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## NUMERICAL STUDY ON THE SENSITIVITY ANALYSIS FOR STRENGTH AND STIFFNESS OF REGULAR CELLULAR SOLIDS

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# NUMERICAL STUDY ON THE SENSITIVITY ANALYSIS FOR STRENGTH AND STIFFNESS OF REGULAR CELLULAR SOLIDS

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**Abstract-** The aim of this study is to present a sensitivity analysis of geometric variables on the mechanical behavior of regular cellular solids. The cellular solids studied are named “spherical” and “elliptical”, if there is part of an empty sphere inside or if there is part of an empty revolved ellipse, respectively. Pure loads are applied separately, namely compression and shear, in order to study the influence of the variables on axial stress and axial displacement (compression) and shear stress and shear strain on shear. The sensitivity analysis is useful to establish limits for the studied variables in an optimization process and to know the influence of the variables on the results. In the case that a variable has little or no influence on the results; it must be evaluated if it is worth using it. In this work, the influence of two geometric variables, namely stacking and radius were studied for two types of regular cellular solids. The solids are composed by 7\*7\*7 base cells. The maximum axial stress and axial displacement were measured on compression for each model. On shear, the maximum shear stress and the maximum shear strain on the plane that is sensitive to the variation of the studied variable were taken. Three models were studied in each case. The influence of the studied geometric variables on the results are presented and discussed. It is found that all the variables have influence on the results, although in a different manner.

(Abstract)

**Keywords-** FEM; Optimization; Periodic Cellular solids; Sensitivity analysis; Optimization cellular solids

## I. INTRODUCTION

Engineers make efforts to reduce the pores from casted parts, welded joints, or coatings, because there is a common idea that pores always act as a defect. With this common thinking, it is hard to accept that metallic structures that include pores can be quite effective for structural applications. Cellular materials have advantages, such as the high specific strength, and the associated reduction of cost that is inherent to its use [1]. In fact, and as it is logical, a cellular material has a fraction of its weight in comparison with the bulk base cell. One can then achieve a reduction of material cost. The influence of geometric variables in the mechanical behavior of cellular solids is being an intensive subject of study in the last years [1]. Some research work has been published about the static behavior of cellular solids. However, most of the work is about metallic foams. Nieh et al. [2] investigated the mechanical properties of open-cell 6101 aluminum foams with different densities (5–10%) and morphologies (4–16 cells cm<sup>-1</sup>) subjected to compression loadings. The authors concluded that the relative density is the main variable controlling the modulus and yield strength of foams. It is also found that the cell shape has effect on the strength of foams. Size effects and the shear responses of aluminum foam were investigated by Rakow and Wass [3]. The authors produced foam samples and subjected them to static and quasi-static low-rate shear deformation using a loading fixture designed for the produced materials. The authors have found that, for the range of densities examined, the shear modulus shows a linear dependence on relative density, while the ultimate strength follows a non-linear power law Andrews et. al tested the

“compressive and tensile behavior of Al foams [4]. The authors compared the uniaxial compressive and tensile modulus and strength of several aluminum foams with models for cellular solids. They found that the closed cell foams have modulus and strengths that are quite lower than the model predicted values, and concluded that the discrepancy is due to micro defects which cause bending instead of stretching of the cell walls [4]. Regular cellular solids were studied by Öschner and Lamprecht [5]. They analyzed uniaxial compression behavior of regular shaped cellular metals. The authors proposed a simple model of a cellular metal for fundamental studies of the mechanical behavior and found that, on the numerical simulation of the macroscopic behavior of cellular metals based on unit cells, the use of periodic boundary conditions provides a dramatic reduction of CPU time and a correct representation of the mechanical behavior.

## II. NUMERICAL PROCEDURE

The models were built on Autodesk Inventor, and then imported into a commercial Finite Element Method program ANSYS 13 in order to perform the Finite Element Method (FEM) analysis. The studied models are of two types:

- A square base cell with an empty sphere inside in which the projections of the holes on the faces are circles in every face, and known as spherical (fig. 1 left)
- Extruded rectangle with an empty revolved ellipse inside, in which the projection of the holes on the faces of the base cell are elliptic in 4 faces and round in 2 faces), and known as elliptical (fig. 1 right)

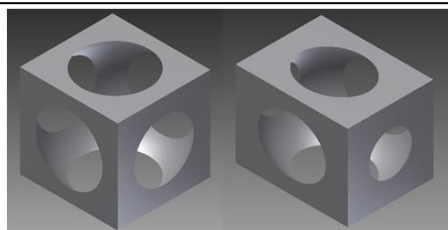


Fig.1– Two types of the considered base cells: spherical (left) and elliptic (right)

For both Spherical and Elliptical, one considered stacking and radius/ratio variation. For Stacking, one considered the variation of the base cell in length, keeping the radius or ratio constant, if it's spherical or elliptical, respectively. For Radius/Ratio, one considered the variation of the base cell in length, in order to keep the stacking constant and changing radius/ratio, depending if it's spherical or elliptic. Only one dimension of the ellipse was changed from one model to another in order to change the ratio. The considered ratio is equal to the higher radius divided by the lower radius. For both cases, the models have different mass, and the results are presented by mass unit. One can see the variation of the base cell in the three models in each case for elliptic ratio (Fig. 2) and elliptic stacking (Fig 3). The models are named a,b and c.

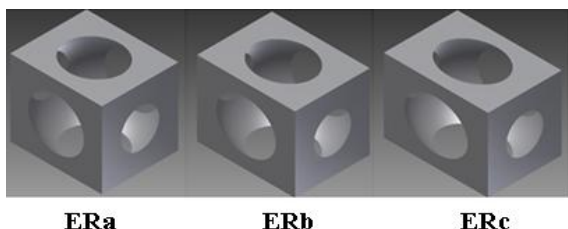


Fig.2 – variation of the base cell for the three models on Elliptic Ratio (ER)

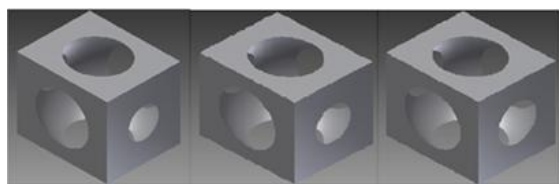


Fig.3 – variation of the base cell for the three models on Elliptic Stacking (ES)

One can see the variation of the base cell in the three models in each case for spherical radius (Fig.4) and spherical stacking (Fig. 5)



Fig.4 – variation of the base cell for the three models on Spherical Radius (SR)

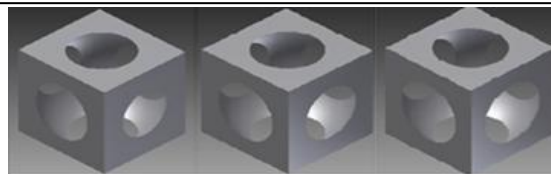


Fig.5 – variation of the base cell for the three models on Spherical Stacking (SS)

The next table (table 1), shows the value of stacking and radius used on the FEM models for spherical cells and the value of stacking and ratio used on the FEM models for elliptic cells.

Table 1 -Value of the geometric variables for the studied cells

Model: Elliptical				
Model	a	b	c	
Stacking ES	8/Pi	9/Pi	10/pi	(mm)
Ratio ER	1,1	1,2	1,3	
Model :Spherical				
Model	a	b	c	
Stacking SS	5,9	6,5	7,1	(mm)
Radius SR	7	8	9	(mm)

The base cell dimensions for elliptic are shown in table 2:

Table 2- Base cell dimensions for elliptic ratio and elliptic stacking

Elliptic ratio (ER)			
Model	Era	ERb	ERc
Mass(kg)	0,341	0,433	0,456
Height h (mm)	10	10	10
Length L (mm)	12	13,1	14,17
Stacking (mm)	8/Pi	8/Pi	8/Pi
Elliptic stacking (ES)			
Model	ESa	ESb	ESc
Mass(kg)	0,438	0,426	0,341
Height h (mm)	10	10	10
Length L (mm)	12,4	12,25	12
Ratio	1,1	1,1	1,1

The base cell dimensions for spherical are shown in table 3:

Table 3- Base cell dimensions for spherical radius and spherical stacking

Spherical radius (SR)			
Model	SRa	SRb	SRc
Mass(kg)	0,43	0,404	0,382
Height h (mm)	11	11	11
Length L (mm)	11	11,6	12,3
Stacking (mm)	7	7	7
Spherical stacking (SS)			
Model	SSa	SSb	SSc
Mass(kg)	0,483	0,456	0,43
Height h (mm)	11	11	11
Length L (mm)	11,65	11,325	11
Radius (mm)	7	7	7

The Fig. 6 shows an example of the base cell dimensions for spherical

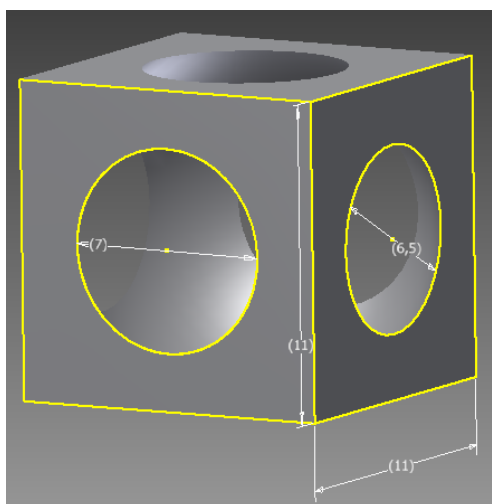


Fig. 6- Base cell dimensions for SSC

The models consist of 7\*7\*7 base cells. In order to load the model uniformly, two plates were added to the model on each side. The model is loaded on one plate, and constrained on the opposite plate outer area. The loads applied were axial compression (load direction perpendicular to an area) and shear (load direction parallel to the area). Both stresses (normal stress in compression and shear stress in shear) and displacements in compression and shear strain in shear were studied. For compression and for ratio and radius study on shear, the loads were applied on the sides of the model, as in the fig. 7, left. For Stacking, in the case of shear, the plates are located on the top and bottom of the model, as one can see in the next fig. (fig. 7, right), because if the plates are on the sides, the shear stress plane, which is yz, is not sensitive to the variation of stacking. The used material is Aluminum, with a density of 2800 Kg/m<sup>3</sup>, a Young Modulus E of 70 GPa, and a Poisson coefficient of 0,33. Solid 4 node tetragonal elements (SOLID285) were used for tetragonal free mesh, having an average dimension of 1 mm. The chosen mesh dimension is low enough to obtain converged results. A Pressure of 1000 Pa was applied on the plate's top area. Results for the entire model were taken for all cases.

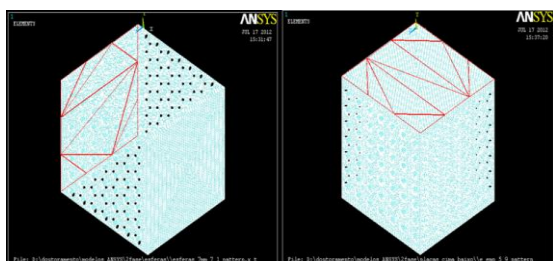


Fig. 7- Loads applied on FEM models: plates on the sides in the case of compression (left) and plates on the top and bottom in the case of shear (right)

### III. RESULTS

#### 3.1 Shear results

Figures 8 and 9 present the variation of the shear stress and shear strain for stacking elliptic (fig. 8) and ratio elliptic (fig. 9). The results are presented by mass unit. For the results charts (Fig 8-15), the letter on the x axis represents the model, as in figures 2-5. The results presented are the global maximum in absolute value, both in terms of stress and strain.

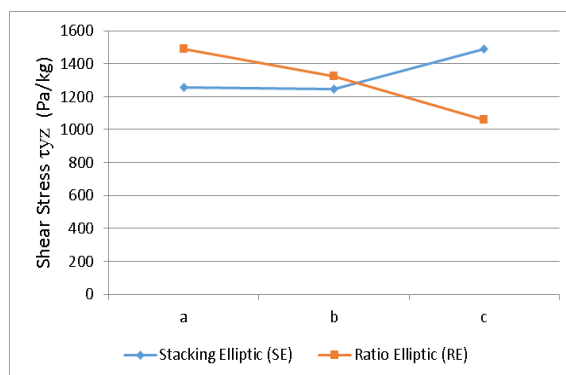


Fig. 8- Variation of the shear stress for elliptic

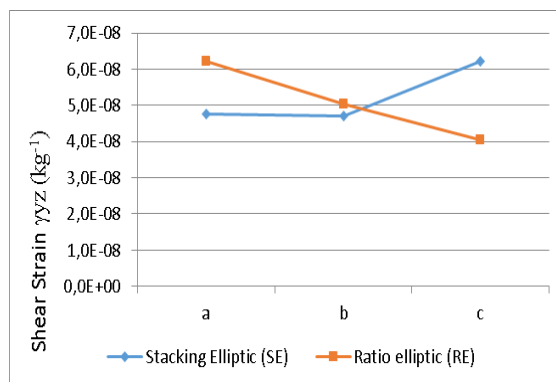


Fig. 9- Variation of the shear strain for elliptic

Table 4 presents a resume of the results obtained in shear for elliptic

Table 4- FEM results by mass unit obtained in shear for elliptic

Shear			
τyz[Pa/kg]	a	b	c
ES	1254,5	1243,8	1488,7
ER	1488,7	1325,6	1061,7
γyz[Kg-1]	a	b	c
ES	4,77E-08	4,72E-08	6,22E-08
ER	6,22E-08	5,03E-08	4,04E-08

Fig. 10 and 11 show the variation of the shear stress (fig. 10) and Shear strain (Fig. 11) for spherical model.

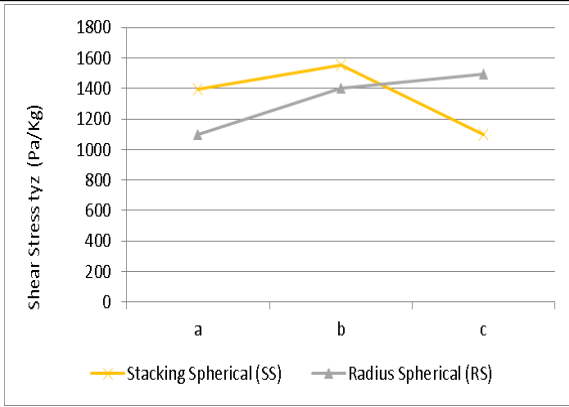


Fig. 10 - Variation of the shear stress for spherical

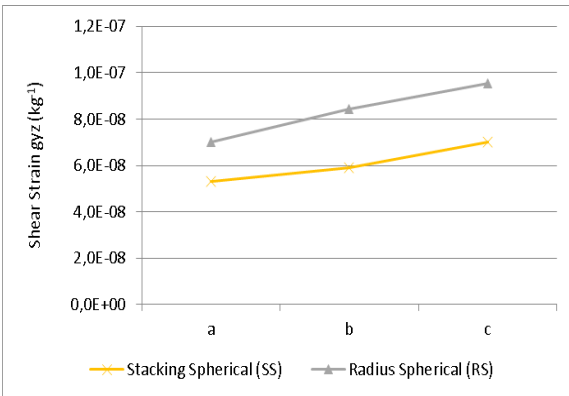


Fig. 11- Variation of the shear strain for elliptic

Table 5 shows FEM results obtained in shear for spherical models

Table 5- FEM results by mass unit obtained in shear for spherical

Shear			
$\tau_{yz}$ [Pa/kg]	a	b	c
SS	1393,395	1555,257	1100,081
SR	1100,1	1404,2	1490,9
$\gamma_{yz}$ [Kg <sup>-1</sup> ]	a	b	c
SS	5,30E-08	5,90E-08	7,00E-08
SR	7,00E-08	8,44E-08	9,56E-08

### 3.2 Compression results

The figures 12 and 13 show the variations of the axial stress (Fig. 11) and axial displacement (fig. 12) for the elliptic model. The results are presented by mass unit. The results presented are the global maximum in absolute value, both in terms of stress and displacements.

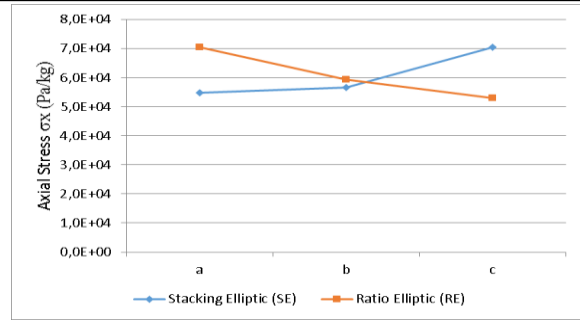


Fig. 12- Variation of the axial stress for elliptic

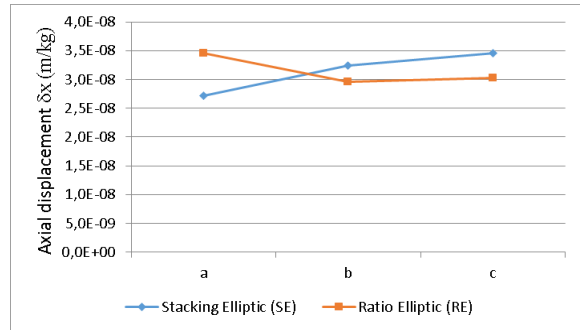


Fig. 13- Variation of the axial displacement for elliptic

Table 6 presents FEM results for elliptic cells for compression loading

Table 6- FEM results for elliptic cells on compression

Compression			
$\sigma_x$ [Pa/kg]	a	b	c
ES	54778,8	56658,0	70319,9
ER	70319,9	59185,5	52930,5
$\delta_x$ [m/kg]	a	b	c
ES	2,72E-08	3,24E-08	3,46E-08
ER	3,46E-08	2,96E-08	3,03E-08

Figures 14 and 15 show the variations of the axial stress (Fig. 14) and axial displacement (fig. 15) for the spherical model. The results are presented by mass unit.

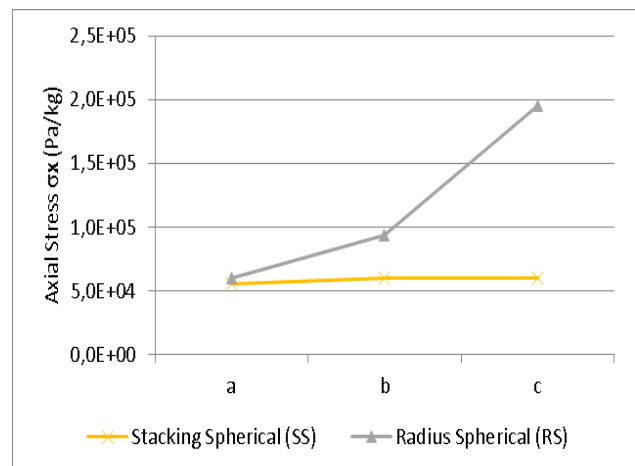


Fig. 14-Variation of the axial stress for spherical model

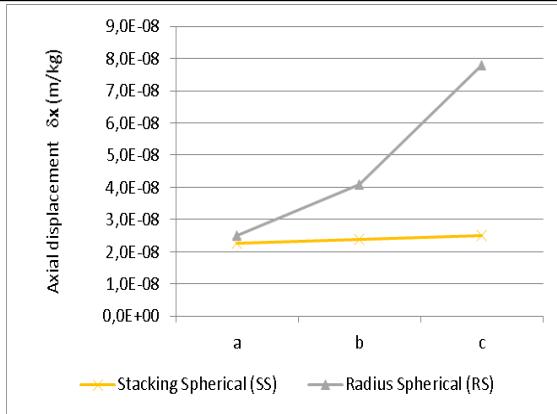


Fig. 15-Variation of the axial displacement for spherical model

Table 7 shows a resume of the results obtained in compression and plotted on the previous charts

Table 7- Results obtained in compression for spherical

Compression			
$\sigma_x$ [Pa/kg]	a	b	c
SS	55371,6	59997,6	60173,5
SR	60173,5	93586,4	194873,0
$\delta_x$ [m/kg]	a	b	c
SS	2,28E-08	2,39E-08	2,51E-08
SR	2,51E-08	4,08E-08	7,80E-08

## IV. DISCUSSION OF RESULTS

### 4.1 Shear

#### 4.1.1 Elliptical

Regarding  $\gamma_{yz}$  results for the elliptic case, one can say that there is a near exponential relation on stacking elliptic, and also there is a decrease of  $\gamma_{yz}$  with the elliptic ratio. Regarding  $\tau_{yz}$  results for elliptic, one can say that  $\tau_{yz}$  decreases slightly from model a to model b and then increases from model b to model c on stacking elliptic, on a near exponential variation, and there is a near linear decrease of  $\tau_{yz}$  with ratio elliptic.

#### 4.1.2 Spherical

Regarding  $\tau_{yz}$  results for spherical model, one can say that on stacking, the results increase between the first model and the second, and then they decrease substantially between the second and the third. On radius spherical, there is a near logarithmic relation. About  $\gamma_{yz}$  results for spherical, one can say that there is a strict increase of  $\gamma_{yz}$  on stacking spherical, although not completely linear or exponential. There is an almost linear increase of  $\gamma_{yz}$  with the spherical radius.

## 4.2 Compression

### 4.2.1 Elliptical

Regarding  $\sigma_x$  results for elliptical model, one can say that  $\sigma_x$  has a strict increase with stacking, being the variation from model b to c higher than from model a to b, and  $\sigma_x$  has a strict decrease with the elliptic ratio.  $\delta_x$  results for elliptical show that stacking elliptic suffers a strict decrease, in an approximate logarithmic relation and ratio elliptic suffers a decrease between the models a and b, and then a slight increase.

### 4.2.2 Spherical

$\sigma_x$  results for spherical model show that  $\sigma_x$  suffers very low variation with the stacking spherical. The axial stress  $\sigma_x$  increases exponentially with the increase of the spherical radius. About  $\delta_x$  results for spherical, one can say that stacking spherical suffers a linear increase, although the variation is quite small and  $\delta_x$  suffers an exponential increase on spherical radius.

## V. CONCLUSIONS

The studied variables have different behavior on shear and compression, as expected. Since the models have different mass, the results by mass unit are the best way to analyze and extract relevant information. All the studied variables are sensitive, either in stiffness and resistance analysis. However, some variables are quite sensitive, while others have much lower sensitivity. Although one can say that all the studied variables have influence on the mechanical behavior of cellular materials, the results help to identify which variables should be considered for further optimization studies. All variables are relevant for optimization studies, both on shear and compression, except stacking spherical (SS) on compression, which has a very low sensitivity on both axial stress and axial displacement. This study provides a preliminary analysis of the sensitivity of the studied geometric variables and can be useful for a development of an optimization routine with the aim of optimizing the variables and then applying the best geometry on cellular solids of this type in various applications.

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