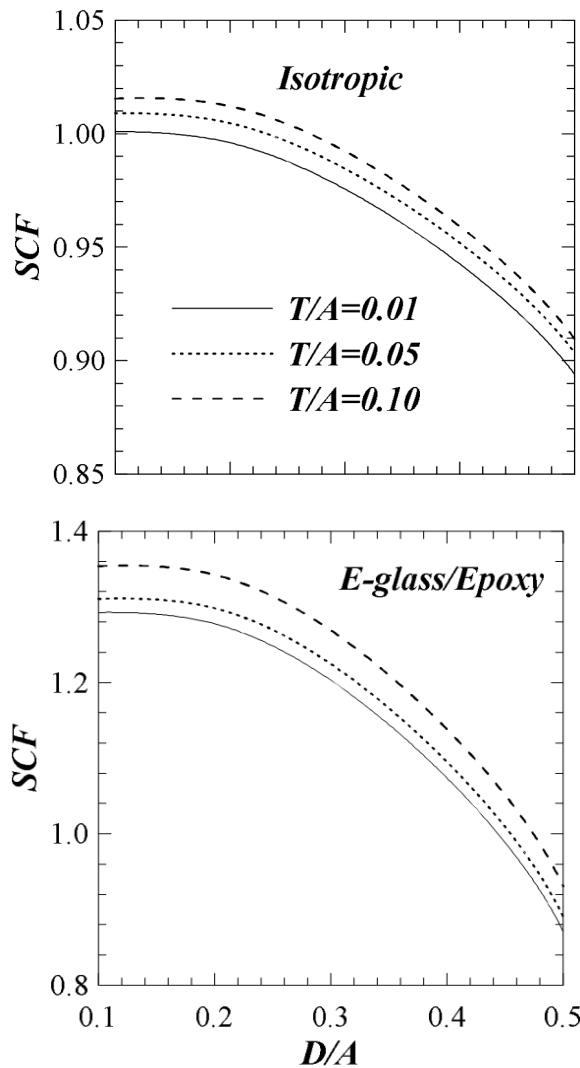


Figure 3. Variations of SCF (for σ_x) versus D/A ratio

Fig. 5 shows variation of SCF (σ_z) versus D/A ratio for isotropic and e-glass/epoxy material. For isotropic material SCF (σ_z) decrease with continuous increase in D/A ratio and increase in SCF with corresponding increase in T/A ratio up to T/A=0.05 and then it decreases.

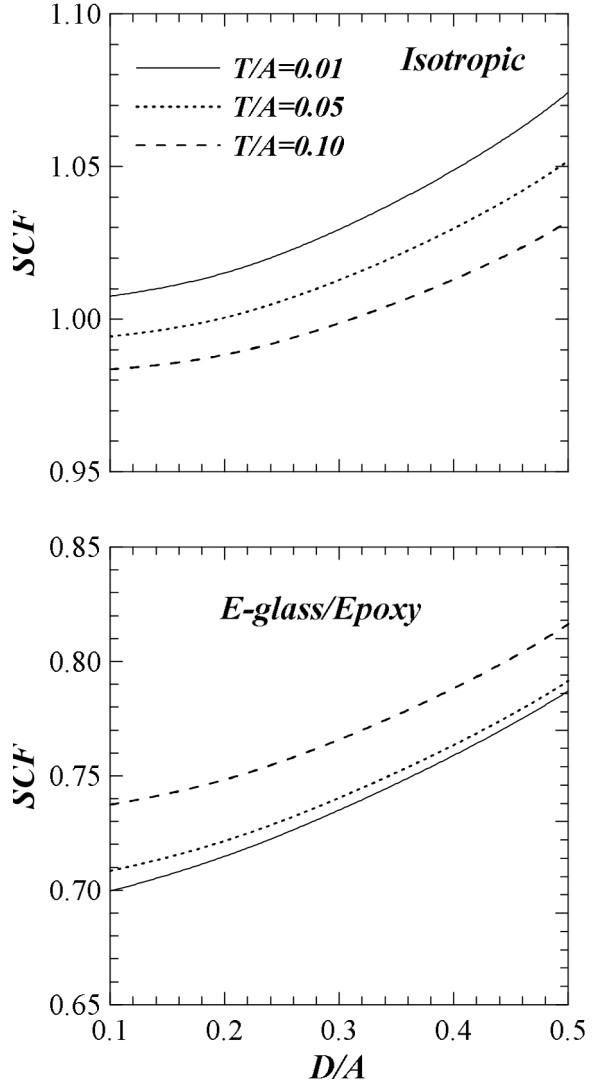


Figure 4. Variations of SCF (for σ_y) versus D/A ratio

E-glass/Epoxy shows continuous increase in SCF (σ_z) with corresponding increase in both D/A and T/A ratios. Isotropic material attains its maximum SCF (σ_z) value as 1.09 at T/A=0.05 and D/A=0.1 ratio, where as minimum SCF value of 0.89 is obtained at D/A=0.5 and T/A=0.01 parameters. E-glass/Epoxy obtains its maximum SCF value as 0.67 at T/A=0.10 and D/A=0.5, whereas minimum SCF value of 0.48 is attained at T/A=0.01 and D/A=0.1 ratios respectively.

Fig. 6 shows variation of SCF (σ_{xy}) versus D/A ratio for isotropic and e-glass/epoxy material.

Following observations can be made from the analysis represented in Fig. 6. Isotropic material follows decrease in SCF (σ_{xy}) with corresponding increase in D/A ratio and increase in SCF (σ_{xy}) with continuous

increase in T/A ratio respectively. Isotropic material show maximum SCF (σ_{xy}) of 2.55 at T/A=0.10 and D/A=0.5 respectively, whereas minimum SCF of 1.33 is attained at T/A=0.01 and D/A=0.5 ratios. E-glass/epoxy follows the decrease in SCF (σ_{xy}) with corresponding increase in D/A ratio. E-glass/epoxy have its maximum SCF of 3.30 obtained for T/A=0.10 and D/A=0.1, whereas minimum SCF of 2.60 is obtained at T/A=0.01 and D/A=0.1 respectively.

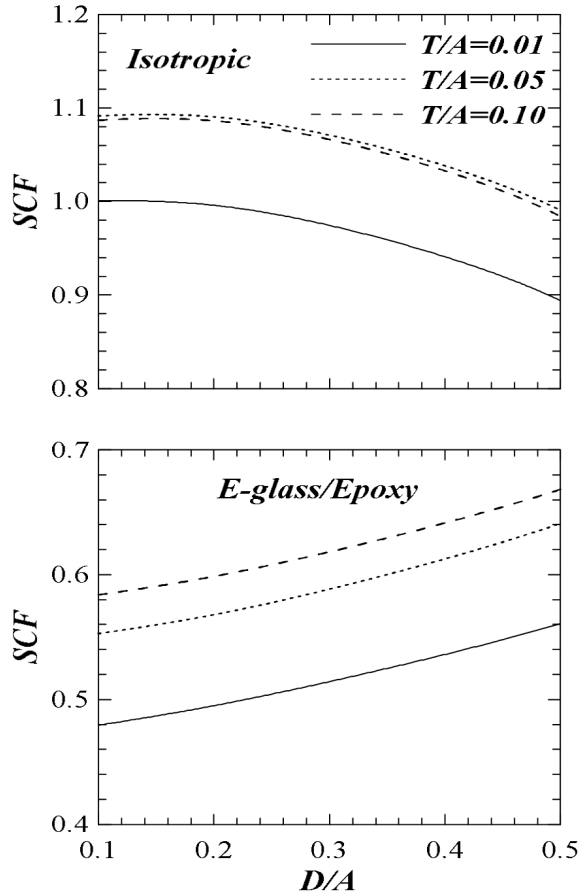


Figure 5. Variations of SCF (for σ_{xy}) versus D/A ratio

Fig. 7 shows variation of U_z/U_z^* versus D/A ratio for isotropic and e-glass/epoxy material. It has been observed that U_z/U_z^* for isotropic material increases initially with increase in D/A ratio up to 0.2 and after D/A=0.2, U_z/U_z^* decrease with corresponding increase in D/A ratios. U_z/U_z^* for isotropic material show maximum SCF value of 1.09 at T/A=0.01 and D/A=0.2 ratios respectively, whereas minimum U_z/U_z^* of 0.89 is obtained at T/A=0.10 and D/A=0.5 parameters. E-glass/epoxy follows opposite trend as compared to isotropic material with increase in U_z/U_z^* for corresponding increase in D/A and T/A parameters. Maximum U_z/U_z^* value of e-glass/epoxy is 0.64 obtained for T/A=0.10 and D/A=0.5 ratios, whereas minimum value of 0.56 attained at T/A=0.01 and D/A=0.1 ratios respectively. Fig. 7 shows variation of U_z/U_z^* versus D/A ratio for isotropic and e-glass/epoxy material. It has been observed that U_z/U_z^* for isotropic material increases initially with increase in D/A ratio up to 0.2 and after D/A=0.2, U_z/U_z^* decrease with corresponding increase in D/A

ratios. U_z/U_z^* for isotropic material show maximum SCF value of 1.09 at T/A=0.01 and D/A=0.2 ratios respectively, whereas minimum U_z/U_z^* of 0.89 is obtained at T/A=0.10 and D/A=0.5 parameters. E-glass/epoxy follows opposite trend as compared to isotropic material with increase in U_z/U_z^* for corresponding increase in D/A and T/A parameters. Maximum U_z/U_z^* value of e-glass/epoxy is 0.64 obtained for T/A=0.10 and D/A=0.5 ratios, whereas minimum value of 0.56 attained at T/A=0.01 and D/A=0.1 ratios respectively.

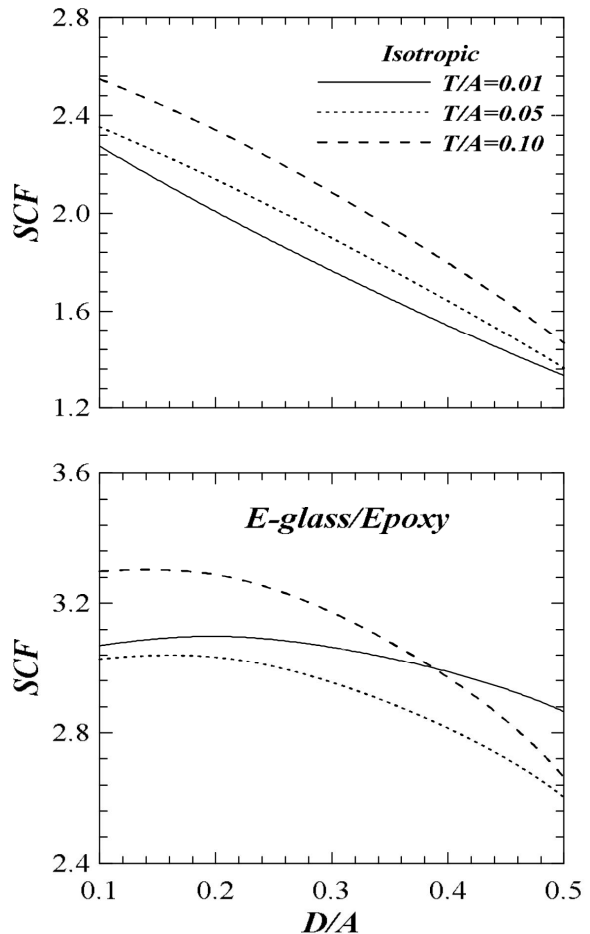
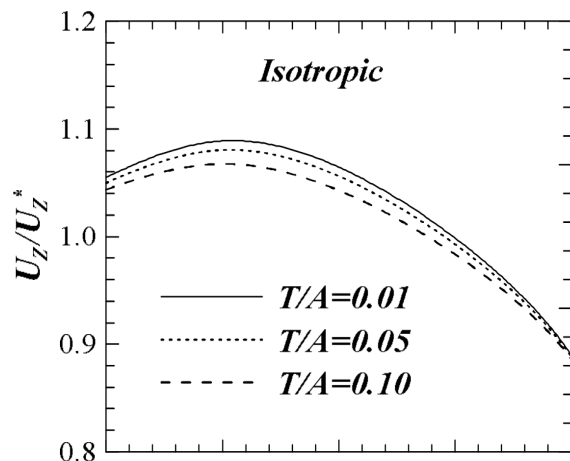


Figure 6. Variations of SCF (for τ_{xy}) versus D/A ratio.



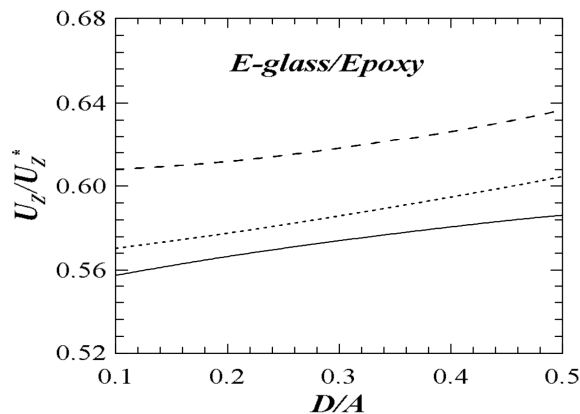


Figure 7. Variations of U_z/U_z^* versus D/A ratio.

V. CONCLUSION

Detailed investigations on the stress fields and the relationships between three dimensional stress concentration factor (SCF) for isotropic and composite plate subjected to transverse loading were conducted using 3D finite element method:-

- The influence of D/A and T/A parameters shows a substantial role for all stresses and ratios of U_z/U_z^* for both materials considered.
- Among all the stresses, SCF (for σ_{xy}) is seen maximum on the hole boundary along the width direction of the plate, whereas remaining stresses were maximum on support edges of the plate.
- Higher E_x/E_y & E_x/G_{xy} ratios, prominently effect the values of deflection and SCF (for σ_x , σ_y , σ_z , σ_{xy}) respectively. Thus both material graph behave differently for all stresses and deflection because of large difference in their material parameters.
- Additionally this can also be concluded that the results obtained are in line with other similar works.

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