

July 2013

## Compressed Sensing for Image Compression Using Wavelet Packet Analysis

Kanike Vijay Kumar

*Department of ECE, G. Pulla Reddy, Engineering college, Kurnool, A. P., India, vijay.kanike1987@gmail.com*

K. Suresh Reddy

*Department of ECE, G. Pulla Reddy, Engineering college, Kurnool, A. P., India, sureshkatam@gmail.com*

Follow this and additional works at: <https://www.interscience.in/ijcsi>



Part of the [Computer Engineering Commons](#), [Information Security Commons](#), and the [Systems and Communications Commons](#)

---

### Recommended Citation

Kumar, Kanike Vijay and Reddy, K. Suresh (2013) "Compressed Sensing for Image Compression Using Wavelet Packet Analysis," *International Journal of Computer Science and Informatics*: Vol. 3 : Iss. 1 , Article 3.

Available at: <https://www.interscience.in/ijcsi/vol3/iss1/3>

This Article is brought to you for free and open access by Interscience Research Network. It has been accepted for inclusion in International Journal of Computer Science and Informatics by an authorized editor of Interscience Research Network. For more information, please contact [sritampatnaik@gmail.com](mailto:sritampatnaik@gmail.com).

# Compressed Sensing for Image Compression

## Using Wavelet Packet Analysis

**Kanike Vijay Kumar & K. Suresh Reddy**

Department of ECE, G. Pulla Reddy, Engineering college, Kurnool, A. P., India  
E-mail : vijay.kanike1987@gmail.com, sureshkatam@gmail.com

---

**Abstract** - Compressed sensing is a recently developed technique that exploits the sparsity of naturally occurring signals and images to reduce the volume of the data using less number of samples, computing the sparsity of the signal. In the traditional/conventional approaches the images are acquired and compressed, where as compressed sensing aims to acquire the “compressed signals” with few numbers of samples and reconstruct the images. This will allow us to acquire the large ground/region with few numbers of input samples. This technique works on the assumption that natural signals/images have inherent sparsity. . In this algorithm, the original image is first decomposes with the wavelet packet to make it sparse, and then retains the low frequency coefficients in line with the optimal basis of the wavelet packet, meanwhile, makes random measurements of all the high frequency coefficients according to the compressed sensing theory, and last restores them with the orthogonal matching pursuit (OMP) method, and does the inverse transform of the wavelet packet to reconstruct the original image, to achieve the image compression.

**Keywords**- *image compression; CS; wavelet packet; optimal basis.*

---

### I. INTRODUCTION

In the traditional signal processing, the signal conversion from analog to digital has been strictly in compliance with the Nyquist sampling theorem, which indicates that the sampling rate must at least reach twice greater than the signal bandwidth in order to accurately reconstruct the original signal. With the increasing capacity of sensor systems for data, the growing amount of data needs to be handled, which presents higher requirements for the ability of signal processing, and brings great challenges to the corresponding hardware devices. In practice, signals are often sampled at a high rate and compressed afterwards so as to reduce the costs of storage, processing and Transmission, which wastes a lot of sampling resources. The theory makes full use of the signal sparsity or compressibility to fulfill the signal acquisition, encoding and decoding. CS indicates that if a signal is sparse or compressible in a certain transform domain, the signal can be exactly or approximately reconstructed by acquiring a small amount of the signal projection values, that is, the signal sampling and compression encoding are achieved in the same step, with non-adaptive measurement encoding on the signal at a rate far below the Nyquist sampling rate, converting from sampling the signal itself to sampling the information contained in the signal. The signal codec framework based on CS is quite different to that based

on traditional theories, which greatly reduces the signal sampling rate, signal processing time, data storage and transmission costs, leading signal processing into a new revolutionary era. Wavelet analysis of good time-frequency characteristics widely used in the image compression field has become one of the mainstream technologies, which has the high decorrelation and energy compression efficiency, and can effectively remove the blocking effect and mosquito noise. However, wavelet transform only decomposes the low-frequency sub-band level by level, which is prone to loss of the image detailed information with a high compression ratio, deteriorating the reconstructed image quality. Wavelet packet analysis is an extension of wavelet analysis. Compared with wavelet transform, wavelet packet transform provides a more sophisticated analysis method for the signal frequency band, which is partitioned with multilevel divisions. High-frequency sub-bands without subdivisions in the multi-resolution analysis are further decomposed to increase the frequency resolution. Wavelet packet analysis can also adaptively select the appropriate frequency band according to the characteristics of the analyzed signal to match the signal so as to increase the time-frequency resolution. The algorithm proposed in this paper combines compressed sensing theory with wavelet packet analysis, which first decomposes the original image with the wavelet packet to make it sparse, and

then retains the low-frequency coefficients in line with the optimal basis of the wavelet packet, meanwhile, makes measurement encoding on all the high-frequency coefficients in accordance with the compressed sensing theory, and last restores them with the orthogonal matching pursuit (OMP) method, and does the wavelet packet inverse transform to reconstruct the original image, to accomplish the image compression.

## II. THE ALGORITHM PRINCIPLE

### A. Compressed Sensing Theory

If  $x \in \mathbb{R}^{N \times 1}$  is a one-dimensional signal, it can be expanded by a group of orthogonal basis (e.g., wavelet packet basis)  $\Psi = \{\Psi_1, \Psi_2, \dots, \Psi_N\}$ , i.e.

$$x = \sum_{k=1}^N \Psi_k y_k = \Psi y \quad (1)$$

Where  $y_k = \langle x, \Psi_k \rangle$ , whose inverse transform is  $x = \Psi^H y$  and  $\Psi \Psi^H = \Psi^H \Psi = I$ , therein  $\Psi \in \mathbb{C}^{N \times N}$ , and  $I$  is the identity matrix. When the expansion of  $x$  only contains the number of  $K \ll N$  nonzero coefficients  $y_k$  on the basis of  $\Psi$ , define  $\Psi$  as the sparse basis of  $x$ .

Generally, the signal itself is not sparse, but its coefficients by a certain transformation can be considered sparse. For example, implement wavelet packet transform to the signal  $x$ , then retain the number of  $K$  larger components of its coefficients, while, set the other  $N - K$  components (which make much less constructions to the signal reconstruction) zeros, and execute wavelet packet inverse transform to reconstruct the approximate signal. In this manner, the signal  $x$  can be considered as of  $K$ -sparsity on the wavelet packet basis of  $\Psi$ .

Towards the signal  $x$ , project it onto a group of measurement vectors  $\Phi = \{\phi_1, \phi_2, \dots, \phi_M\}$ , and obtain its  $M$  linear measurement, i.e.

$$s = \Phi x \quad (2)$$

Where  $\Phi \in \mathbb{R}^{M \times N}$ , and each line of  $\Phi$  can be regarded as a sensor, which multiplies the signal  $x$  and picks up a part of its information. In accordance with the  $M$  measurements  $s$  and vectors  $\Phi$ , the original signal can be reconstructed.

With the substitution of Equation (1) into Equation (2), deduce the following equation

$$s = \Phi \Psi y = \Theta y \quad (3)$$

Where  $\Theta = \Phi \Psi$  is a  $M \times N$  matrix.

As stated above, CS reduces the  $N$ -dimensional signal  $x$  to the  $M$ -dimensional measurement signal  $S$ . In Equation (2), the number of unknowns  $N$  is greater than the number of equations  $M$ , therefore, the direct solution of Equation (2) to reconstruct  $X$  cannot be the define solution. However, In Equation (3),  $y$  is of  $K$ -sparsity, only containing the number of  $K$  nonzero coefficients, and  $K < M \leq N$ , thus,  $y$  can be derived from solving the inverse problem of Equation (3) by the existing sparse decomposition algorithms, and then  $x$  can be reconstructed through Equation (1).

The literature points out, in order to ensure the algorithm convergence,  $\Theta$  in Equation (3) must satisfy the RIP (restricted isometric property) criterion, that is, for any vector  $u$  of strict  $K$ -sparsity,  $\Theta$  satisfies

$$1 - \varepsilon \leq \frac{\|\Theta u\|_2}{\|u\|_2} \leq 1 + \varepsilon. \quad (4)$$

Where  $\varepsilon > 0$ . An equivalent criterion to RIP is that the measurement matrix  $\Theta$  and the sparse matrix  $\Psi$  are irrelevant to each other.

For the CS theory, its inverse process of reconstruction is to solve the following optimization issue based on the  $l_0$  norm

$$\min \|y\|_0, s.t. \Phi \Psi y = s. \quad (5)$$

But the solution of  $l_0$  norm is a NP-hard (non-deterministic polynomial) issue, so convert it to the solution of the following optimization issue based on the  $l_1$  norm

$$\min \|y\|_1, s.t. \Phi \Psi y = s. \quad (6)$$

The current commonly used methods for solving the issue above include MP (matching pursuit), OMP (orthogonal matching pursuit), CP (chaining pursuit), GP (gradient projection) and so on.

### B. Wavelet Packet Analysis

Given the orthogonal scale function  $\phi(t)$  and wavelet function  $\psi(t)$ , the scale relationship between them is described as follow

$$\left. \begin{aligned} \phi(t) &= \sqrt{2} \sum_{k \in \mathbb{Z}} h_k \phi(2t - k) \\ \psi(t) &= \sqrt{2} \sum_{k \in \mathbb{Z}} g_k \phi(2t - k) \end{aligned} \right\} \quad (7)$$

Where  $h_k$  and  $g_k$  are filter coefficients of the multi-resolution analysis.

With further extension of the two-scale equations above, the recursive relations are presented below

$$W_{2n}(t) = \sqrt{2} \sum_{k \in \mathbb{Z}} h_k W_n(2t - k) \quad (8)$$

$$W_{2n+1}(t) = \sqrt{2} \sum_{k \in \mathbb{Z}} g_k W_n(2t - k)$$

When  $n = 0$ ,  $W_0(t) = \phi(t)$ , and  $W_1(t) = \psi(t)$ . The function set  $\{W_n(t)\}$ ,  $n \in \mathbb{Z}$  defined as above is the wavelet packet determined by  $W_0(t) = \phi(t)$ . Thus, the wavelet packet  $\{W_n(t)\}$ ,  $n \in \mathbb{Z}$  containing the scale function  $W_0(t)$  and the mother wavelet  $W_1(t)$  is a function set of certain correlations.

The idea of wavelet packet tree is similar to that of wavelet decomposition, but the wavelet packet transform provides a more sophisticated and flexible analysis method for the signal frequency band, which makes decompositions on all the smooth and detailed sub-bands level-by-level, without redundancies and omissions, consequently achieving better analyses of time frequency localization to the signals containing a large number of intermediate and high frequency information. In this way, wavelet packet analysis can be designed to find the optimal description of the original signal. The 2-D wavelet packet tree with 2-level decomposition, in comparison with the wavelet tree, is shown in Fig. 1.

Image compression based on the wavelet packet tree must select a good basis function to effectively indicate the characteristics of the original image. To select an ideal wavelet packet basis, first define a cost function of the given sequence, and then find the basis that minimizes the cost function from all the wavelet packet bases. For a given sequence, the least cost means its most effective representation, and the corresponding basis is called the optimal basis of wavelet packet. This paper utilizes the Shannon entropy criterion to determine the optimal wavelet packet basis, and Shannon entropy is computed by the following formula

$$H(x) = - \sum_i |x_i|^2 \cdot \log |x_i|^2, \text{ s.t. } \log 0 = 0. \quad (9)$$

or

$$\eta(x) = - \sum_i P_i \cdot \log P_i. \quad (10)$$

Where  $P_i = |x_i|^2 / \|x\|^2$ , and  $[x_n]$  is the expansion coefficient sequence of the signal based on an orthogonal wavelet packet.

### III. THE ALGORITHM IMPLEMENTATION

Sparsity or compressibility of the signal to be processed is the premise to apply CS. In general, the signal itself is not sparse, but its coefficients by a certain transformation can be considered sparse or compressible. The algorithm proposed in this paper first decomposes the original image with the wavelet packet to make it sparse, and then carries out the non-adaptive measurement encoding on the corresponding wavelet packet coefficients on the basis of CS, to complete the image compression. The specific implementation steps of the algorithm are as follows:

- 1) Select an appropriate wavelet function and set a required decomposition level, then execute the wavelet packet foil decomposition on the original image.
- 2) Determine the optimal basis of the wavelet packet in the light of the Shannon entropy criterion.
- 3) As the main information and energy of the original image are concentrated in the low-frequency sub-band by the wavelet packet transform, which plays a very important role in the image reconstruction, all the low-frequency coefficients are compressed losslessly in order to reduce the loss of the useful information.
- 4) According to the theory of CS, select an appropriate random measurement matrix, and make measurement encoding on all the high-frequency coefficients in line with the optimal basis of the wavelet packet, and obtain the measured coefficients.
- 5) Restore all the high-frequency coefficients with the method of OMP from the measured coefficients.
- 6) Implement the wavelet packet inverse transform to all the restored low-frequency and high-frequency coefficients, and reconstruct the original image.

### IV. THE ALGORITHM SIMULATION

In accordance with the above steps, implement the algorithm simulation using MATLAB to four selected test images of much detailed information with the size of 256 X 256, as shown in Fig. 2. In the experiment, select the wavelet function 'sym8' of approximate symmetry to do the 2-level wavelet packet decomposition of the input image, and select the random Gaussian matrix obeying the (0, 1/N) distribution as the measurement matrix. As the low-frequency sub-band by the wavelet packet decomposition is the approximation of the original image at different scales, which is not considered sparse, the algorithm only proceeds to the measurement encoding on all the high-frequency coefficients of the wavelet packet decomposition, so as

to reduce the loss of the useful information. The relation curves between the number of measured coefficients and PSNR of the reconstructed image are shown in Fig. 3, which also illustrates the comparison of the algorithm proposed in this paper with the other two image compression algorithms both based on CS. In Fig. 3, 'algorithm 1' executes measurement encoding on all the coefficients of wavelet transform together, and 'algorithm 2' implements measurement encoding only on the high-frequency coefficients of wavelet transform, and 'algorithm 3' represents the algorithm proposed in this paper.

**V. RESULTS**

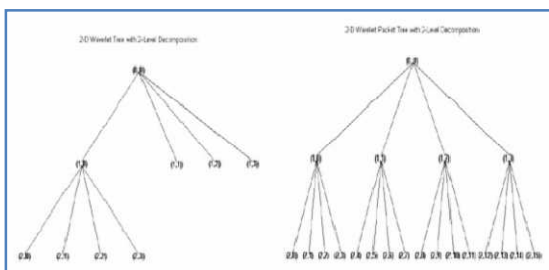


Fig.1 : (a) Wavelet tree; (b) Wavelet packet tree with full decomposition.

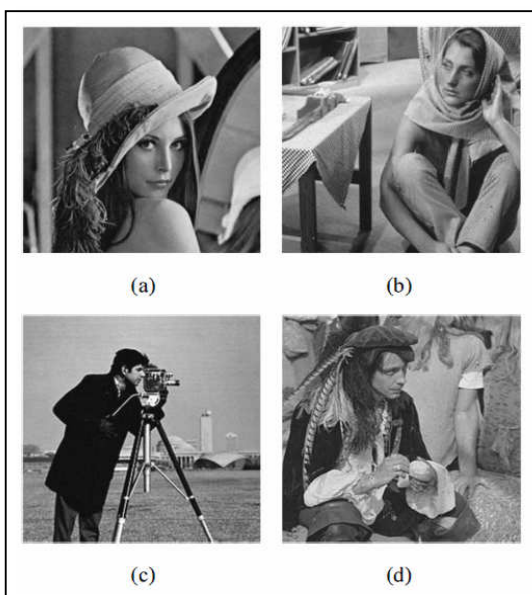


Fig. 2 : Four original test images used to simulate the algorithms:  
 (a) Lena: (b) Barbara: (c) Cameraman: (d) Pirate.

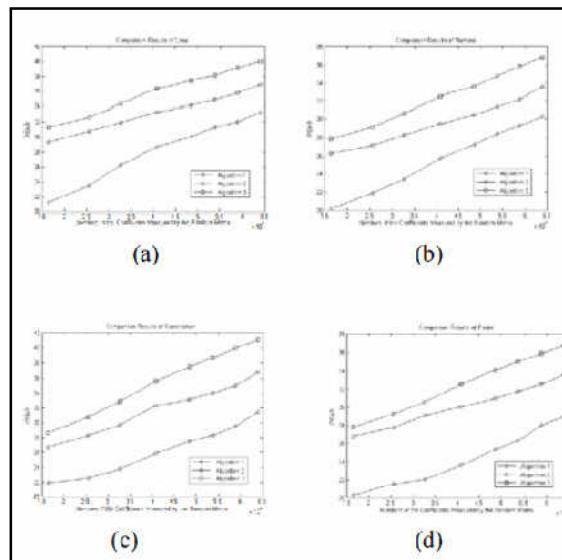


Fig. 3 : Comparison results of four test images with three algorithms: (a) comparison results of Lena; (b) comparison results of Barbara; (c) comparison results of Cameraman; (d) comparison results of Pirate.

**VI. CONCLUSION**

As seen from the simulation results above-mentioned, the algorithm proposed in this paper is of simpleness, high efficiency, and easy implementation for image compression and its performance with the better reconstructed image quality is much superior to the other two algorithms in comparison. Moreover, the selections of measurement matrix and reconstruction algorithm in the compressed sensing theory, wavelet function and determining criterion on the optimal basis in the wavelet packet analysis, and so on, all affect the algorithm performance, and better selections can further improve the algorithm performance.

**REFERENCES**

- [1] DONOHO D. Compressed sensing [J]. IEEE Transactions on Information Theory, 2006, 52(4): 1289-1306.
- [2] CANDES E. Compressive sampling[A]. Proceedings of the International Congress of Mathematicians[C]. Madrid, Spain, 2006, 3: 1433-1452.
- [3] FIGUEIREDO M A T, NOWAK R D, WRIGHT S J. Gradient projection for sparse reconstruction: Application to compressed sensing and other inverse problem [J]. Journal of Selected Topics in Signal Processing: Special Issue on Convex

- Optimization Methods for Signal Processing, 2007,1(4): 586-598.
- [4] CANDES E, ROMBERG J, TAO T. Robust uncertainty principles: Exact signal reconstruction from highly incomplete frequency information[J]. IEEE Transactions on Information Theory, 2006, 52(4): 489-509.
- [5] DONOHO D, TSAIG Y. Extensions of compressed sensing[J]. Signal Processing, 2006,86(3): 533-548.

