

April 2014

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### Recommended Citation

Sudhakar, P. (2014) "Detection of MDPSK in Fading Channels using Gaussian Particle Filter," *International Journal of Power System Operation and Energy Management*: Vol. 3 : Iss. 2 , Article 4.

Available at: <https://www.interscience.in/ijpsoem/vol3/iss2/4>

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# Detection of MDPSK in Fading Channels using Gaussian Particle Filter

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## Abstract-

We propose Gaussian particle filtering approach for solving the problem of corrupting the MDPSK signals by fading as well as noise. Particle filtering is a powerful tool for non linear problems but it faces sampling degeneration problem which leads to re-sampling process. Gaussian Particle Filtering doesn't need re-sampling process because it approximates the posterior distribution as Gaussian. GPF is preferable than PF for fading channels.

**Key Words:** GPF, MDPSK, Fading Channels, non-Gaussian Noise

## I.INTRODUCTION

In digital wireless systems multi path fading is the most common phenomena. The random attenuations and delays are due to constructive and destructive combination of number of multi paths received at the receiver. This type of fading effects the message coded signals transmitted through wireless channel and causes signal variations. So, it is difficult to demodulate the modulated signals in fading transmission channels in the presence of non Gaussian additive noise. Many filters have been proposed to solve it. [3], [5], [6].

In general particle filtering uses stochastic grid approximation for the conditional probability distribution of the symbols with particles. It needs more computational efforts because of re-sampling process [4], [1]. In this paper Gaussian particle filter is analyzed

theoretically to solve the problem of demodulation of M-ary differential phase shift keying (MDPSK) in fading channels. Here GPF is used to estimate the posterior distribution of the symbols in M-ary DPSK signals in fading channels with non Gaussian additive noise. The fading channel problems can be written in the form of Dynamic State Space (DSS) model. The model represents the unknown variable  $x_n$  as the distribution  $p(x_n|x_{n-1})$  where  $n$  is the time. The observations  $y_n$  in the application are usually noisy and distorted versions of  $x_n$ . the distribution  $p(y_n|x_n)$  represents the observation equation conditioned on the unknown state variable  $x_n$  which is to be estimated. We denote by  $x_{0:t}$  and  $y_{0:t}$  the signal and observations up to time  $n$ , respectively, i.e.,  $x_{0:n}=\{x_0,x_1,\dots,x_n\}$  and  $y_{0:n}=\{y_0,y_1,\dots,y_n\}$  Our aim is to estimate recursively in time,

- The filtering distribution or the marginal posterior of the state at time  $n$  can be written as

$$P(x_n | y_{0:n}) = C_n p(x_n | y_{0:n-1}) p(y_n | x_n) \quad (1)$$

Where  $C_n$  is the normalizing constant given by

$$C_n = (\int p(x_n | y_{0:n-1}) p(y_n | x_n) dx_n)^{-1} \quad (2)$$

- The predictive distribution can be expressed as

$$p(x_{n+1} | y_{0:n}) = \int p(x_{n+1} | x_n) p(x_n | y_{0:n}) dx_n \quad (3)$$

GPF approximates posterior mean and covariance of the unknown state using importance sampling. The assumption of additive Gaussian noise can be relaxed to be non Gaussian and non additive in the case of GPF. GPF is quite similar to particle filter by the fact that particles are obtained using importance sampling. However re-sampling is not required, unlike particle filtering (PF), in the GPF.

This results in reduce complexity of GPF and is a major advantage.

## II. CHANNEL SPECIFICATION AND ESTIMATION

*Digitally modulated signals:* Let  $r_t$  indicates one of the combination of  $N$  possible set of information of  $k$  bits  $r_t \in \mathbb{R}$ ,  $t = \{1, 2, \dots, N\}$ . where  $t$  is a discrete time index. The modulated signal transmitted may be represented as  $s_{mod}(t) = \text{Re}\{s_t(r_{1:t}) \exp(j2\pi f_c t)\}$ , where  $f_c$  is a carrier frequency, and  $s_t(\cdot)$  function performs converting from digital sequence to waveform. It is assumed that  $r_t$  is homogeneous,  $M$ -state, Markov chain with known probabilities  $p_{ij} = \Pr\{r_{t+1}=j | r_t=i\}$ ,  $i, j \in \mathbb{R}$ , and also  $\sum = 1$  for each  $i$ , and initial distribution  $p_i = \Pr\{r_1=i\}$ ,  $\sum = 1$  for  $i \in \mathbb{R}$ . [3]

*Fading Channel observations:* A Rayleigh fading channel can explained by a multiplicative discrete time disturbance  $g_t$ . The fading channel is modeled as an ARMA( $q, q$ ) process where  $q$  is the order of the filter [6]. The ARMA coefficients  $a$  (AR

part) and  $b$  (MA part) are chosen so that the cut-off frequency of the filter matches the normalized channel Doppler frequency  $f_d T$  where  $T$  is the symbol rate,  $f_d T$  are familiar values. The additive complex noise corrupts the output of the filter by a zero-mean Gaussians with  $k$  components. For identifying the variance of the distribution of the noise samples drawn, we introduce a latent variable  $z_t$ ,  $z_t \in \mathbb{R}_z = \{1, 2, \dots, k\}$ ,  $t = \{1, 2, \dots\}$  such that  $\Pr(z_t=j) = \lambda_j$ , for  $j = 1, 2, \dots, k$ ,  $\sum \lambda = 1$ . Conditional upon  $r_{1:t}$  and  $z_{1:t}$ , the problem can be formulated in the following linear state space form  $x_{t:t-q+1} = A x_{t-1:t-q} + B v_t$ ,  $y_t = C(t_{1:t}) x_{t:t-q+1} + D(z_t) w_t$ , (4) where  $y_t$  is the output of the filter,  $x_t$  is defined so that  $g_t = b_T x_{t:t-q+1}$ ,  $A$  is a function of  $a$ ,  $B = (1, 0, 0, \dots, 0)^T$ ,  $C(r_{1:t}) = s_t(r_{1:t}) b^T$  and  $D = \cdot$ . We assume  $x_{0:1-q} \sim N_c(\cdot, P_0)$ , where  $P_0 > 0$ , and let  $v_t \sim N_c(0, 1)$ ,  $w_t \sim N_c(0, 1)$  be mutually independent for all  $t > 0$ . The symbols  $r_t$  the channel

characteristics  $x_t$  and latent variables are unknown for  $t > 0$  whereas  $A, B, C(r_{1:t}), D(z_t)$ , and  $P_0$  are known for each  $r_{1:t} \in \mathbb{R}_t$ ,  $z_t \in \mathbb{R}_z$ .

*Estimation Objectives:* Obtain the maximum a posterior estimates of the symbols  $\arg p(r_t | y_{1:t})$  and  $\arg p(r_{1:t} | y_{1:t})$ , where  $L > 0$ . Since these conditional probabilities involve a prohibitive computational cost exponential in the number of observations, these do not admit any analytical solution.

## III. GAUSSIAN PARTICLE FILTER

The GPF approximates the filtering and predictive distributions in (1) and (3) by Gaussian densities using the particle filtering methodology [7], [4], [2]. The basic idea of Monte Carlo methods is to consider a collection of particles for representing the distribution  $P(x_n)$  of a random variable,  $x_n$ .  $M$  particles  $X = \{(\cdot), (\cdot), \dots, (\cdot)\}$  are generated under certain conditions known as importance sampling distribution  $\pi(x_n)$ . The

weighted factors for the particles is given as a set of values  $W = \{w(1), w(2), \dots, w(M)\}$ , where  $w(i) = p(x_n) / \pi(x_n)$ . The set  $\{X, W\}$  represents samples from the posterior distribution  $p(x_n)$ .

**A. Measurement Update**

After receiving the  $n^{th}$  observation  $y_n$ , the filtering distribution is given by  $P(x_n | y_{0:n}) = C_n \int p(x_n | y_{0:n-1}) p(y_n | x_n) dx_n \approx C_n p(y_n | x_n) N(x_n; \hat{\mu}_n, \hat{\Sigma}_n)$ . (5) The GPF measurement update step approximates the above density as Gaussian, i.e.,  $p(x_n | y_{0:n}) = N(x_n; \hat{\mu}_n, \hat{\Sigma}_n)$ . In general, analytical expressions for the mean  $\hat{\mu}_n$  and covariance  $\hat{\Sigma}_n$  of  $P(x_n | y_{0:n})$  are not available. However, for the GPF update, Monte Carlo estimates of  $\hat{\mu}_n$  and  $\hat{\Sigma}_n$  can be computed from the samples  $\{x_n^{(i)}\}$  and their weights, where the samples are obtained from an importance sampling function  $\pi(x_n | y_{0:n})$ . This leads to measurement update algorithm as follows

1. Choose the samples from importance function  $\pi(x_n | y_{0:n})$  & denote them as  $\{x_n^{(i)}\}$ .
2. Calculate the respective weights by  $\hat{w}_n^{(i)} = \frac{p(y_n | x_n^{(i)}) N(x_n^{(i)}; \hat{\mu}_n, \hat{\Sigma}_n) \pi(x_n^{(i)} | y_{0:n})}{\pi(x_n^{(i)} | y_{0:n})}$
3. Normalize the weights as  $w_n^{(i)} = \hat{w}_n^{(i)} / \sum \hat{w}_n^{(i)}$ .
4. Estimate the mean and covariance by  $\hat{\mu}_n = \sum w_n^{(i)} x_n^{(i)}$  and  $\hat{\Sigma}_n = \sum w_n^{(i)} (x_n^{(i)} - \hat{\mu}_n)(x_n^{(i)} - \hat{\mu}_n)^T$  (6)

GPF – time update algorithm

1. Draw samples from  $N(x_{n+1}; \hat{\mu}_n, \hat{\Sigma}_n)$  and denote them as  $\{x_{n+1}^{(i)}\}$ .
2. For  $j=1, 2, \dots, M$ , sample from  $p(x_{n+1} | x_n = x_{n+1}^{(j)})$  to obtain  $\{x_{n+1}^{(i,j)}\}$ .
3. Compute the mean  $\hat{\mu}_{n+1}$  and covariance  $\hat{\Sigma}_{n+1}$  as  $\hat{\mu}_{n+1} = \sum w_n^{(i)} \sum (\mu_{n+1}(x_{n+1}^{(i,j)})) (\mu_{n+1}(x_{n+1}^{(i,j)}))$  (7)

**B. Time Update**

Assuming that the samples from  $p(x_{n+1} | x_n)$  can be drawn at any time  $n$ . with the value  $p(x_n | y_{0:n})$  approximated as Gaussian, we can obtain predictive distribution  $p(x_{n+1} | y_{0:n})$  and approximate as Gaussian. The

predictive distribution is given by  $p(x_{n+1} | y_{0:n}) = \int p(x_{n+1} | x_n) p(x_n | y_{0:n}) dx_n \approx \int p(x_{n+1} | x_n) N(x_n; \hat{\mu}_n, \hat{\Sigma}_n) dx_n$ . (8)

A Monte Carlo approximation for the predictive distribution is given by

$$P(x_{n+1} | y_{0:n}) \approx \sum p(x_{n+1} | x_n^{(i)}) \quad (9)$$

where  $x_n^{(i)}$  are particles from  $N(x_n; \hat{\mu}_n, \hat{\Sigma}_n)$ . From these observations,  $M$  samples denoted as  $\{x_{n+1}^{(i)}\}$  are obtained from  $p(x_{n+1} | x_n^{(i)})$ .

**IV. CONCLUSION**

The Gaussian particle filter provides much better performance for demodulation of M-ary differential phase shift keying(MDPSK) signals under conditions of Rayleigh fading channels in non-Gaussian additive noise than Particle filtering. The parallelizability update of the filtering and predictive distributions as Gaussians which leads to the absence of resampling process makes it convenient for real time applications. Gaussian particle filter can also be implemented for nonlinear systems with Gaussian noise.

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