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Mathematical Formulation of Multi-Area Unit Commitment Problem

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Abstract- **Multi-Area Unit Commitment Problem is to determine the optimal commitment strategy for generating units located in multiple areas that are interconnected via tie–lines and joint operation of generation resources can result in significant operational cost savings. This research paper presents the mathematical formulation for Multi-Area Unit Commitment Problem along with tie-line concept of interconnected power system. The objective of this paper is to describe the multi-area unit commitment problem, mathematical formulation and tie line concept along with transmission interconnections constrains. Also, standard IEEE data for 26-Generating units of four area system along with 24-hours load demand is given as appendix , which can be used as quick reference by other researchers.**

Keywords- ***Multi-Area Unit Commitment Problem (MAUCP), Multi-Area Economic Dispatch (MAED), Tie Line, Transmission Interconnection Constraints***

I. INTRODUCTION

Unit commitment is a complex optimization task for planning and operation of a power system network. In the modern power system networks, there are various generating resources like thermal, hydro, nuclear etc. Also, the load demand varies during a day and attains different peak values. Thus, it is required to decide which generating unit to turn on and at what time it is needed in the power system network and also the sequence in which the units must be shut down keeping in mind the cost effectiveness of turning on and shutting down of respective units. The entire process of computing and making these decisions is known as unit commitment (UC).

II. UNIT COMMITMENT PROBLEM

The scheduling of the units together with the allocation of the generation quantities which must be scheduled to meet the demand for a specific period represents the Unit Commitment Problem (UCP). The Unit Commitment Problem is to determine a minimal cost turn-on and turn-off schedule of a set of electrical power generating units to meet a load demand while satisfying a set of operational constraints. The production cost includes fuel, startup, shutdown, and no load costs. The operational constraints that must be taken into account include, 1. The total power generated must meet the load demand plus system losses. 2.There must be enough spinning reserve to cover any shortfalls in generation. 3. The loading of each unit must be within its minimum and maximum allowable rating. 4.The minimum up and down times of each unit must be observed. The unit commitment is aimed at devising a proper generator commitment schedule for a power system over a period of one day to one week. The main objective of unit commitment is to minimize the total production cost over the study period & to satisfy the constraints imposed on the system such as power generation-load balance, spinning reserve, operating constraints, minimum up time & minimum down time, etc. Several conventional methods are available to solve the unit commitment problem. But all these methods need the exact mathematical model of the system & there may be a chance of getting stuck at the local optimum.

III. MULTI-AREA UNIT COMMITMENT

A multi-area unit commitment (MAUC) represents two or more interconnected regions of a power system. In multi area system, several generation areas are interconnected by tie lines. The objective of MAUC is to achieve the most economic generation to meet out the local demand without violating tie-line capacity limits constraints [1]. In an interconnected multi area system, joint operation of generation resources can result in significant operational cost savings [2]. It is possible by transmitting power from a utility, which had cheaper sources of generation to another utility having costlier generation sources. The total reduction in system cost shared by the participating utilities[4]. The exchange of energy between two utilities having significant difference in their marginal operating costs. The utility with the higher operating cost receives power from the utility with low operating cost. This arrangement usually on an hour to hour basis and is conducted by the two system operators. In the competitive environment, customer request for high service reliability and lower electricity prices. Thus, it is an important to maximize own profit with high reliability and minimize overall operating cost [3].

A multi-area power system is expected to provide a series of advantages such as: Substantial operational cost savings;

Increased safety of the power system; Increased competitiveness; Enhanced environmental benefits. For many techniques, is very difficult to find an ‘optimal’ solution to the MAUC problem for a large scale power system, within a very low computational time. In an interconnected multi-area system, joint operation of generation resources can result in significant operational cost savings. Multi-area joint operation can be implemented at different levels. At the first level, an independent unit commitment decision is made for each area, and the committed units are then jointly economically dispatched to satisfy the multi-area energy requirements. At the second level, both the capacity commitment decision and the energy dispatch decision are made jointly for all the participating areas in the interconnected multi-area system.

IV. MULTI-AREA UNIT COMMITMENT PROBLEM FORMULATION

For Interconnected Multi-Area Power System, the objective function for the multi-area unit commitment (MAUC) is to minimize the overall generation cost of available generation sources by satisfying load demand curve, which can be mathematically written as:

$$\min_{I,P} \sum_{k=1}^{N_A}\sum_{j=1}^{t}\sum_{i=1}^{N_k}\left[I_{i,j}^{k} F_j^{k}(Pg_{i,j}^{k}) + I_{i,j}(1-I_{i,j-1})S_i(X_{i,j-1}^{off}))\right] \tag{1}$$

Subject to various constraints are to be met for optimization such as system power balance constraints, Spinning reserve constraints, Minimum Up and Down time constraints, import/export constraints and initial on/off condition of each unit.

In Multi-Area Unit Commitment Problem, the total operating cost of electrical energy includes fuel cost, start up cost and shut down cost. The fuel costs are calculated using the data of unit heat rate and fuel price information, which is normally a quadratic equation of power output of each generator at each hour determined by Economic Dispatch (ED) and mathematically can be written as:

$$F(Pg_i^k) = a_i^k (Pg_i^k)^2 + b_i^k (Pg_i^k) + c_i^k$$ Rs./Hour or \$/Hour , where, k=1,2,3,4,…………,NA

(2)

and the incremental production cost can be written as:

$$\lambda = 2a_i^k Pg_i^k + b_i^k \quad \text{or} \quad Pg_i^k = \lambda - \frac{b_i^k}{2a_i^k} \tag{3}$$

The startup cost of each thermal unit is an exponential function of the time that the unit has been off and can be mathematically described as:

$$S(X_{i,j}^{off}) = A_i + B_i(1 - e^{X_{i,j}^{off}}) \tag{4}$$

To decompose the problem function, the equation (4) can be rewritten as:

$$\min_{P} \sum_{j=1}^{t}[F(Pg_{i,j})] \tag{5}$$

Where,

$$F(Pg_{i,j}) = \sum_{k=1}^{N_A} F^k(Pg^k{}_{i,j}) \tag{6}$$

Each $F^k(P^k_{g_{i,j}})$ for k=1,2,3,4,……..,N$_A$ is represented in the form of schedule table, which is the solution of mixed variable optimization problem

$$\min_{I,P} \sum_{i}[I_{i,j}^k F_i^k(Pg_{i,j}^k) + I_{i,j}(1-I_{i,j-1})(S_i(X_{i,j}^{off}))] \tag{7}$$

Subject to constraints of equation (9) , (11) and (12) and initial on/off condition of each unit.

V. MULTI-AREA UNIT COMMITMENT CONSTRAINTS

A. *System Power balance constraints*

The sum of generation of all the committed units at jth hour must be greater than or equal to the demand at a particular hour ‘j’. Mathematically,

$$\sum_{k} Pg_{j}^{k} = \sum_{k} D_j^k + W_j; \quad j = 1,2,3,....,t \tag{8}$$

Where,

$$\sum_{k} Pg_j^k = \sum_{k} Pg_{i,j}^k$$

i.e. sum of real power generated by each thermal unit must be sufficient enough to meet the sum of total demand of each area while neglecting transmission losses.

B. *Spinning Reserve constraints in each area*

Spinning reserve is the term used to describe the total amount of generation available from all units synchronized (i.e. spinning) on the system, minus the present load and losses being supplied. Spinning reserve must be carried so that the loss of one or more units does not cause too far a drop in system frequency. In order to maintain certain degree of reliability an excess capacity of generation is essential to immediately take over when a running unit fails, or unexpected load occurs, which can be mathematically given as:

$$\sum_{i} Pg^k_{i,j\max} \ge D_j^k + R_j^k + E_j^k - L_j^k; \quad j = 1,2,3,.....,t \tag{9}$$

C. *Generation limits of each unit*

The output generated by the individual units must be within the maximum and minimum generation limits i.e. Each generation unit has output range, which is represented as:

$$Pg^k_{j_{\min}} \le Pg^k_{i,j} \le Pg^k_{j_{\max}}\ ; \quad i = 1,2,3,....,N_k; \quad j = 1,2,3,.....,t; \tag{10}$$

D. *Minimum Up and Down Time constraints*

A committed unit can be turned OFF only after it satisfies its minimum up time values, at the same time, a reserved unit can be turned ON only after it satisfies, its minimum down time. This is due to the fact that the temperature of a thermal unit can be increased or decreased only gradually. Minimum up time means once the unit is running, it should not be turned off immediately and mathematically can be described as:

$$(X^{off}_{i,j-1} - T^{on}_i) \times (I_{i,j-1} - I_{i,j}) \ge 0 \tag{11}$$

Minimum down time means once the unit is decommitted, there is a minimum time before it can be recommitted and mathematically described as:

$$(X^{off}_{i,j-1} - T^{off}_i) \times (I_{i,j} - I_{i,j-1}) \ge 0 \tag{12}$$

E. *Import/Export constraints*

Multi-area unit commitment with no import/export constraints is basically identical to single area unit commitment. However, any realistic procedure to determine multi-area unit commitment must include import/export limits of power transfer for the following reasons:

(i) Physical transmission line limitations
(ii) Fuel Availability
(iii) Area Security consideration
(iv) Regulatory restrictions

In Multi- area unit commitment problem, power generation limits caused by tie line constraints are as follows:

(a) Upper limits: $\sum_i Pg^k_{i,j} \le D^k_j + E^k_{j_{\max}}$ (13)

(b) Lower limits: $\sum_i Pg^k_{i,j} \ge \sum_k D^k_j - L^k_{j_{\max}}$ (14)

(c) **Import/Export balance:**

$$\sum_i E^k_j - \sum_k L^k_j + W_j = 0 \qquad k = 1,2,3,....,N_A \tag{15}$$

F. *Area Generation Limits*

Some regions within the control area, called zones, may also have spinning and operating reserve constraints, particularly if transmission interconnecting that region with the rest of the system is constrained, which is mathematically given as:

$$\sum_i Pg^k_{i,j} \le \sum_i Pg^k_{i_{\max}} - R^k_j \qquad ;k = 1,2,3,......N_A \qquad ;j = 1,2,3,.....,t \tag{16}$$

$$\sum_i Pg^k_{i,j} \ge \sum_i Pg^k_{i_{\min}} \qquad ;k = 1,2,3,.....N_A \qquad ;j = 1,2,3,.....,t \tag{17}$$

VI. TIE-LINE CONSTRAINTS

To illustrate the tie-line flow in a multi-area system, the four area system given below is studied:

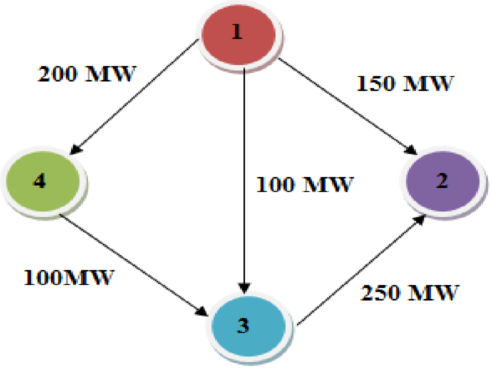

Figure-1: Multi-Area connection and tie-line limitations

An economically efficient area may generate more power than the local demand, and the excessive power will be exported to other areas through the tie-lines. For example; assume area 1 has excess power, the line flows would have directions from area 1 to other areas, and the maximum power generation for area 1 would be the local demand in area 1 plus the sum of all the tie-line capacities connected to area 1. If we fix the area 1 generation at its maximum level, then the maximum power generation in area 2 could be calculated in a similar way to area 1.
Since tie-line imports power at its maximum capacity, this amount should be subtracted from the generation limit of

area 2. According to the system power balance equation some areas must have a power generation deficiency, and require generation imports. The minimum generation level of these areas is the local demand, minus all the connected tie-line capacities. If any of these tie lines is connected to an area with higher deficiencies, then the flow directions should be reversed.

VII. MULTI-AREA ECONOMIC DISPATCH

The objective of Multi-area Economic Dispatch (MAED) is to determine the allocation of generation of each unit in the system and power exchange between areas so as to minimize the total production cost. In Multi-area Economic Dispatch (MAED), The objective is to select λ_{sys} at every hour to minimize the operation cost, which must satisfy the mathematical equation:

$$Pg^k_j = D^k_j + E^k_j - L^k_j$$

(18)

Where, $Pg^k_j = \sum_{i=1}^{N_k} Pg^k_{i,j}$

(19)

Since, the local demand D^k_j is determined in accordance with the economic dispatch within the pool, changes of Pg^k_j will cause the spinning reserve constraints of equation $\sum_i Pg^k_{i,j\max} \ge D^k_j + R^k_j + E^k_j - L^k_j$ to change accordingly and redefine equation $\min_{I,P} \sum_i [I^k_{i,j} F^k_i (Pg^k_{i,j}) + I_{i,j}(1 - I_{i,j-1})(S_i(X^{off}_{i,j}))]$.

Generating units may operate in one of the following modes when commitment schedule and unit generation limits are encountered.

(a) Coordinate Mode: The output of the ith generator is determined by the principle of equal incremental fuel cost, which is mathematically represented by $\lambda_{\min,i} \le \lambda_{sys} \le \lambda_{\max,i}$

(b) Minimum mode: The output of the ith generator is the minimum value i.e. $\lambda_{\min,i} > \lambda_{sys}$

(c) Maximum Mode: The output of the ith generator is the maximum value i.e. $\lambda_{\max,i} < \lambda_{sys}$

(d) Shut down mode: The output of the ith generator is zero i.e. Unit i is not in operation $Pg_i = 0$

Besides limitations on individual unit generations, in a multi-area system, the tie-line constraints in equation (11), (12) and (14) are to be preserved (sealed). The operation of each area could be generalized into one of the modes as follows:

(i) **Area coordinate mode:** $\lambda^k = \lambda_{sys}$

$$D^k_j - L^k_{\max} \le \sum_i Pg^k_{i,j} \le D^k_j + E^k_{\max} \quad \text{or}$$

$$-L^k_{\max} \le \sum_i Pg^k_{i,j} - D^k_j \le E^k_{\max}$$

(20)

Limited export mode: When the generating cost in one area is lower than the cost in the remaining areas of the system, that area may generate its upper limits according to following equations:

$$\sum_i Pg^k_{i,j} \le D^k_j + E^k_{j\max} \quad \text{or}$$

$$\sum_i Pg^k_{i,j} \ge \sum_i Pg^k_{i\min} \quad ; \quad k = 1,2,3,.....N_A \quad ; \quad j = 1,2,3,.....,t$$

(21)

Therefore,

$$\lambda^k < \lambda_{sys}$$

Where, λ^k is the optimal equal incremental cost which satisfies the generation requirement in each area k.

(ii) Limited import mode: An area may reach its lower generation limit according to following equations:

$$\sum_i Pg^k_{i,j} \ge \sum_k D^k_j - L^k_{j\max} \quad \text{or}$$

$$\sum_i Pg^k_{i,j} \ge \sum_i P^k_{g_{i\min}} \quad ; k = 1,2,3,.....N_A \quad ; j = 1,2,3,.....,t$$

(22)

because of higher generation cost. Therefore,

$\lambda^k_{\min} > \lambda_{sys}$

The proper generation schedule in Multi-Area will result by satisfying tie-line constraints and minimizing the system generation cost.

VIII. TIE-LINE CONCEPT FOR MULTI-AREA POWER SYSTEM

Modern power systems are divided into various areas. Each of these areas are interconnected to its neighboring areas through transmission lines. The transmission lines that connect an area to its neighboring area are called Tie-Lines. Power sharing between two areas occurs through these tie-lines. Load frequency control regulates the power flow between different areas while holding the frequency constant.

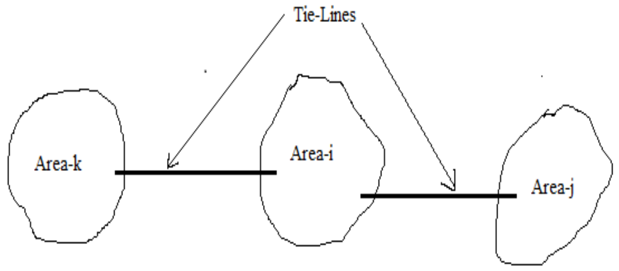

Figure-2: Tie-Lines Connection in Interconnected Multi-Area Power System

In interconnected Multi-Area Power System, the system frequency rises when the load decreases if ΔP_{ref} is kept at zero. Similarly the frequency may drop if the load increases. However it is desirable to maintain the frequency constant such that $\Delta f=0$. The power flow through different tie-lines are scheduled. For example- area- i may export a pre-specified amount of power to area- j while importing another pre-specified amount of power from area- k . However it is expected that to fulfill this obligation, area- i absorbs its own load change, i.e., increase generation to supply extra load in the area or decrease generation when the load demand in the area has reduced. While doing this area- i must however maintain its obligation to areas j and k as far as importing and exporting power is concerned. In interconnected Multi-Area Power System, along with MAUC and demand satisfactions another important aspect is load frequency control (LFC), which has the has the following objectives:

- Hold the frequency constant ($\Delta f = 0$) against any load change. Each area must contribute to absorb any load change such that frequency does not deviate.
- Each area must maintain the tie-line power flow to its pre-specified value.
- Each area must maintain the tie-line power flow to its pre-specified value.

The first step in the LFC is to form the area control error (ACE) that is defined as

$$ACE = (P_{tie} - P_{sch}) + B_f \Delta f = \Delta P_{tie} + B_f \Delta f \tag{23}$$

The change in the reference of the power setting ΔPref, i , of the Area- i is obtained by the feedback of the ACE through an integral controller of the form

$$\Delta P_{ref,i} = -K_i \int ACE \quad dt \tag{24}$$

The ACE is negative if the net power flow out of an area is low or if the frequency has dropped or both. In this case the generation must be increased. This can be achieved by increasing ΔPref, i . This negative sign accounts for this inverse relation between ΔPref, i and ACE. The tie-line power flow and frequency of each area are monitored in its control center. Once the ACE is computed and ΔPref, i is obtained from (24), commands are given to various turbine-generator controls to adjust their reference power settings.

Abbreviations

i= index for units
j=index for time
$P^k_{g_i}$ =Power generation of unit i in area K
$F(P^k_{g_i})$ =Production cost of unit i in area K
a^k_i, b^k_i, c^k_i =Cost function parameters of unit i in area K
$X^{off}_{i,j}$ =Time duration for which unit i have been off at jth hour
$P^k_{i,j}$ =Power generation of unit i in area K at jth hour
$I^k_{i,j}$ =Commitment state (0 for OFF, 1 for ON)
$Pg^k_{i,j}$ =Power generation of unit i in area K at jth hour
D^k_j =Total system demand of area K at jth hour
R^k_j = Spinning reserve of area K at jth hour
E^k_j =Total export power to area K at jth hour
$Pj^k_{\max}$ =Maximum power generation in area K at jth hour
$Pj^k_{\min}$ =Minimum power generation in area K at jth hour
T^{on}_i =Minimum up-time of unit i
T^{off}_i =Minimum down time of unit i
$Lj^k_{\max}$ =Maximum total import power in area K at jth hour
W_j =Net power exchange with outside system
λ_{sys} =Marginal cost of supplying the last incremental energy to meet entire system demand
$Pg^k_{i\min}$ =Minimum power generation at area K at ith hour
$Pg^k_{i\max}$ =Maximum power generation at area K ith hour
P_{tie} =Tie-Line Power through tie-line
P_{sch} = Scheduled Power through tie-line
B_f = Frequency bias constant
Ki = The integral gain

CONCLUSION

In this paper, researchers have presented the concept of Multi-Area unit commitment along with Multi Area Economic Dispatch and Tie-Line Concept and their mathematical formulation for typical Multi-area unit commitment problem and multi area economic dispatch.. Also, standard IEEE data for 26-Generating units of four area system along with 24-hours load demand is given in appendix , which can be used as quick reference by other researchers.

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APPENDICES

Data Sheet for Multi-Area Unit Commitment [11]

Table-1 Generating Unit Characteristics

Unit No.	Minimum Up Time [Hour]	Minimum Down Time[Hour]	Initial Condition [Hour]	Minimum Generation [MW]	Maximum Generation [MW]
1	0	0	-1	2.40	12
2	0	0	-1	2.40	12
3	0	0	-1	2.40	12
4	0	0	-1	2.40	12
5	0	0	-1	2.40	12
6	0	0	-1	4.00	20
7	0	0	-1	4.00	20
8	0	0	-1	4.00	20
9	0	0	-1	4.00	20
10	0	-2	3	15.20	76
11	3	-2	3	15.20	76
12	3	-2	3	15.20	76
13	3	-2	3	15.20	76
14	3	-2	-3	25.00	100
15	4	-2	-3	25.00	100
16	4	-2	-3	25.00	100
16	4	-3	5	54.25	155
18	4	-3	5	54.25	155
19	5	-3	5	54.25	155
20	5	-3	5	54.25	155
21	5	-4	-4	68.95	197
22	5	-4	-4	68.95	197
23	5	-4	-4	68.95	197
24	8	-5	10	140.00	350
25	8	-5	10	140.00	350
26	8	-5	10	140.00	350

Table 2. Cost functions for Generating Units in Area 1[11]

Unit No.	Generation Cost Coefficient a [$/MW^2]	Generation Cost Coefficient b [$/MW]	Generation Cost Coefficient c [$]	Start Up cost coefficient A [$]	Start Up cost coefficient B [$]	Start up time constant [tau]
1	24.36	25.237	0.012	0	0	1
2	24.379	25.255	0.0121	0	0	1
3	24.395	25.273	0.0125	0	0	1
4	24.42	25.299	0.0129	0	0	1
5	24.434	25.321	0.013	0	0	1
6	117.121	37	0.006	20	20	2
7	117.239	37.132	0.0062	20	20	2
8	117.358	37.307	0.0064	20	20	2
9	117.481	37.49	0.0066	20	20	2
10	81	13.322	0.0046	50	50	3
11	81.028	13.244	0.0047	50	50	3
12	81.104	13.3	0.0049	50	50	3
13	81.176	13.35	0.0052	50	50	3
14	217	18	0.0042	70	70	4
15	217.1	18.1	0.0044	70	70	4
16	217.2	18.2	0.0047	70	70	4
16	142.035	10.394	0.0043	150	150	6
18	142.229	10.515	0.0045	150	150	6
19	142.418	10.637	0.0047	150	150	6
20	143.497	10.708	0.0048	150	150	6
21	256.101	22	0.0025	200	200	8
22	257.649	22.1	0.0026	200	200	8
23	258.176	22.2	0.0026	200	200	8
24	175.057	10.462	0.0016	300	200	8
25	305.036	7.486	0.0019	500	500	10
26	306.91	7.493	0.0019	500	500	10

Table 3. Cost functions for Generating Units in Area 2[11]

Unit No.	Generation Cost Coefficient a $[\$/\text{MW}^2]$	Generation Cost Coefficient b [$/MW]	Generation Cost Coefficient c [$]	Start Up cost coefficient A [$]	Start Up cost coefficient B [$]	Start up time constant [tau]
1	24.389	25.547	0.0123	0	0	1
2	24.411	25.675	0.0125	0	0	1
3	24.638	25.803	0.013	0	0	1
4	24.76	25.932	0.0134	0	0	1
5	24.488	26.061	0.0136	0	0	1
6	117.755	37.551	0.0059	20	20	2
7	118.108	37.664	0.0066	20	20	2
8	118.458	37.777	0.0066	20	20	2
9	118.821	37.89	0.0073	20	20	2
10	81.136	13.327	0.0047	50	50	3
11	81.298	13.354	0.0049	50	50	3
12	81.464	13.38	0.0051	50	50	3
13	81.626	13.407	0.0053	50	50	3
14	217.895	18	0.0043	70	70	4
15	218.355	18.1	0.0051	70	70	4
16	218.775	18.2	0.0049	70	70	4
16	142.735	10.695	0.0047	150	150	6
18	143.029	10.715	0.0047	150	150	6
19	143.318	10.737	0.0048	150	150	6
20	143.597	10.758	0.0049	150	150	6
21	259.131	23	0.0026	200	200	8
22	259.649	23.1	0.0026	200	200	8
23	260.176	23.2	0.0026	200	200	8
24	177.057	10.862	0.0015	300	200	8
25	310.002	7.492	0.0019	500	500	10
26	311.91	7.503	0.0019	500	500	10

Table 4. Cost functions for Generating Units in Area 3[11]

Unit No.	Generation Cost Coefficient a [$/MW^2]	Generation Cost Coefficient b [$/MW]	Generation Cost Coefficient c [$]	Start Up cost coefficient A [$]	Start Up cost coefficient B [$]	Start up time constant [tau]
1	24.451	26.547	0.0123	0	0	1
2	24.395	26.675	0.0125	0	0	1
3	24.738	26.803	0.013	0	0	1
4	24.861	26.932	0.0134	0	0	1
5	24.988	27.061	0.0136	0	0	1
6	118.755	38.551	0.0069	20	20	2
7	119.108	38.664	0.0076	20	20	2
8	119.458	38.777	0.0076	20	20	2
9	119.821	38.89	0.0083	20	20	2
10	82.136	14.327	0.0047	50	50	3
11	82.298	14.354	0.0059	50	50	3
12	82.464	14.481	0.0061	50	50	3
13	82.626	14.407	0.0063	50	50	3
14	218.895	19	0.0053	70	70	4
15	219.355	19.1	0.0061	70	70	4
16	219.775	19.2	0.0059	70	70	4
16	143.735	11.695	0.0056	150	150	6
18	144.029	11.715	0.0057	150	150	6
19	144.318	11.737	0.0058	150	150	6
20	144.597	11.758	0.0059	150	150	6
21	259.131	24	0.0036	200	200	8
22	259.649	24.1	0.0036	200	200	8
23	260.176	24.2	0.0036	200	200	8
24	177.057	11.862	0.0015	300	200	8
25	310.002	7.692	0.0019	500	500	10
26	311.91	7.703	0.0019	500	500	10

Table 5. Cost functions for Generating Units in Area 4 [11]

Unit No.	Generation Cost Coefficient a $[\$/\mathrm{MW}^2]$	Generation Cost Coefficient b [$/MW]	Generation Cost Coefficient c [$]	Start Up cost coefficient A [$]	Start Up cost coefficient B [$]	Start up time constant [tau]
1	24.389	25.202	0.0123	0	0	1
2	24.411	25.255	0.0125	0	0	1
3	24.638	25.273	0.013	0	0	1
4	24.76	25.342	0.0134	0	0	1
5	24.888	25.366	0.0136	0	0	1
6	117.755	37.012	0.0059	20	20	2
7	118.108	37.055	0.0066	20	20	2
8	118.458	37.098	0.0066	20	20	2
9	118.821	37.156	0.0073	20	20	2
10	81.136	13.261	0.0047	50	50	3
11	81.298	13.278	0.0049	50	50	3
12	81.464	13.295	0.0051	50	50	3
13	81.626	13.309	0.0053	50	50	3
14	217.895	17.5	0.0043	70	70	4
15	218.355	17.6	0.0051	70	70	4
16	218.775	17.7	0.0049	70	70	4
16	142.735	10.21	0.0047	150	150	6
18	143.029	10.268	0.0047	150	150	6
19	143.318	10.307	0.0048	150	150	6
20	143.597	10.375	0.0049	150	150	6
21	259.131	22.5	0.0026	200	200	8
22	259.649	22.6	0.0026	200	200	8
23	260.176	22.7	0.0026	200	200	8
24	177.057	10.462	0.0015	300	200	8
25	310.002	7.492	0.0019	500	500	10
26	311.91	7.503	0.0019	500	500	10