Dual-Hop OSTBC Transmissions Over Fading Channels under Receiver Phase Noise for Regenerative Systems

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Abstract - Space-time coding for fading channels is a Communication technique that realizes the diversity benefits of multiple transmit antennas. Previous work in this area has focused on the narrowband flat fading case where spatial diversity only is available. In this paper, we investigate the use of space-time coding in OFDM based broadband systems where both spatial and frequency diversity are available. We consider a strategy which basically consists of coding across OFDM tones and will therefore be called *space-frequency coding*. For a spatial broadband channel model taking into account physical propagation parameters and antenna spacing, we *derive the design criteria for space-frequency codes* and we show that space-time codes designed to achieve full spatial diversity in the narrowband case will in general not achieve full space-frequency diversity. Specifically, we show that the Alamouti scheme across tones fails to exploit frequency diversity. For a given set of propagation parameters and given antenna spacing, we *establish the maximum achievable diversity order.* Finally, we provide simulation results studying the influence of delay spread, propagation parameters, and antenna spacing on the performance of Space-frequency codes.

I. INTRODUCTION AND OUTLINE

Two of the major impairments of wireless communications systems are *fading* caused by destructive addition of multipath in the propagation medium and *interference from other users.* Diversity provides the receiver with several (ideally independent) replica of the transmitted signal and is therefore a powerful means to combat fading and interference. Common forms of diversity are time diversity (due to Doppler spread) and frequency diversity (due to delay spread). In recent years the use of spatial (or antenna) diversity has become increasingly popular. Spatial diversity is particularly attractive since it can be provided without loss in spectral efficiency. *Receiver diversity,* i.e., the use of multiple antennas on receiver side is well-studied subject **[l].** Driven by mobile wireless applications, where it is difficult to deploy multiple antennas in the handset, *transmit diversity* or equivalently the use of multiple antennas on the transmit side become inactive area research**.** *Space time coding* evolve **as** one of the most promising transmit diversity techniques **[9]-[13].**Most of the previous work on transmit diversity has been restricted to single-carrier systems operating over narrow band channels' where spatial diversity only is available. In this paper, we consider multi-antenna (multiple transmit and one or several receive antennas) broadband channels (i.e. channels with delay spread) where both spatial diversity

(due to multiple antennas) and frequency diversity (due to delay spread) are available. *Orthogonal frequency division multiplexing* (OFDM) **[14]-[16]** *significantly reduces receiver complexity* in wireless broadband multi-antenna systems **[17, 181.** We therefore study the use of space-time coding in OFDM-based multi-antenna systems. Contributions. We consider a strategy which basically consists of employing a space-time code across OFDM tones and will therefore be called *space-frequency coding.* Our contributions are **as** follows. For a spatial delay spread channel model taking into account physical propagation parameters and antenna spacing, we derive the *design criteria for space frequency codes* and we show that space-time codes designed to achieve full spatial diversity in the narrow band case will in general not yield full space frequency diversity. The requirement of exploiting frequency diversity **as** well imposes additional constraints on the codes and makes the design considerably more complicated than in the narrowband case. For a given set of channel parameters and given antenna spacing, we derive the *maximum achievable diversity order* and we discuss the impact of antenna spacing and propagation parameters on diversity. Using the design criteria established in this paper, we show that the *Alamouti scheme* across tones *fails* exploit the frequency-diversity available in the delay spread case. We provide simulation results demonstrating the performance of known space-time

codes employed **as** space-frequency codes under various propagation conditions. Organization **of the paper.** The rest of this paper is organized **as** follows. In Section **2,** we introduce the channel model, we briefly describe broadband OFDM-based multi antenna systems and we discuss space frequency coding. In Section **3,** we derive the design criteria for space-frequency codes and we discuss their relation to previously established design criteria for the narrowband (slow and fast fading) case. In Section **4,** we provide some simulation results. Finally, Section *5* contains our conclusions.

II. CHANNEL MODEL, OFDM, AND SPACE-FREQUENCY CODING

We shall first introduce the channel model, and then briefly describe OFDM-based multi-antenna systems and space-frequency coding.

2.1. OFDM-based Multi-Antenna Systems

In an OFDM-based multi-antenna system the data streams are OFDM-modulated before transmission. The OFDM modulator applies an N-point IFFT to N consecutive data symbols and then prepends the cyclic prefix (CP) (which is a copy of the last *L* samples of the symbol) to the symbol, so that the overall OFDM symbol length is $M = N + L$. In the receiver, the individual signals are passed through an OFDM demodulator which first discards the CP and then applies an FFT. Organizing the transmitted data symbols into frequency vectors **c k** = [cf) cr) ... with **ct)** denoting the data symbol transmitted from the i-th antenna on the k-th tone, the reconstructed data vector for the k-th tone is given by $k \, Jr \, k = 6 \, H \, (e^{jsk}) ck \, \text{-I-} \, nk, k = 0, 1, ..., N - 1,$ **(5)** where **nk** is complex-valued additive white Gaussian noise satisfying with **I Med**en opting the identity matrix of size MR. The data symbols **ct'** are taken from a finite complex alphabet chosen such that the average energy of the constellation elements is **1.**

2.2. Space-Frequency Coding

The bit stream to be transmitted is encoded by the Space-frequency encoder into blocks of size *MT* **x** N. One data burst therefore consists of N vectors of size *MT* **x 1**or equivalently one spatial OFDM symbol. The channels assumed to be constant over at least one OFDM symbol. Assuming perfect channel state information, the maximum likelihood (ML) decoder computes the vector sequence minimization is overall possible code words. We finally note that an interesting experimental performance analysis of a space-frequency coded OFDM system appears in **[20].**

III. DESIGN CRITERIA

In this section, we shall derive the design criteria for space- frequency codes assuming that the receiver has perfect channel state information.

3.1. Pair wise Error Probability

Two different space-frequency code words of size MT **x**N and assume that **C** was transmitted. For a given channel realization **H(ej%k),**t he probability that the receiver decides erroneously in favor of the signal E is given by **[21]** denotes the squared Euclidean distance between the two Code words **C** and **E.** Using the Chern off bound $Q (z) < e-x2/2$ Next, we need to compute the expected pair wise error probability by averaging overall channel realizations taking into account the channel model presented in Sec. **2.1.** For this we define **Yk = H** (e'gk)(Ck - e h) for $k = 0,1, ..., N-1$ and $T \, T \, y$ = [Y? YT ... YN-11 .With this notation we get *d2* **(C, EIH(ej** *sk)=) l lY/I2*aSince the **Hi** were assumed to beiid . gaussian it follows from (1) that the H (ej%k) for $k = 0,1, ..., N - 1$ are gaussian **as** well and hence the MRN **x 1** vector *Y* is gaussian.The average over all channel realizations of the right-hand side in (8) is fully characterized by the eigen values of the covariance matrix of **Y** [22] defined **as** $Cy = E \{ Y \, YH\}.$In[23] it is shown that **L-1Cy = [D'(C - E)T(C - E)*DiH]** C3 **RI,** **(9)** *I=O* where **D** = diag {e-j*k} fz:, **A ⊛ B** denotes the Kronecke r product of the matrices **A** and B and the superscript *stands for element wise conjugation. Denoting the non zero eigen values of **CY as5 Xi(Cy)** *(i = 0,1, ..., r(Cy) - 1)* the following result can be established **[23]**where' **P(C -+ E) =** *EH* { *P* **(C +** **EIH(eJsk))}** is the pair wise error probability averaged over all channel realizations. Next, using the following property of Kronecke r products**(A ⊛ B)(F ⊛ G) =** **(AF) ⊛ (BG)** and the factorizations **Ri = R:/2Ry/2** *(1 = 0,1, ..., L - l),* it can be shown **[23]** that $C \, y$ = **G(C, E) GH(C, E) (11)** with the *NMR x MTMRL* matrix **G(C, E) = [(C - E)* @ R;I2 [D(C - E)*] @ R:I2**... **[DL-'(C-E) T]@** *R2/_2,.]* **(12)**Based on **(lo), (11)** and **(12)** we are now able to derive the design criteria for space-frequency codes.

3.2. Design Criteria and Diversity Order

In the following, we assume that $N > MTL$. The design criteria for space-frequency codes follow from **(10) as** the well-known *rank and determinant criteria* derived in**[ll,** 91 for the single-carrier narrowband case with the matrix **B(c,e)** defined in Eq. 6 of **[l l]** replaced by **GT(C,** E).Note that for L = 1 and & = **I** the matrix **GT(C,E)** reduces to **(C - E) @ 1 ~** Kn**o**w, using the fact that every eigen value of the *MT* x *MT* matrix **(C - E)(C - E)*** is and eigen value of the *MTMR* x *MTMR* matrix **[(C - E)(C -E)*] @ 1h.1**w~ ith multiplicity *MR,* it follows that the design criteria in an OFDM system with no delay spread are equivalent to those in a single-carrier-based narrowband system. This is intuitively clear since for L = **1** there is no frequency diversity Let us next use **(12)** to establish some results on the maximum achievable diversity order in broadband space

frequency coded OFDM systems. The maximum rank of the *NMR* x *MTMRL* matrix **G(C,E)** is *MTMRL.* Since $r(R'12)$ =**r(R)** we can establish the following **[23]:**

Theorem 1. The ***maximum achievable diversity order*** in a space-frequency coded broadband OFDM system is given**by***L-1dmaz = s Cr (Ri)* **I** *MTMRL,* where **s** is the minimum rank of **(C - E)** over all pairs of code words **C** and **E.** From **(12)** it follows that the I-th multipath can potentially add a diversity order of *sr (R1).* Since the rank of *R I* will be governed by the angle spread of the I-th scattered cluster and the antenna spacing at the receiver, we conclude that the achievable diversity order critically depends on the propagation environment and the antenna spacing .In **[23]** it is shown that in order to achieve MTMRL-fold diversity it is necessary to have **r(D'(C - E)*)** = *M T* for *i* = **0,1,** ..., *L* - **1** and for all pairs of code words **C**an d **E.** Furthermore, it is required that ~ *(R I*=*) M R* for*1* = 0, **1,** ..., *L* - **1.** These conditions guarantee that the individual *NMR* x *MTMR* matrices **[D'(C - E)*]** @ *R, (i = 0,* 1, ..., *L* - **1)** have full rank. Finally, the space-frequency code has to be designed such that the stacked matrix **G(C,** E) has full rank **as** well .In the following, in order to simplify the discussion we restrict our attention to the case of no spatial fading correlation and a uniform power delay profile, i.e., **Ri** = **1~~** for *I* = **O , l ,** ..., *L* - **1.** In this case we obtain from (9)**CY** = **[D'(C - E)T(C - E)*DIH]** @ **I M** which implies that the rank of **CY** will be *MR* times the rank of the *N* x *N* matrix **Ry** = **F(C, E) FH(C, E)**with the *N* x *MTL* matrix **F(C,E)** = **[(C-E)T D(C-E)*** ... **DL-'(C-E)*]. (13)**

The coding gain in this case can be obtained by making use of the fact that every eigen value of **Ry** is an f **CY** with multiplicity *MR.* The question of what diversity order a given space-frequency code achieves now reduces to the question of finding the minimum rank of t he *N* x *MTL* matrix **F(C,E)** over the set of all possible code words **C** and **E.** In order to achieve full(i.e.*MT MRL* fold)diversity the matrix **F(C,E)** has to be full rank for every pair of code words **C** and **E.** This is the case when**(C - E)** has rank *MT* for all code words **C** and **E** and each of the blocks **B; =** **Di(C -** *E)T* is linearly independent of the other **Bi** with 1 # *i* for every pair **C** and **E.** While a space-time code designed to achieve full diversity in then arrow band case will have full rank **(C -E)** and hence full rank **B,** (i = 0,1, ..., *L* - 1) for all code words and **E,** the guaranteed. Note that ensuring linear independence of the blocks B, amounts to ensuring that the space-frequency code exploits the avail frequency diversity can, however ,make the following statement: ***Theorem 2.*** **A** space-time code designed to provide adversity order of **s** in a single-carrier-based narrow band system provide sat least the same diversity ordering broadband OFDM system. The proof of Theorem **2** follows immediately by noting that

if **(C - E)** has minimum rank s over the se t of all possible code words **C** and **E,** then the minimum rank of **F(C,E)** will be at least s. **A** result similar to

Theorem 2 was reported in **[24]** for single-carrier systems operating in delay-spread environments. We emphasize, However, that a space-time code achieving full spatial diversity in a narrowband environment will in general not achieve full space-frequency diversity in the OFDM broad band case. The next subsection provides an example corroborating this statement. In **[23]** it is furthermore shown that space-time codes designed for the narrowband ***fast fading*** case will in general not be guaranteed to achieve full space-frequency diversity in the OFDM broadband case. In fact they may not even achieve full spatial diversity **[23].** We note, however, that even though space-time codes designed for the fast fading case and smart greedy space-time codes**[ll]** will be suboptimum in a space-frequency setting, they can be expected to exploit at least some of the available frequency diversity. Summarizing, we conclude that using existing space-time codes in a space-frequency setting will in general be suboptimum. New designs are needed taking into account the criteria derived in this paper. Some first signs of space-frequency codes are provided in [23]. The systematic design of good space-frequency codes remains an important open research problem.

3.3. Space-Frequency Alamouti Scheme

In the previous subsection we found that space-time codes designed to achieve full spatial diversity in the narrow band case will in general not achieve full space frequency diversity. We shall next provide an example illustrating his effect. Specifically, we consider the case of two transmit antennas *(MR is arbitrary)* and employ the Alamouti scheme [lo] across OFDM tones, i.e., the matrices**(C - E)** have the following structure With *di* = ci - **ei** *(i = 0,1, ..., N* - 1). From_ *N* -*.1* -(**C - E)(C - E)H** = *ldiI2* **12i=O**at follows that all possible code difference matrices **C** and **E** will be of full rank and hence the code achieves full spatial diversity in the narrowband case. This is a well-known fact and has been proved in [lo]. Next, take the minimum distance error event where only one out of the *N* **di** is non zeros ay *do.* In that case the matrix **(C - E)T** will have non zero entries only in the two top rows. Therefore, all the matrices **Di(C-E)T** *(i* = 0,1, ..., *L-* 1) will have nonzero entries in the two top rows only and hence it follows from (13)that the matrix **F(C,** E) will have rank 2 irrespectively of the amount of delay spread (and hence frequency diversity)in the channel. We conclude that while the Alamout scheme continues to achieve second-order diversity in the broadband OFDM case, it fails to exploit the additionally available frequency diversity.

This once again shows that the broadband OFDM case calls for new code designs.

IV. SIMULATION RESULTSI

In this section, we provide simulation results demonstrating the performance of known space-time codes employed **as** space-frequency codes and studying the influence of physical propagation parameters on the performance of space-frequency codes. We simulated an OFDM system with MT = 2, N = 64 tones and CP of length 16 using the two transmit antenna 16-state 4-PSK code proposed in [ll]. The signal-to-noise-ratio (SNR) was defined **as** SNR = 10 log *(2)*. All results were obtained averaging over 10,000 independent Monte Carlo trials, where each realization consisted of one burst (i.e. one OFDM symbol).

Simulation Example 1. In the first simulation example we study the impact of delay spread on the performance eof space-frequency codes. For MR = 2 and uncorrelated spatial fading Fig. **2** shows the symbol error rate **as** a function of SNR for the low delay spread case (i.e. one path with*/pol2* = 1) and for the high delay spread case (i.e. six paths with $IP; \quad l2$ = l), respectively. We can see that the presence of delay spread drastically improves the performance of the space-frequency code. From the slopes of the two curves we can conclude that the space-frequency code does indeed seem to be able to exploit at least some of the available frequency diversity, but certainly not all of it. Recall that since $MT = MR$ = 2 and six paths with path gain 1 represent the channel provides 24-th order diversity

Simulation Example 2. In Sec. 3.2 we have demonstrated that the rank of the correlation matrices **RI** has critical impact on the available diversity. Furthermore, we showed in Sec. 2.1 that the rank of the correlation matrices is driven by the angle spread of the individual scattered clusters and by the antenna spacing at the receiver. In this simulation example, we study the influence of spatial fading correlation on the performance of space-frequency codes. We assumed the following power delay profile [l 0.77 0.561.Fig. 3 shows the symbol error rate for MR = 2 and MR = **3**and low and high spatial fading correlation, respectively. The solid curves correspond to MR = **2** whereas the dashed dotted curves correspond to MR = 3. The upper two curves show the symbol error rate in the case of high spatial fading correlation which was achieved by making the cluster angle spreads small and choosing the relative antenna spacing to be A = 1/4. Low spatial fading correlation was achieved by setting A = 1/2 and choosing large cluster angle spreads. We can clearly see that in the case of low spatial fading correlation the performance of the code is significantly better than in the case of high spatial fading correlation. This is

consistent with our findings (and intuition) that high spatial fading correlation reduces the diversity order and hence yields degraded performance **in** terms of symbol error rate. We can furthermore see that adding a receive antenna has correlation. This is so, since due to the small cluster angles spreads and hence large spatial fading correlation only a small increase in spatial diversity can be expected from additional receive antennas.

V. CONCLUSION

We studied space-frequency coded broadband OFDM Systems where both spatial and frequency diversity are available. Considering a strategy which consists of coding across OFDM tones and employing a spatial broadband channel model taking into account physical propagation parameter sand antenna spacing, we *derived the design criteria **for** space-frequency codes.* We furthermore showed that space-time codes designed to achieve full spatial diversity in the narrowband case will in general not achieve full space-frequency diversity in the broadband case. Space time codes designed for the fast fading case and smart greedy space-time codes [ll] can be expected to be able to exploit at least some of the available frequency diversity .Nevertheless, these codes will generally be sub optimum when employed **as** space-frequency codes. Hence, new designs taking into account the design criteria derived in this paper are needed. We further more *established* frequency coded ODM systems and we studied the impact of spatial fading correlation on the performance of space-frequency codes.

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