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## BLIND SOURCE SEPARATION OF AUDIO SIGNALS USING WVD-KR ALGORITHM

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# BLIND SOURCE SEPARATION OF AUDIO SIGNALS USING WVD-KR ALGORITHM

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**Abstract-** Under-determined blind source separation aims to separate  $N$  non-stationary sources from  $M$  ( $M < N$ ) mixtures. Paper presents a time-frequency approach (TF) to under-determined blind source separation of  $N$  non-stationary sources from  $M$  mixtures ( $M < N$ ). It is based on Wigner-Ville distribution and Khatri-Rao product. Improved method involves a two step approach which involves the estimation of the mixing matrix where negative values of auto WVD of the sources are fully considered and secondly auto-term TF points are extracted. After extracting the auto-term TF points source WVD values at every TF point are computed using a new algorithm based on Khatri-Rao product. Thus sources are separated with the proposed approach no matter how many active sources there are as long as  $N \leq 2M-1$ . Simulation results are presented to show the superiority of the proposed algorithm by comparing it with the existing algorithms.

**Keywords-** Under-determined blind source separation, Wigner-ville distribution, Khatri-rao product.

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## I. INTRODUCTION

Blind source separation (BSS) aims to find unknown sources from a number of known mixtures without knowing the mixing system. The problem of under-determined blind source separation has received considerable research interest over the last decade due to the potential applications in various areas including speech enhancement, digital communications, financial time series analysis etc. The term 'blind' means there is no prior knowledge of the sources and the structure of the transfer medium. Today there exist a variety of BSS algorithms which exploit one of the following properties of the sources higher-order statistics (HOS), second order statistics (SOS), the non-stationarity and sparsity. When the number of sources is larger than that of the mixtures BSS problem is called an under-determined BSS problem, this is generally more difficult than the complete BSS problem where number of sources are equal to that of mixtures. So many algorithms are provided earlier to tackle the under-determined blind source separation problem. Girolami proposed Expectation – maximization algorithm (EM) [1] to learn sparse and over complete representations. It is used in UBSS based on subspace representations. In blind separation of sub- and super- Gaussian sources from under-determined mixtures the underlying sources are assumed to have two orthogonal components one in row space and other in null space of a mixing matrix, the distribution parameters are estimated based on expectation-maximization (EM) algorithm. Proposed algorithm provides estimate of mixing matrix with higher accuracy and can separate both sparse and non-sparse signals with higher SIR than conventional algorithms but the above algorithm cannot provide direct separation of sources as the orthogonal components are estimated first. Clustering algorithm

[2] is another algorithm that deals with the problem of UBSS, in this the TF plane is divided into different blocks and source separation is then carried out in block by block manner. The mixing matrix is adaptively estimated with a gradient type algorithm or with the k-means clustering method. TF representation [3]-[5] is used to exploit source non-stationarity in UBSS, it provides a useful signal processing tool for characterizing signals in both time and frequency planes. The novel integration of the two approaches in the proposed algorithm overcomes the problem of artificial sources thereby provides improved separation performance. But proposed one often leads to phantom sources which makes thresholding difficult because of the approximate classification technique employed.

In the proposed paper sparsity constraints [6]-[8] are relaxed and developed a TF-based UBSS algorithm which is called Wigner-ville distribution Khatri-rao product (WVD-KR) algorithm. In this sources are recovered exactly at every auto-term TF points [9]-[10] no matter how many active sources are present provided the number of sources are less than twice of that of the sensors i.e.,  $N \leq 2M-1$ .

## II. PROBLEM STATEMENT

Most traditional BSS algorithms aim to solve well-determined and over-determined BSS. Well-determined BSS includes the number of sources equals with the number of mixtures and over-determined BSS involves number of sources less than number of mixtures. In early TF based UBSS algorithms the sources were assumed to be completely sparse in the TF domain [9] i.e., disjoint which is not practical. Earlier approaches solved BSS

with arbitrary mixed sources these methods can separate all separable single sources and all inseparable mixtures which implies that it cannot separate all sources when used to UBSS.

### III. WIGNER-VILLE DISTRIBUTION

Wigner-ville distribution provides a high concentration of signal energy in TF domain which makes some non-stationarity signals such as speech signals much sparser in TF domain. Auto WVD shows how the energy of the signal varies with time and frequency. It has the property of high TF concentration and has been widely used. If  $z(t) \in L^2(\mathbb{R})$  is an analog signal auto WVD is defined as a radial Fourier transform of the product  $z^*(t - \tau/2) z(t + \tau/2)$  where  $L^2(\mathbb{R})$  is the collection of signals with finite energy.

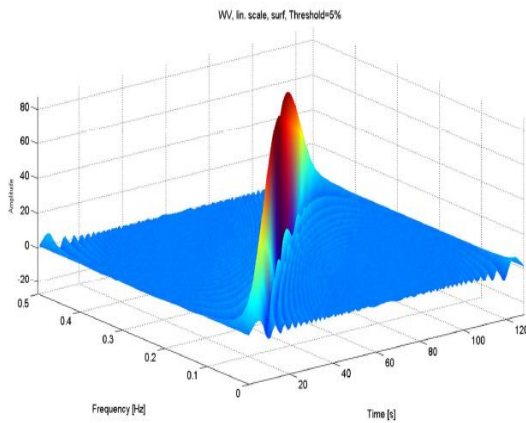


Figure 1. Wigner-ville distribution

Auto WVD has many good properties such as real valued property and TF shift property. The TF shift property means that

$$\begin{aligned} \text{If } z(t) = 0, t \notin (t_1, t_2), \text{ then } \rho_{zz}(t, f) = 0, t \notin (t_1, t_2) \\ \text{If } Z(f) = 0, f \notin (f_1, f_2), \text{ then } \rho_{zz}(t, f) = 0, f \notin (f_1, f_2) \end{aligned}$$

where  $Z(f)$  is the Fourier transform of  $z(t)$ , this guarantees the sparsity of the signal in TF domain which is essential for the mixing matrix estimation in UBSS. Negative valued region in TF region is fully considered for the mixing matrix estimation. The approach includes autocorrelation function which is used for calculating the power spectrum. The signal is compared to itself for all possible relative shifts, or lags in forming autocorrelation function.

$$r_{ss}(\tau) = \int s(t) s(t+\tau) dt \quad (1)$$

where  $\tau$  is the shift of the signal with respect to itself The Wigner-Ville (and all of Cohen's class of distribution) uses a variation of the autocorrelation function where time remains in the result, called instantaneous autocorrelation function

$$R_{ss}(t, \tau) = s(t+\tau/2) s^*(t-\tau/2) \quad (2)$$

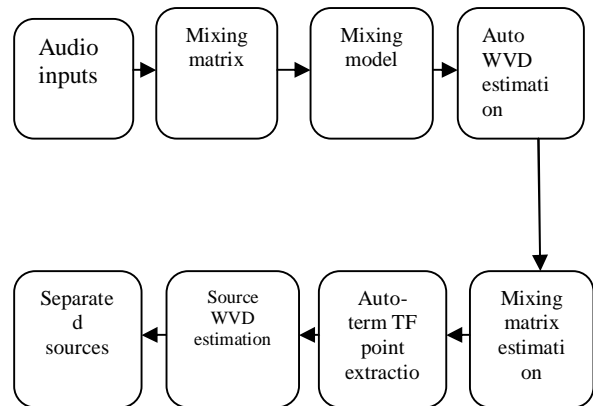
where  $\tau$  is the time lag and  $*$  represents the complex conjugate of the signal

### IV. WVD-KR ALGORITHM

A. WVD-KR Algorithm is the TF based algorithm based on Wigner-ville distribution and Khatri-rao product.

1. Make the mixing model.
2. Estimate the WVD of sources and mixtures.
3. Compute the estimate of mixing matrix.
4. Extract the auto-term TF points.
5. After extraction of auto-term TF points compute the source WVD value at each TF point is using an algorithm based on Khatri-rao product.

Block diagram shows the sequence of process which includes auto WVD estimation, auto-term TF point extraction and source WVD estimation.



Block diagram of the proposed algorithm

#### a. Mixing model

The instantaneous linear mixing model is given by

$$x(t) = As(t), \quad (3)$$

where  $s(t) = [s_1(t), \dots, s_N(t)]^T$  is the  $N \times 1$  underlying source vector,  $x(t) = [x_1(t), \dots, x_M(t)]^T$  is the  $M \times 1$  mixture vector,  $A = [a_1 \dots a_N]$  is the  $M \times N$  unknown real-valued mixing matrix.

#### b. Auto WVD estimation

Wigner-ville distribution provides a high concentration of signal energy in TF domain which makes some non-stationarity signals such as speech signals much sparser TF domain. Auto WVD shows how the energy of the signal varies with time and frequency.

The Wigner-Ville Distribution (WVD) is defined as

$$W_s(t, \omega) = 1/2\pi \int s(t + \tau/2) s^*(t - \tau/2) e^{-j\omega\tau} d\tau, \quad (4)$$

where  $\tau$  is the time lag. Auto WVD shows how the energy of a signal varies with time and frequency. It has the property of high TF concentration and has been widely used. The auto WVD seems like a kind of probability function.

#### c. Mixing matrix estimation

STFD matrices of the sources and mixtures is found out STFD matrix of the sources

$$W_s(t, f) = \begin{bmatrix} \rho_{s1s1}(t, f) \rho_{s1s2}(t, f) \dots \rho_{s1sN}(t, f) \\ \rho_{s2s1}(t, f) \rho_{s2s2}(t, f) \dots \rho_{s2sN}(t, f) \\ \dots \\ \rho_{sNs1}(t, f) \rho_{sNs2}(t, f) \dots \rho_{sNsN}(t, f) \end{bmatrix} \quad (5)$$

The diagonal elements of  $W_s(t, f)$  are auto WVDs and the off-diagonal elements are cross WVDs of the sources. STFD matrix of the mixtures is given by

$$W_x(t, f) = A W_s(t, f) A^T \quad (6)$$

The traditional methods for estimating the mixing matrix using WVD are based on the assumption that there exists a SSD for each source. SSD is a set of points in the TF domain where only one source is active, the trace of  $W_x(t, f)$  equals its unique non-zero eigen value, which should be the maximum of all the eigen values if all of them are non-negative. The criterion to decide whether or not a TF point  $(t, f)$  belongs to SSD.

$$|\lambda_{\max}\{W_x(t, f)\} / \text{trace}\{W_x(t, f)\} - 1| \leq \varepsilon_2 \quad (7)$$

Where  $\varepsilon_2$  is a threshold set to be a positive real value. Thus applying k-means clustering techniques to all of the eigenvectors of the mixtures can get the estimate of the mixing matrix.

#### d. Auto-term TF points extraction

TF representation is used to exploit source non-stationarity in UBSS, All the TF points that satisfy the following criterion will be identified as auto-term TF points and are extracted

$$\text{trace}(U W_x(t, f) U^T) / \|U W_x(t, f) U^T\| \geq \varepsilon_4 \quad (8)$$

where  $U$  is the whitening matrix which is usually estimated as the inverse square root of the covariance matrix of the mixtures.  $\varepsilon_4$  is a threshold close to 1

#### e. Source WVD value estimation:

After extracting auto-term TF points WVD value of sources at every auto-term TF point is computed using new algorithm based on Khatri-Rao product.

## V. RESULTS AND DISCUSSIONS

In the simulations source signals are separated from the under-determined mixtures. For speech and audio signals a sparse representation can be obtained in the TF domain. First mixing model is made with four speech signals and three mixtures which is made using the randomly generated mixing matrix. Second a  $4 \times 3$  case is considered to estimate the mixing matrix  $A$  with the proposed improved method. Four speech signals with 44 KHz sampling frequency and 70,000 samples are used as sources and mixed into three mixtures with randomly generated mixing matrix  $A$ . Linear mixing model provides the speech signals and mixed signals.

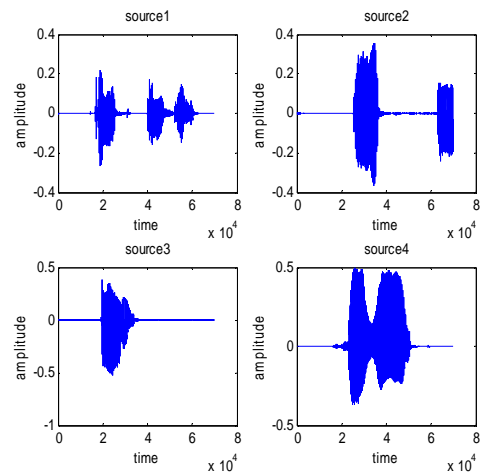


Figure 3. Audio inputs (Speech Signals)

#### Mixing matrix

$$A = \begin{bmatrix} 0.8147 & 0.6324 & 0.9575 \\ 0.9058 & 0.0975 & 0.9649 \\ 0.1270 & 0.2785 & 0.1576 \\ 0.9134 & 0.5469 & 0.9706 \end{bmatrix}$$

Three mixtures are made from four speech signals using the randomly generated mixing matrix.  $4 \times 3$  case is considered to estimate the mixing matrix  $A$  with the proposed improved method.

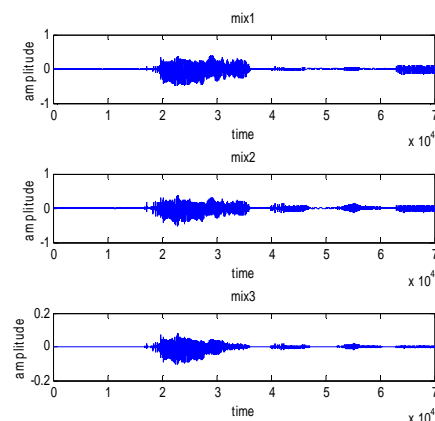
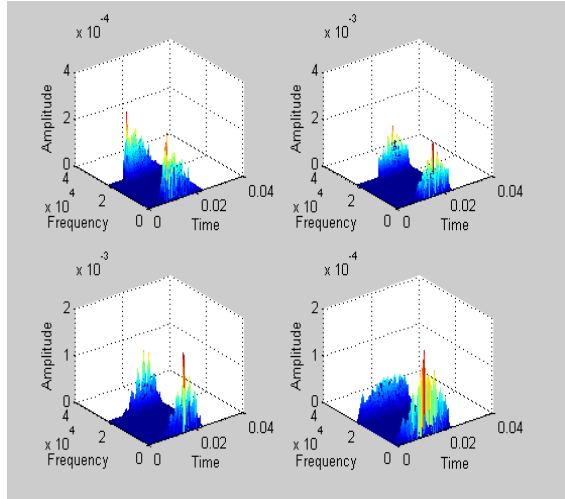


Figure 4. Mixed signals

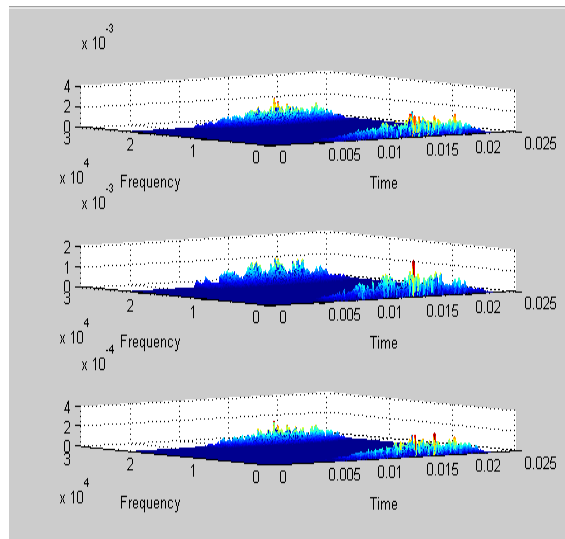
**Auto WVD estimation:**

Auto WVD shows how the energy of the signal varies with time and frequency. It has the property of high TF concentration which makes some non-stationary signals, such as speech signals, much sparser in the TF domain which implies better separation.



**Figure 5. WVD Spectrum of Speech signals**

From the Wigner-Ville distribution of the sources and mixtures spatial TF distribution (STFD) matrices of the sources and mixtures are computed which gives the mixing matrix estimate.



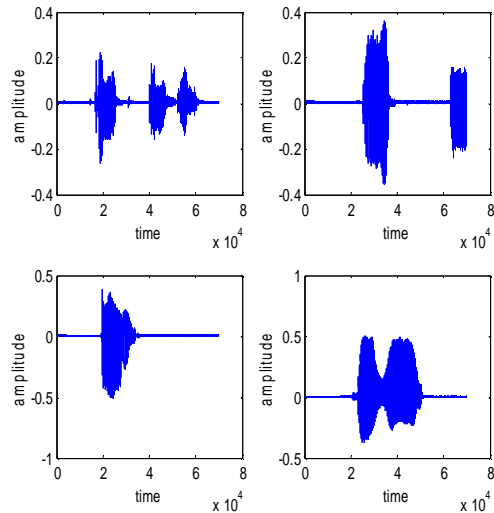
**Figure 6. WVD Spectrum of mixtures**

**Mixing matrix estimate:**

STFD matrix of the sources and STFD matrix of the mixtures is calculated. Eigen vectors of the mixtures is found out and thus ,applying k-means clustering techniques to all of the eigen vectors of the mixtures can get the estimate of the mixing matrix. Column normalization is done. Source WVD value estimation: After extracting auto-term TF points WVD value of sources at every auto-term TF point is found out using new algorithm based on Khatri- rao product.

**Separated sources:**

After extracting auto-term TF points WVD value of sources at every auto-term TF point is found out using a new algorithm based on Khatri-Rao product and WVD value of sources at every TF point gives the separated sources from the mixtures.

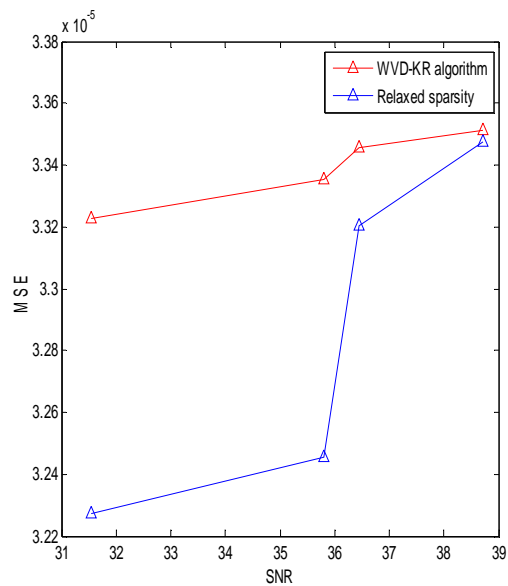


**Figure 8. Separated sources**

Mean Square Error is used to evaluate the recovery accuracy of the sources.

$$MSE_{dB} = 10 \log_{10} \left( \sum_{i=1}^N \sum_{k=1}^T \|s_i[k]\|^2 / \min_{1 \leq p \leq N} \sum_{i=1}^N \sum_{k=1}^T \|s_i[k] - s_p[k]\|^2 \right) \quad (9)$$

where  $s_i$ ,  $i=1, \dots, N$  are the source estimates,  $T$  is the number of samples. Performance comparison of UBSS-FAS with other algorithms can be estimated using SNR.



**Figure 9. Performance Comparison**

TABLE I. Recovered Sources

Recovered sources	Speech signals			
	<i>Source1</i>	<i>Source2</i>	<i>Source3</i>	<i>Source4</i>
SNR	31.5599	35.8203	36.4450	38.6961
MSE	0.3331	0.3336	0.3336	0.3342

## VI. CONCLUSION

The proposed under-determined BSS algorithm was applied in TF domain which provides better separation. A sparser representation of speech and audio signals can be obtained in TF domain. UBSS approach based on WVD and Khatri-Rao product to separate  $n$  sources from  $M$  mixtures has many advantages, first negative auto WVD values of the sources are fully considered for the estimation of the mixing matrix and secondly algorithm does not restrict the number of sources which are active at same auto-term TF points as long as  $N \leq 2M-1$ . Mean Square Error is used to evaluate the recovery accuracy of the sources. Simulation results provide the effectiveness of the proposed algorithm with the conventional algorithms through the performance analysis inferred from the SNR and MSE values of the recovered sources.



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