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STATISTICAL METHODS TO ESTIMATE NATURAL FREQUENCY OF AIR CONDITIONER PIPING

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Abstract - Resonance is a critical phenomena in the designing of any structure. Piping structure, which is one of the several elements of air conditioning and refrigeration system, undergo severe vibration. In practice different approaches such as experimental, analytical, numerical and statistical methods are used to study vibration characteristics of a structure. Statistical method acts as a first hand tool to judge the behaviour of a system. Statistical methods minimise redesign process and expensive methods of controlling vibrations, saving significant amount of effort, cost and time. In the present study, statistical methods are used to estimate fundamental natural frequency of a pipe by selecting different geometrical parameters. The natural frequency obtained can be used to check resonance. Geometrical parameters, which significantly affect the natural frequency, are estimated by using Design of Experiments (DOE) and Analysis of Variance (ANOVA) techniques. A linear equation that predicts fundamental frequency is formulated using regression analysis. Frequencies predicted by regression model are compared to frequencies obtained from ANSYS Mechanical©. A good correlation is found with error less than 5%.

Keywords *- natural frequency; air conditioner piping; design of experiments; regression model; analysis of variance.*

I. INTRODUCTION

Suction and discharge piping of HVAC system, carrying fluid from evaporator coil to compressor and from compressor to condenser coil are prone to vibration from compressor. Since resonance results in high displacement amplitude that leads to premature failure of the piping structure, appropriate selection of geometrical parameters, which decide pipe's natural frequencies are required to avoid resonance.

Several past studies have been carried out to analyze vibrations in air conditioning piping. Statistical methods (DOE and Regression analysis) applied to cantilever beam in order to avoid resonance are demonstrated in [1]. Results from finite element methods and experimental methods for an air conditioning piping are compared in [2]. Design optimization techniques for an air conditioning piping system to reduce the cost of pipes are discussed in [3]. Modal analysis of an air conditioning piping using finite element methods is explained in [4].

Present study aims at developing a mathematical model to estimate fundamental frequency in terms of geometrical parameters of a pipe, reducing time required to design a pipe. Loops and bends are used to design and optimize the flexibility of pipe.

A typical piping of an air conditioning system along with different control parameters are taken for the present investigation and is as shown in fig. 1.

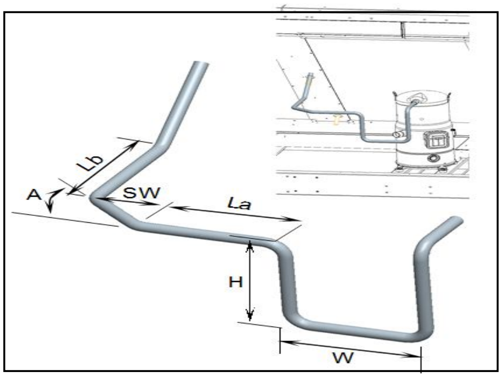

Fig.1 : Typical piping structure along with different parameters

In the fig.1 H is the loop height, W is the loop width, La and Lb are lengths, A is slant angle measured with respect to longitudinal axis of length La and slant width, SW is the distance between longitudinal axis of length Lb and an axis perpendicular to longitudinal axis of length La and parallel to axis of Lb. Tube diameter and thickness are kept constant. Pipe is made of copper with Young's modulus = 1.15e5 MPa, poisson's ratio = 0.33 and density = 8.94e-9 tonne/mm^3.

Two statistical methods are proposed in this paper. The first method is based on Design of Experiment (DOE) and Analysis of Variance (ANOVA). It identifies contribution of significant parameters affecting the natural frequency, to which more attention must be paid in order to avoid resonance. The second method uses the concept of a regression model to determine a correlation which can predict fundamental natural frequency for the given parameters. The accuracy of the natural frequency obtained using the regression model is compared to the results generated by ANSYS Mechanical©.

II. STATISTICAL DESIGN OF EXPERIMENTS

Statistical DOE refers to the process of planning the experiment so that appropriate data that can be analyzed by statistical methods will be collected, resulting in valid and objective conclusions (Montgomery, 2006). A statistical tool is always preferred for drawing the meaningful conclusion from an experimental design data. There are two aspects to any experimental problem; the design of the experiment and the statistical analysis of the data. When many factors control the performance of any system then it is essential to find out significant factors which need special attention either to control or optimize the system performance [1]. Taguchi's concept of Orthogonal Array (OA) as a part of statistical DOE is used in such situations to plan the set of experiments and ANOVA technique is used to find out the significant factors.

These techniques have been used in the current study to investigate significant factors affecting the natural frequency of a piping structure out of *H, W, La, Lb, A* and *SW*. The first step in constructing an orthogonal array to fit a specific case study is to count the total degree of freedom that tells the minimum number of experiments that must be performed to study all the chosen control factors. The number of degrees of freedom associated with a factor is equal to one less than the number of levels for that factor (Madhav Phadke, 1989). Therefore degrees of freedom (DOF) of factors are (*A*(2), *La*(2), *Lb*(2), *H*(2), *W*(2), *SW*(2)). Degrees of freedom of interactions are (*A X La*(4), *A X Lb*(4), *La X Lb*(4)). Considering all the factors and their interactions, there are 24 degrees of freedom. Hence this experiment is carried out using L27, orthogonal array for 13 factors each at 3 levels to design the experiments for finding out the fundamental frequency of a given pipe under the simultaneous variation of six different loop parameters at three levels as shown in Table I.

TABLE I : FACTORS AND THEIR LEVELS FOR EXPERIMENT

		Levels		
Sr. No.	**Control Factors**	**1**	**2**	**3**
1	*A* (degrees)	15	37.5	60
2	*La* (mm)	120	200	280
3	*Lb* (mm)	120	200	280
4	*H* (mm)	120	200	300
5	*W* (mm)	120	200	300
6	*SW* (mm)	50	75	100

In total, L27 has 26 degrees of freedom. The remaining (26-24) two degrees of freedom are used for error. The standard linear graph for L27 matches the requirement as shown in fig. 2.

The design of experiments based on the L27 and linear graph is shown in Table II for the present case.

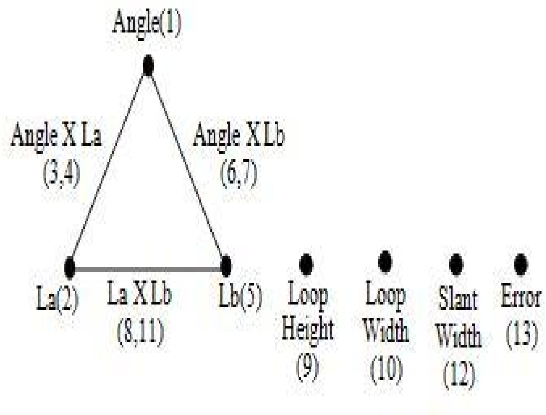

Fig. 2 : Standard Linear Graph for L27

Inability to distinguish effect of factors and interactions is called confounding (Madhav Phadke, 1989). As it is expected that factors *A*, *La* and *Lb* to interact, no factors are assigned to columns (3,4), (6,7) and (8,11). This is done to avoid confounding. The fundamental natural frequencies (*F*) of a given pipe for the combination of parameters are as shown in Table II. These are found by conducting modal analysis in ANSYS Mechanical ©.

III. ANALYSIS OF VARIANCE

The results obtained for natural frequency from the modal analysis in ANSYS Mechanical© are analyzed by using the statistical tool ANOVA. It determines the relative effect of the individual factors and their interactions on the natural frequency of pipe. The analysis by using ANOVA technique is done analytically. An equation for total variation may be written as

$$SS_T = SS_A + SS_{La} + SS_{Lb} + SS_H + SS_W + SS_{SW} + SS_{AXLa} + SS_{AXLb} + SS_{LaXLb} + SS_E \quad (1)$$

Where SS_T is total sum of squares. SS_A, SS_{La}, SS_{Lb}, SS_H, SS_W and SS_{SW} are sum of squares for angle, *La*, *Lb*, loop height, loop width and slant width respectively. SS_{AXLa}, SS_{AXLb}, SS_{LaXLb} are sum of squares of *A X La*, *A X Lb* and *La X Lb* interactions respectively. SS_E is sum of square of the error. If *T* is the sum of all (*N*) natural frequencies, the total sum of squares is given by,

$$SS_T = \left[\sum_{i=1}^{N} F_i^2 \right] - \frac{T^2}{N} \quad (2)$$

Sum of squares of loop height (*H*) factor is given as

$$SS_H = \left[\sum_{i=1}^{K_H} \frac{H_i^2}{N_{Hi}} \right] - \frac{T^2}{N} \qquad (3)$$

Where, K_H is the number of levels of loop height factor. H_i and N_{Hi} are the sum and number of observations respectively under i^{th} level. Similarly, sum of squares of other five factors can also be calculated. Sum of squares of interaction of *La* and *Lb* is given by,

$$SS_{LaXLb} = \left[\sum_{i=1}^{n} \left(\frac{(LaXLb)_i^2}{N_{(LaXLb)i}} \right) \right] - \frac{T^2}{N} - SS_{La} - SS_{Lb} \qquad (4)$$

where $(LaXLb)_i$ and $N_{(LaXLb)i}$ are the sum and number of observations (natural frequencies) respectively under i^{th} condition of the combinations of factors *La* and *Lb* and *n* is the number of possible combinations of the interacting factors *La* and *Lb*. Similarly, the sum of squares for other two interactions can also be found out. The results obtained from ANOVA are given in table III. From this table it can be seen that *H*, *W*, *A*, *La*, *Lb* and *SW* contribute 39.49%, 26.25%, 4.97%, 5.24%, 12.35% and 3.15% respectively. Fig. 3 gives main effects plot which highlights that loop height is the most significant factor amongst all the factors, as loop height is having the highest slope followed by loop width and length *Lb*. Slant width is least significant.

TABLE II : ORTHOGONAL ARRAY L27 WITH ANSYS MECHANICAL© RESULTS

Expt. No.	***A* (1)**	***La* (2)**	***A* & *La* (3,4)**	***Lb* (5)**	***A* & *Lb* (6,7)**	***H* (9)**	***W* (10)**	***La* & *Lb* (8,11)**	***SW* (12)**	***F* (Hz) from ANSYS**
1	15	120		120		120	120		50	86.42
2	15	120		200		200	200		75	47.85
3	15	120		280		300	300		100	27.84
4	15	200		120		200	200		100	50.23
5	15	200		200		300	300		50	29.99
6	15	200		280		120	120		75	52.07
7	15	280		120		300	300		75	30.38
8	15	280		200		120	120		100	54.47
9	15	280		280		200	200		50	38.53
10	37.5	120		120		200	300		75	43.49
11	37.5	120		200		300	120		100	38.23
12	37.5	120		280		120	200		50	50.01
13	37.5	200		120		300	120		50	40.40
14	37.5	200		200		120	200		75	50.68
15	37.5	200		280		200	300		100	31.38
16	37.5	280		120		120	200		100	53.29
17	37.5	280		200		200	300		50	36.36
18	37.5	280		280		300	120		75	34.75
19	60	120		120		300	200		100	33.38
20	60	120		200		120	300		50	44.20
21	60	120		280		200	120		75	48.30
22	60	200		120		120	300		75	44.05
23	60	200		200		200	120		100	45.41
24	60	200		280		300	200		50	30.47
25	60	280		120		200	120		50	53.87
26	60	280		200		300	200		75	30.63
27	60	280		280		120	300		100	28.67

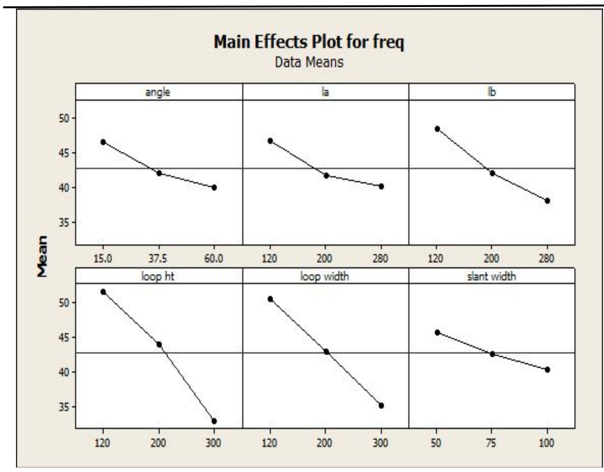

Fig. 3. Main Effects Plot for Freqency

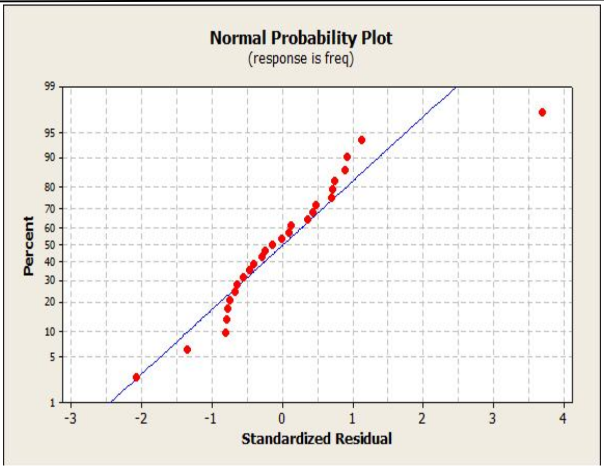

Fig. 4. Normal probability plot of standardized residuals

TABLE III : RESULTS OF ANALYSIS OF VARIANCE(ANOVA)

Source of Variation	**DOF**	**Sum of Squares**	**% contribution**
H	2	1581.79	39.49
W	2	1051.38	26.25
A	2	199.25	4.97
La	2	209.98	5.24
Lb	2	494.51	12.35
SW	2	126.03	3.15
A X *La*	4	107.17	2.68
A X *Lb*	4	92.40	2.31
La X *Lb*	4	66.03	1.65
error	2	77.14	1.93
Total	26	4005.68	

A. *Model Adequacy Checking*

Before the conclusions from the analysis of variance are adopted, the adequacy of the underlying model should be checked (Montgomery, 2006). For this purpose the residual analysis is used as the primary diagnostic tool in the present study. Fig. 4 shows the normal probability plot of standardized residuals for frequency. This plot concludes that there are only two observations which have standardized residual more than 2, rest of the residuals are well within the acceptable limits. Hence, the model is termed as adequate for present case.

IV. REGRESSION ANALYSIS

There is a strong interplay between design of experiments and regression analysis (Montgomery, 2006). Regression analysis is a statistical technique for modeling and investigating the relationship between two or more variables. Regression models are categorized depending upon number of independent variables or regressors in it. If the regressor is only one, then it is termed as a simple linear regression model and if regressors are two or more, then it is termed as a multiple regression model [1]. So for the present case multiple regression analysis is done as there are six regressors, namely *A*, *La*, *Lb*, *H*, *W* and *SW*.

The linear regression equation to predict natural frequency (*f*) is modeled as

$$F = \beta_0 + \beta_1 A + \beta_2 La + \beta_3 Lb + \beta_4 H + \beta_5 W + \beta_6 SW \quad (5)$$

Where $\beta_0, \beta_1, \beta_2, \beta_3, \beta_4, \beta_5$ and β_6 are called regression coefficients which are calculated by database provided to MINITAB© software by performing experiments in ANSYS Mechanical©. Database contains results from Table II. The coefficients of regression equation are obtained as

$\beta_0 = 116$, $\beta_1 = -0.145$, $\beta_2 = -0.0408$, $\beta_3 = -0.0649$, $\beta_4 = -0.104$, $\beta_5 = -0.0846$ and $\beta_6 = -0.105$.

Regression model has R square value of 90.4%. All interactions are having p-value more than 0.05. Hence they are neglected in regression equation and regression equation is formed by considering all the factors having p-value less than 0.05. For validating the results obtained from regression model the natural frequencies for random values within the database limits are calculated by using the regression equation and are compared with values obtained from ANSYS Mechanical©. The errors are reported in Table IV. Fig. 5 gives a graphical comparison of frequency values predicted by regression model and

ANSYS Mechanical©. It can be seen that the natural frequencies obtained from ANSYS Mechanical© and the regression equation are in close agreement.

TABLE IV : COMPARISON OF FREQUENCY VALUES BETWEEN REGRESSION MODEL AND ANSYS©

Sr. No.	*A*	*La*	*Lb*	*H*	*W*	*SW*	*F*, Hz (Regression model)	*F*, Hz (ANSYS)	Error (Hz)	% Error
1	40	216.8	172.6	200	200	100	41.93	43.3	1.37	3.16
2	54	276.8	120	120	140	100	54.26	53.22	-1.05	-1.97
3	25	186	223	168	133	60	55.29	55.05	-0.24	-0.44
4	55	230	160	150	196	80	47.68	46.70	-0.97	-2.08
5	45	130	255	260	230	55	35.35	34.77	-0.58	-1.67

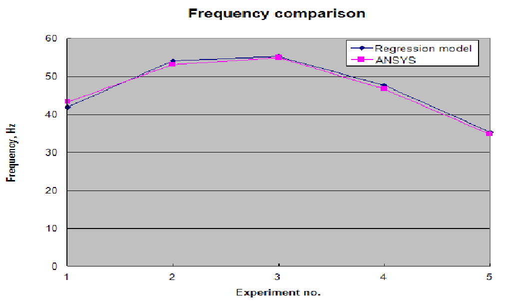

Fig. 5 : Frequency comparison between Regression model and ANSYS©

V. CONCLUSION

In the current study, two statistical techniques namely ANOVA and Regression analysis are performed for a discharge pipe of an air conditioning unit. The results obtained from ANOVA suggest that loop height, loop width and length *Lb* contribute 39.49%, 26.25% and 12.35% respectively in the total variation of natural frequency of pipe for the first mode. Together these three factors account for 78.09% of total variation in fundamental natural frequency. Hence these three parameters are critical in deciding the natural frequency of the pipe. It is also concluded from the results of ANOVA that contributions of interactions between angle, *La* and *Lb* are very less. Further, the linear regression equation determined from the regression model is found to give reasonably accurate natural frequencies for random values of different parameters within the database limit. Maximum error reported is 3.16%. Hence it can be concluded that a linear equation is sufficient for the present case and there is no need for higher order regression equation. The present study thus can be extended for determining the subsequent natural frequencies for complete tube analysis in the context of free vibration.

VI. ACKNOWLEDGMENT

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