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# APPROXIMATION OF HYSTERESIS DENSITY FUNCTION IN STRETCH SENSOR™

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**Abstract**— Stretch Sensor™ developed by Images Scientific Instruments Inc, USA, is a unique polymer component that changes resistance when stretched. When sensor is stretched and released it exhibits hysteresis and large relaxation time. For identification of hysteresis and relaxation, Preisach model is a very well-known method. Experiments are carried out using tension tester and the experimental data is used for identification. Modified Preisach model is used for relaxation identification and the experimental data is discretized for analysis of relaxation. Identification is based on first reversal curve of major hysteresis loop and noise error of sensor. It has been observed that if sensor is used in pre-stretch conditions, relaxation time is reduced. Also more the iterations of stretch, hysteresis is reduced and sensor output error is also reduced. Hysteresis and relaxation time cannot be eliminated because they are inherent properties of polymer but can be compensated under specific conditions. Compensation is useful for calibration of the sensor.

**Index Terms** — Hysteresis, Identification procedure for parameters, Preisach model, Relaxation.

## I. INTRODUCTION

The stretch sensor used for experimentation is a unique polymer component that changes resistance when stretched. The stretch sensor exhibits hysteresis and relaxation which is inherent property of sensor. The sensor shows hysteresis and relaxation in the variation of electrical properties with mechanical displacement. It makes the calibration of these materials difficult. Experiments are carried out on conductive polymer stretch sensors to study the variation of electrical resistance with displacement. A mathematical model, based on the modified the Preisach [1] approaches, has been used to model the variation of electrical resistance with displacement in a conductive polymer. After that, compensators based on the modified Preisach model, developed by many researchers can be used. The compensator removes the effect of hysteresis and relaxation from the output obtained from the conductive polymer sensor. This helps in calibrating the material for its use in stretch sensing.

## II. THE STRETCH SENSOR™

Stretch Sensor™ developed by Images Scientific Instruments Inc., USA [2], is a unique polymer component that changes resistance when stretched. An unstretched sensor has a nominal resistance of 1.0 KΩ per linear inch. As the stretch sensor is stretched the resistance gradually increases. When the sensor is stretched 50%, its resistance will approximately double to 2.0 KΩ per inch. The sensor is a flexible cylindrical cord 0.060 – 0.070 inch in diameter. Recommended operating range is 40-50% elongation for repeatable operation. The stretch sensor has a few resistive artifacts. When stretched into position and released, the resistance may increase slightly upon release, before decaying to its resting resistive value. The decay of the resistive value to its resting value

takes place over time. The initial release will typically bring the resistance value down to approximately +10% of its initial resting value. Resistive value continues to decay to its nominal resting value, see Fig. 1.

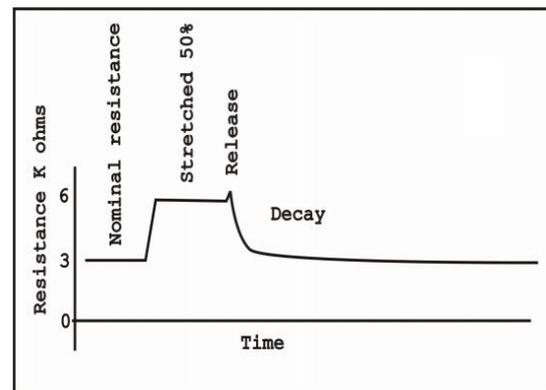


Fig. 1: Stretch characteristic of sensor

Stretch sensor is used in applications like Virtual Reality gloves and suits, robotics, wearable clothes, breadth sensing, and more [3].

## III. EXPERIMENTAL SETUP AND DATA

Conductivity tests are carried out on the conductive polymer sensor to study the variation of its electrical resistance with displacement. Potential divider type conductive test using Wheatstone Bridge is used.

### A. Tension Tester

To conduct the tests, a tension tester is developed. A tension tester is a mechanical setup to create tension or to elongate a material under test. The tension tester developed is an angular type tension creator, which creates an angular tension in the Stretch Sensor. Experimental setup used for this test is shown in Figure 2.

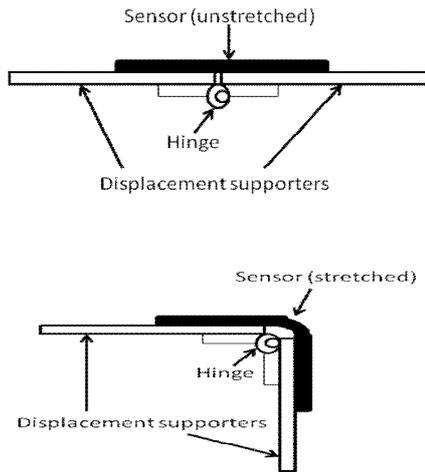


Fig. 2: Tension tester

The experimental setup has two non-conducting displacement supporters on which the Stretch Sensor is mounted. The two displacement supporters are coupled with a hinge so that one of the supporters can be rotated around the hinge to create angular displacement.

Details of the experimental setup are shown in Fig. 3 and Fig. 4.

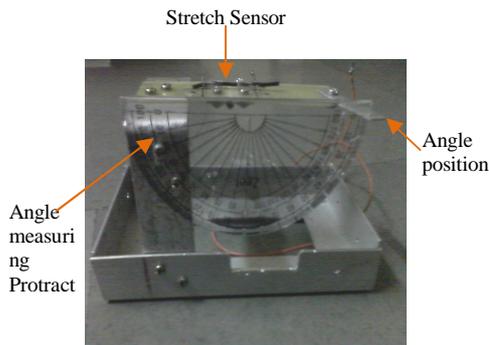


Fig.3: Experimental setup (front view)

Using this setup, angular displacement is applied to the sensor and its resistance is measured at different angles of stretch.

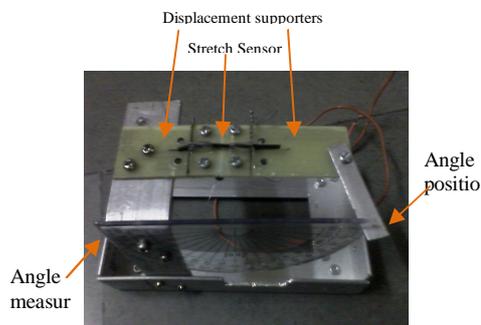


Fig. 4: Experimental setup (top view)

To measure the angle of displacement; a transparent angle measuring protractor is mounted as shown in Fig. 3. The edge of the protractor is notched at 10° each between 0° to 90°. On the rotating supporter an angle positioner is hinged, so as to lock that the

positioner into the notches. The arrangement to lock the positioner in the notch is to have fixed angle rotation for repetitive measurement.

The two ends of the Stretch Sensor are connected to a Wheatstone Bridge. The Wheatstone bridge gives differential voltage output corresponding to the resistance of the portion of the stretch material.

As the output of the bridge is differential, we have to convert it single ended signal for measurement purpose. The conversion is carried out by using op-amp based differential amplifier.

*B. Experimental data*

Angular displacement is applied to the sensor so as to stretch the sensor in steps of 10° starting from 0° to 90° and vice versa, as shown in Fig.5. The corresponding change in voltage was measurement at each step.

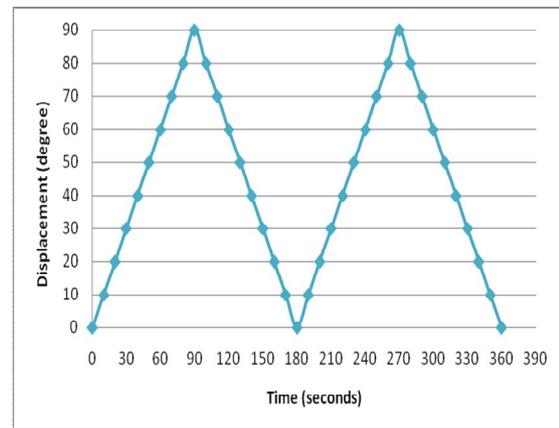


Figure 5: Displacement vs. Time

The displacements were recorded relative to an extended position, not returning to this rest position after each required displacement. Five samples were averaged at each point to improve noise immunity. A reading is taken at a reference / relative position and then at the displacement position. These values are plotted on the y axis and applied displacement is on x axis. The units are linearly related to change in resistance but the exact correlation depending upon the setup of the Wheatstone bridge and Amplifier.

Displacements were made in steps of 10° from the extended end, e.g. 10°, 20°, 30°, 40°, 50°, 60°, 70°, 80° and 90° in forward direction, shown as ▶ in Fig. 6, after reaching 90°, displacement in reverse direction is made, shown as ■ in Fig. 6. Plot of only two sets of reading is shown.

The Table I, shows readings from experimental tests with input angular displacement applied

continuously, i.e. without any time delay between in each step of input.

Table I: Readings from experimental tests with no time delay

Degree	Set 1		Set 2		Set 3	
	$\alpha 1$	$\beta 1$	$\alpha 2$	$\beta 2$	$\alpha 3$	$\beta 3$
0	1.26	1.87	1.87	2.5	2.5	2.7
10	1.73	2.14	2.2	2.6	2.67	2.75
20	2.07	2.27	2.35	2.65	2.76	2.81
30	2.32	2.44	2.5	2.7	2.8	2.89
40	2.52	2.57	2.65	2.75	2.85	3.1
50	2.63	2.65	2.8	2.83	2.9	3.24
60	2.75	2.77	3	3.02	2.97	3.3
70	2.85	2.88	3.17	3.19	3.2	3.36
80	2.92	2.9	3.27	3.29	3.31	3.4
90	3	3	3.4	3.4	3.46	3.46

Plot of two sets of reading is shown in Figure 6.

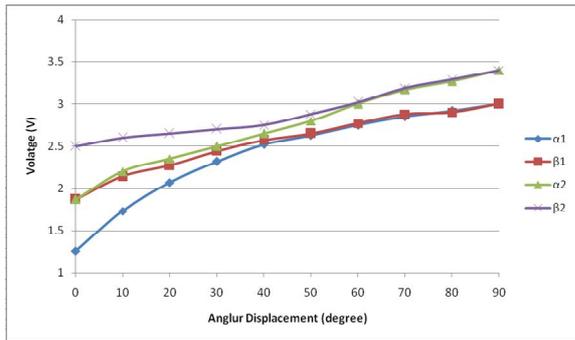


Fig. 6: Plot of Output voltage vs. Angular displacement with no time delay

Next displacements are made in steps of 10<sup>0</sup> but with time delay of 10 seconds between each step of input. Fig. 7 shows plot of only two sets of reading.

Table II: Readings from experimental tests with time delay of 10 seconds between steps of input

Degree	Set 4		Set 5		Set 6	
	$\alpha 6$	$\beta 6$	$\alpha 7$	$\beta 7$	$\alpha 8$	$\beta 8$
0	1.30	1.85	1.45	1.79	1.37	1.63
10	1.76	1.99	1.76	1.98	1.62	1.87
20	2.09	2.18	2.07	2.13	1.94	2.00
30	2.30	2.31	2.25	2.26	2.14	2.22
40	2.46	2.49	2.40	2.46	2.34	2.36
50	2.5	2.6	2.5	2.57	2.4	2.4

	9		5		3	7
60	2.66	2.7	2.65	2.68	2.55	2.59
70	2.85	2.81	2.74	2.78	2.65	2.69
80	2.88	2.89	2.80	2.86	2.74	2.78
90	2.92	2.92	2.98	2.98	2.82	2.82

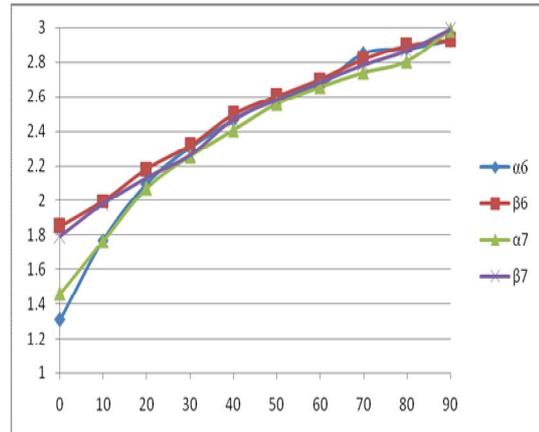


Fig. 7: Plot of Output voltage vs. Angular displacement with time delay of 10 seconds between steps of input

Next the input or displacement was kept constant and reading of voltage was noted for 10 seconds, readings shown in Table II. Fig. 8 shows the plotted graph of the output with displacement kept constant at 90<sup>0</sup>.

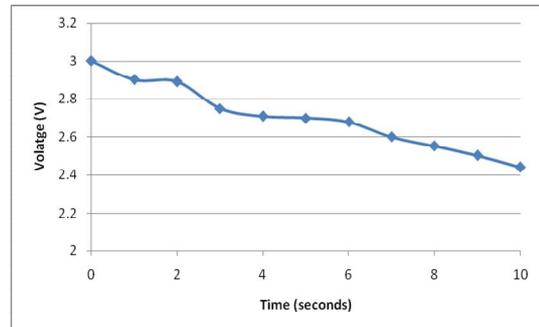


Fig. 8: Plot of relaxation when input constant

Table III: Readings for constant input

Time (seconds)	Volts
0	3
1	2.9
2	2.89
3	2.75
4	2.71
5	2.7
6	2.68

7	2.6
8	2.55
9	2.5
10	2.44

### C. Observations

1. Fig. 6 and Fig. 7 shows that electrical voltage when plotted against angular displacement gives rise to an anti-clockwise hysteresis loop.
2. Width of the loop does not necessarily remain constant with number of cycles.
3. The loop moves upward with number of cycles.
4. Fig. 8 shows that when the input is kept constant the value of voltage relaxes.

Using the above data, mathematical models based on classical Preisach and modified Preisach approaches are used.

## IV. MATHEMATICAL MODEL FOR HYSTERESIS AND RELAXATION

The stretch sensor exhibits hysteresis and large relaxation time and residue which are inherent property of sensor, so Mathematical Models for Hysteresis and Relaxation is used to identify the properties / parameters of sensor.

Hysteresis represents a property of systems that show dependence on input history applied to it i.e. output at any time instant depends on the input applied to the system both at the present time as well as the previous history; consequently same instantaneous value of the input can give different outputs depending on the entire input history. Variation of the output with cyclic input in a system with hysteresis gives rise to a loop, called 'hysteresis loop'. Hysteretic phenomena are encountered in various branches of engineering.

Different mathematical models have been developed, depending on the type of hysteretic behavior, and successfully applied in fields of magnetism, tactile sensors, etc. Hysteresis models are either physics based or empirical. Physics based models, being mainly microscopic and semi-microscopic, involve application of an energy principle and thermodynamic, electromagnetic or other laws depending on the behavior at the grain level [4]. The main disadvantage of such models is the requirement of a large number of material parameters. To develop a mathematical model for the identification of stretch sensor, empirical models was of more interest because it requires less number of parameters and are suitable for practical applications. Therefore review of the existing hysteresis models is done, namely the Preisach model.

Preisach model was first suggested in 1935 by Ferenc (Franz) Preisach in the German academic journal "Zeitschrift für Physik" [1], a well-known model to represent path-dependent behavior of magnetic materials based on some hypothesis concerning the physical mechanisms of magnetism. In this model, the output from a system with hysteresis is considered as a weighted combination of the outputs from a number of elementary hysteresis operators.

The existing path-dependent Preisach model considers the relaxation parameters to be dependent upon the parameters controlling the static hysteresis. This increases computational complexity in the identification of hysteresis and relaxation. Here, a modified dynamic Preisach model which considers the static hysteresis and relaxation to be independent phenomenon is used.

## V. THE PREISACH MODEL

### A. The Classical Preisach Model

Classical Preisach model considers the hysteresis loop to be a weighted combination of output from different independent hysteresis operators. Each independent hysteresis operator  $\gamma_{\alpha\beta}$  is a mechanical unit that up switches to a value +1 when the value of input  $u(t)$  is  $\alpha$  and is increasing and downswitches to a value -1 when the value of input  $u(t)$  is  $\beta$  and is decreasing. Variables  $\alpha$  and  $\beta$  are called the upswitching and downswitching values respectively. It will be assumed subsequently that  $\alpha \geq \beta$ , which is quite natural from the physical point of view. Fig. 9 shows the behavior of  $\gamma_{\alpha\beta}$ . The ascending branch  $abcde$  is followed when the input increases monotonically and the descending branch  $edfba$  is followed when the input decreases monotonically. Each operator  $\gamma_{\alpha\beta}$  has a weight  $\mu(\alpha\beta)$  associated with it.

According to the Preisach model output  $f(t)$  is given by Eq. 6.1

$$f(u(t)) = \int_T \gamma_{\alpha\beta} \mu(\alpha, \beta) d\alpha d\beta = \int_{T_1} \mu(\alpha, \beta) d\alpha d\beta - \int_{T_2} \mu(\alpha, \beta) d\alpha d\beta \quad (1)$$

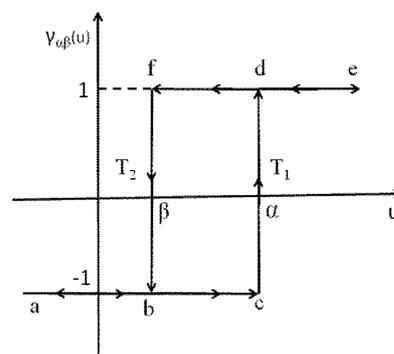


Fig. 9: Hysteresis Operator ( $\gamma_{\alpha\beta}(u)$ ) in Preisach model

Fig. 10 show the geometric interpretation of the Preisach model. The space spanned by  $\alpha$  and  $\beta$  is called Preisach-Mayergoyz (PM) [5] space.  $\Delta ABC$  is the region where  $\alpha > \beta$ . This is the feasible region of PM space.

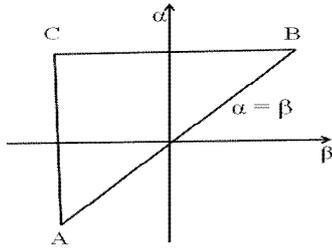


Fig. 10: Preisach-Mayergoyz space

### B. Modification of the Hysteresis Operator

In stretch sensor case, both the input displacement and the output electrical resistance and voltage, always remain positive. So,  $\alpha$ ,  $\beta$  and output from each operator  $\gamma_{\alpha\beta}$  always remain positive. Hence, in the downswitched state, output from each operator is zero. The hysteresis operator considered in the model is shown in Fig. 11. Here, at  $\alpha$ , the operator upswitches by a value +1. But at  $\beta$  it does not downswitch fully. It downswitches by a value  $\eta$  which is an additional parameter associated with each hysteresis operator [6]. Due to this partial downswitching at  $\beta$ , the loop keeps shifting upward with the number of cycles as shown in Fig. 11. If the value of  $\eta$  is more than 1 the loop will shift downward.

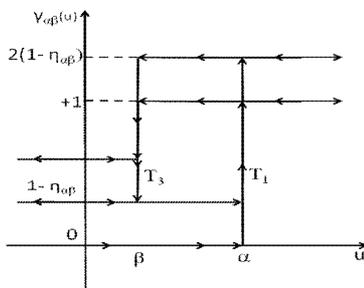


Fig. 11: Modified Hysteresis Operator

As the operators do not downswitch fully, at any time instant, output of an operator at partially downswitched state after  $n$  number of partial downswitchings will be  $n(1 - \eta)$  and of an operator at on stage after  $n$  number of partial downswitchings will be  $n(1 - \eta) + 1$ . Hence, static part of the output of the system  $f_s$  at any time  $t$  can be written as:

$$f_s(u(t)) = \int_{T_1} [n(\alpha, \beta)(1 - \eta(\alpha, \beta)) + 1] \mu(\alpha, \beta) d\alpha d\beta + \int_{T_3} n(\alpha, \beta)(1 - \eta(\alpha, \beta)) \mu(\alpha, \beta) d\alpha d\beta \quad (2)$$

Upward shifting of the hysteresis loop has been taken care of by a modification of the hysteresis operators,  $\eta$ .

The hysteresis operators described above cannot

model a relaxation of the output at a constant input. Hence the relaxation is accounted for by the addition of a dynamic relaxation operator [6].

### C. Addition of a Dynamic Hysteresis Operator

The relaxation has upswitching value  $\zeta$ , but no downswitching value. When the input monotonically increases and reaches a value  $\zeta$ , an operator  $h_\zeta$  upswitches to a value  $\sum_{j=1}^P A_j$  and then decays exponentially as given in Eq. 3. Operator  $h_\zeta$  is called as Dynamic relaxation operator [6]. Here  $A_j(\zeta)$ ,  $b_j(\zeta)$ ,  $c_j(\zeta)$  ( $j = 1, \dots, P$ ) are unknown parameter associated with each dynamic operator. These need to be determined from experimental data a known bivariate probability density function [15].

$$h_\zeta(t) = \begin{cases} 0 & \text{if } t < t_0 \\ \sum_{j=1}^P A_j e^{-b_j(t-t_0)c_j} & \text{if } t \geq t_0 \end{cases} \quad (3)$$

For a given input history at time  $t$ , the dynamic part of the output will be given by Eq. 6.13.

$$f_d(t) = \int_{T_d} \sum_{i=1}^{n_\zeta} \sum_{j=1}^P A_j e^{-b_j(t-t_{\zeta_i})c_j} d\zeta \quad (4)$$

Here,  $T_d$  refers to the domain where the operators are in the upswitched state. The shape and size of  $T_d$  depends upon the input history. Here,  $n_\zeta$  refers to the total number of times  $h_\zeta$  has been upswitched and  $t_{\zeta_i}$  refers to the time instant at which  $h_\zeta$  was upswitched for the  $i$ th time. Total value of the output from the system is the combination of static and dynamic part of the output obtained from Eq. 2 and Eq. 4.

$$y = f_s + f_d \quad (5)$$

### D. Identification of Hysteresis Parameters

The stretch sensor parameters can be found out by minimizing the error of the output predicted by the mathematical model with respect to the experimental data.

An error function can be defined as:

$$E = \sum_{i=1}^N (V_e(i) - V_c(i))^2 \quad (6)$$

Here,  $V_e(i)$  is the electrical voltage obtained from the experiment and  $V_c(i)$  is the electrical voltage obtained from Eq.6. By minimizing this error, the sensor parameters can be found out.

Evaluating Eq. 6, requires the numerical evaluation of double integrals, which is time-consuming and impractical. Second, the determination of the weight function  $\mu(\alpha, \beta)$  requires differentiations of experimental data. These differentiations may

strongly amplify errors (noise) inherently present in the experimental data.

Therefore another approach of Preisach model by numerical implementation is considered. It is based on the explicit formula for the integrals. This formula is able to involve the weighting function without any differentiation of the experimental data.

For the hysteresis operators, PM space is discretized from set of first-order reversal curves of hysteresis loop [7]. A square mesh covering the limiting triangle is created as shown in Figure 12.

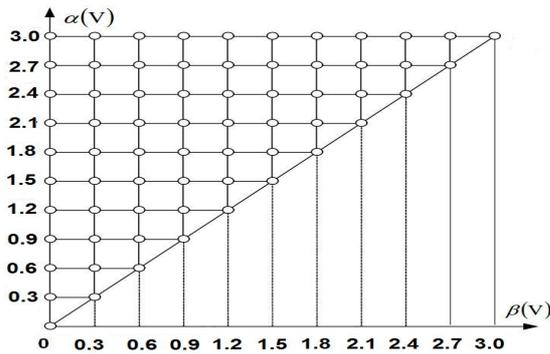


Fig. 12: Mesh of discrete outputs of Preisach model

In the mesh, a finite number of Preisach function is stored. The limiting triangle is separated to a discrete area for a pre-defined interval [8] and values at the mesh nodal points are acquired in a look-up table [7].

To obtain the value of the points inside a mesh, interpolation functions are used, which is calculated from the known values of the four corner nodal points of the cell.

Dynamic relaxation is also accommodated in discretization by adding a “pseudo” column illustrated with dashed line is added to the left side of the  $\alpha(V)$  coordinate [7], as shown in Fig. 13.

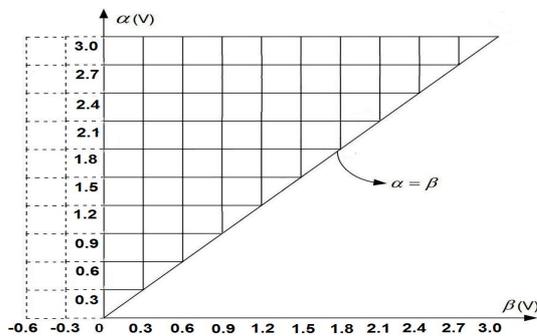


Fig. 13: Limiting triangle with a “pseudo” column

The input sequence (after discretization) is fed into the discretized Preisach operator and the state of each operator [9], is computed, shown in Figure 14.

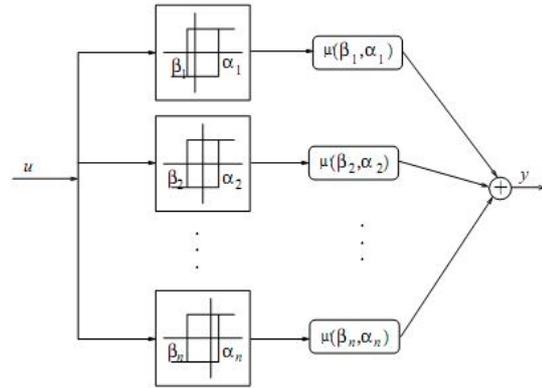


Fig. 14: The discretized Preisach operator

The output of the Preisach model at time instant  $n$  is expressed as:

$$y = \sum_{k=1}^K \mu(\beta_n, \alpha_n) \gamma_{\alpha\beta k}(n) \quad (7)$$

## VI. DATA, CALCULATIONS AND RESULTS

Parameter identification is performed by using data from Table I and calculations based on the equations 2, 4 and 5; for static and dynamic parts of the Preisach model.

Static part of the Preisach model is calculated using Eq.

Using values from Table 5.1, Set 1:

$$\alpha_1 = 1.26$$

$$\beta_1 = 1.87$$

$$\eta = \alpha_1 - \beta_1 = 0.61$$

$\mu(\alpha_1, \beta_1)$  is calculated using bivariate density function [10]

$$\mu(\alpha_1, \beta_1) = 7.59$$

$$f_s = 13.51$$

Dynamic part of the Preisach model is calculated using Eq.

$$f_d = -11.8034$$

Total system output,  $y = f_s + f_d$  (5)

Therefore,

$$y = 1.7066$$

Error was calculated between experimental data and calculated data using Eq.

$$E = 1.87 - 1.7066 = 0.1634 \text{ volts} \approx 8.6^0$$

Corresponding errors to experimental data have been tabulated in Table IV.

Table IV: Results

Data set no.	Error in %
1	10.4
2	8.4
3	8.1
4	7.2
5	7.1
6	6.8
7	6.5
8	6.2
9	6.1
10	5.9

As the number of iterations of reading is increased the error is reduced. If stretch sensor is used in pre-stretch condition, relaxation error will reduce. First cycle of operation of the sensor shows large error but as operation cycles goes on increasing, the error is reduced by almost 4%.

Then by predicting the dynamic hysteresis operator, relaxation time can be identified upon. Calculations using this dynamic hysteresis operator can be utilized to analyze the relaxation time and accordingly the error is added in the reading of the sensor and can be used for compensation of these errors [11].

As data of the sensor is prone to noise and human error in taking reading, hence discretization of the data is very useful to reduce noise effect and normalize the data. Addition of dynamic hysteresis operator is eased by discretization.

## VII. CONCLUSION

Experiments have been carried out on conductive polymer stretch sensor to identify parameters of stretch sensor. Phenomena of hysteresis and relaxation have been observed in the variation of electrical resistance with angular displacement in conductive polymer.

The dynamic hysteresis operator and relaxation time based on the modified Preisach model is used in identification for the effect of hysteresis and relaxation. Due to hysteresis and large relaxation time, the stretch sensor cannot be used in critical applications.



The size of square mesh of PM space can be reduced further, to get better dynamic hysteresis operator identification, so that relaxation residue can be predicted more accurately. In this experiment, only the major loop and the first reversal curve of hysteresis loop has been considered. If the minor loops of hysteresis are also considered, then the more detailed identification of parameters of the sensor can be done and error estimation can be further processed more accurately.

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