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# Analysis of Distortion Parameters of Eight node Serendipity Element on the Elements Performance

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***Abstract*** - Often the result produced using FEM technique on the distorted shaped element gives poor result. Moreover if the results are poor then the design will fail. Eight noded serendipity elements is the most widely used element in 2-D analysis of structures. But despite its benefits it remains distortion sensitive. In the majority of computer programs, automatic mess generation is an integral part of the program. For complex shapes, the automatic grid generation will result in distorted quadrilateral shapes. Thus the solution obtained by using these meshes will produce erroneous results so it becomes necessary to incorporate the distortion measures in to these automatic mesh generations to limit the errors. Here in this paper distortion parameters are defined in terms of the coefficient of the element's shape polynomials and are tested for a range of distortion.

***Keywords****-FEM; serendipity element; distortion parameters*

## I. INTRODUCTION

The present day FEM stands on three legs: mathematical models, matrix formulation of the discrete equation and computing tools to do the numerical work. The third is the one that has undergone the most dramatic changes. In early days of FEM, Hermikoff and McHenry introduced the idea of modeling the continuum by replacing it with structural elements [1]. Later Courant in 1943 established a variation solution of the Poisonous equation using what we know today as linear triangular elements. He calculated the torsional stiffness and observed the convergence by refining the mesh [2].

Almost a decade later, Levy in 1953 recommended the derivation of stiffness matrix separately and assembling them for better accuracy [3]. Argyris in a series entitled *Energy theorem and structural analysis* has given different procedures for stress and displacement analysis [4]. Taig in 1961 introduced the isoparametric concept through the quadrilateral element. His ideas systematically interpreted by Irons resulted in a major breakthrough in the formulation of isoparametric elements in FEM. He later proposed that isoparametric elements can also have curved edges thus opening a wide field of modeling of curved boundaries [5].

Melosh classified the errors in FEM as firstly due to idealization by modeling of curved surfaces as the flat surfaces, secondly discretization errors due to replacing the continuous structure by a finite number of small pieces which can be minimized on refining and manipulation errors due to round off and truncation [6].

Eight noded serendipity element is the most widely used element in 2-D analysis of structures. The element was first defined by Zhu & Zienckiewicz in 1968. This element was an extension of isoparametric family of elements proposed by Taig. Irons have proposed the numerical integration technique for the isoparametric elements thus making it possible to evaluate the element stiffness [5]. The eight noded quadrilateral was different from the bilinear elements in number of ways, the bilinear is contained to have only straight sides. The eight nodded quadrilateral have curved edges also. This element also satisfies the two necessary criteria for conformability i.e. the element is able to represent the constant strain exactly & the displacement remains continuous across the element even when the edges are curved.

However this element despite of its so many benefits remains distortion sensitive. Bathe proposed some tests for isoparametric element and showed that it performs well when distorted slightly [7]. The problem still undefined is that how much distortion is allowable in eight nodded quadrilateral and how the parameter should incorporate the distortion measure. So here in this paper we will make an attempt to define the parameters in terms of the coefficient of the element's shape polynomials and test the element for a range of distortion in each of them.

## II. FINITE ELEMENTS BASED ON DISPLACEMENT FIELDS

The most widely used elements in structural mechanics are based on assumed displacement fields. Thus the x, y, z displacement components u, v, w of an arbitrary points within an element are interpolated from displacements Δ as

$$(u\ v\ w)T = N\,\Delta \qquad (1)$$

Matrix N is called the shape function matrix. It contains the interpolation polynomial. Δ is the nodal displacement vector. Compatibility is satisfied within elements because the polynomial displacement field is continuous. Compatibility is enforced at the node. At the interface, however, elements may or may not be compatible depending upon their assumed displacement fields. Equilibrium prevails at the nodes. At element boundaries, equilibrium is rarely satisfied. A step change is seen as we move from one element to another in general.

## III. ISOPARAMETRIC FAMILIES OF ELEMENTS

Isoparametric elements were introduced in 1966 by Irons. They are able to have curved sides and they use an intrinsic coordinate system for formulating the element stiffness matrix. The same interpolation scheme is used to define both the geometry and the displacement field of an element.

Plane stress is a condition that prevails in a flat plate which lies in a x-y plane and is loaded only in its own plane and without z direction restraint, so that

$$u = \sum_{i=1}^{m} N_i u_i \qquad (2)$$

$$x = \sum_{i=1}^{m} N'_i u_i \qquad (3)$$

Where Ni, Ni' are shape functions in terms of intrinsic coordinates. The element is isoparametric if m = n and Ni = Ni' and the same nodal points are used to define the both element geometry and element displacement. The element is sub parametric if m>n, and super parametric if m<n. The isoparametric element can correctly display rigid body and constant strain modes.For plane stress problems the three unknown fields are displacement field U(x y 0), stress field σ(σxx σyy σxy), strain field ε(εxx εyy εxy). The in-plane components of all the above fields are assumed to be uniform throughout the thickness of the plate. Consequently the dependence of z disappears and all such component becomes the function of x and y only.

## IV. DISTORTION PARAMETER

A flat quadrilateral has four shape parameters, aspect ratio, skew angle and two tapers. The parameters were based on a step by step drawing procedure for constructing a flat quadrilateral. In this paper we had taken a new look at the definition parameters, that they can be expressed in terms of coefficients in simple polynomials which express the shape of the quadrilateral. These polynomial coefficients have a simple physical meaning and are functions of the corner point coordinates of the flat or projected quadrilateral. A simple representation of the shape parameters for a four noded quadrilateral is given first and than extend to the eight node quadrilateral.

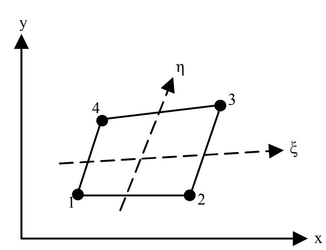



Fig. 3.2 shows a flat quadrilateral in general local xy-system. The shape of the quadrilateral can be written in the form of interpolation function as restraint, so that

$$x = \sum_{i=1}^{4} N_i(\xi,\eta)\, x_i \qquad (4)$$

$$y = \sum_{i=1}^{4} N_i(\xi,\eta)\, y_i \qquad (5)$$

Where

$$N_1 = \frac{(1-\xi)(1-\eta)}{4}$$
$$N_2 = \frac{(1+\xi)(1-\eta)}{4}$$
$$N_3 = \frac{(1+\xi)(1-\eta)}{4} \quad (6)$$
$$N_4 = \frac{(1-\xi)(1+\eta)}{4}$$

And $\xi$ and $\eta$ are non dimensional coordinates with limits -1 to +1. Above equations shows that the eight parameters are needed to define a quadrilateral. In this case, the corner point's coordinates are ($x_1$, $x_2$, $x_3$, $x_4$) ($y_1$, $y_2$, $y_3$, $y_4$).

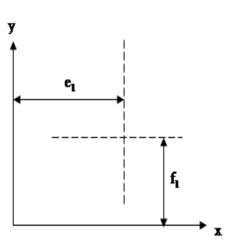
### A. *Interpretation of coefficient $e_1f_1$ to $e_4f_4$*

The use of interpolation functions is very elegant but the mathematical elegance can hide the practical significance of various quantities. An alternative form of the shape representation is to use the simple and basic polynomials.

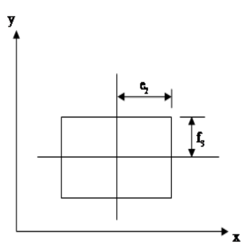
$$x = e_1 + e_2\xi + e_3\eta + e_4\xi\eta \quad (7)$$

$$y = f_1 + f_2\xi + f_3\eta + f_4\xi\eta \quad (8)$$

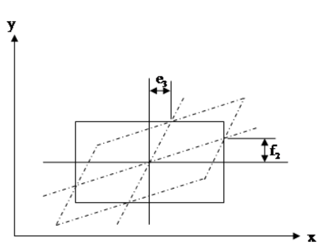
where e and f coefficient are related to the nodal coordinates $x_1$ to $x_4$ and $y_1$ to $y_4$ by linear relation



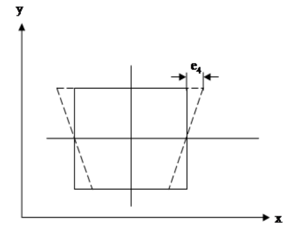

$x = e_1,\ y = f_1$



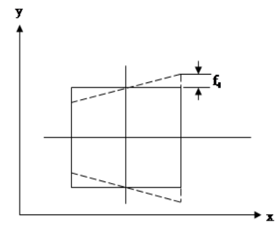

$x = e_2\xi,\ y = f_3\eta$




$x = e_3\eta,\ y = f_2\xi$




$x = e4\xi\eta$




$y = f4\xi\eta$

Fig. 2 : Interpretation of coefficient $e_1f_1$ to $e_4f_4$

There are eight parameters ($e_1$ to $e_4$ and $f_1$ to $f_4$). The physical significance of the e and f coefficients is known. It is clear that $e_1$ and $f_1$ define an origin (translational of axis). $e_2$ and $f_3$ define the size of the rectangle (aspect ratio), $e_3$ and $f_2$ gives two rotations (skew and rotation of axis) and $e_4$ and $f_4$ give tow tapers.

### B. *Interpretation of coefficient $e_5f_5$ to $e_8f_8$*

Similarly for eight node quadrilateral the shape function can be expressed in the simple polynomial.

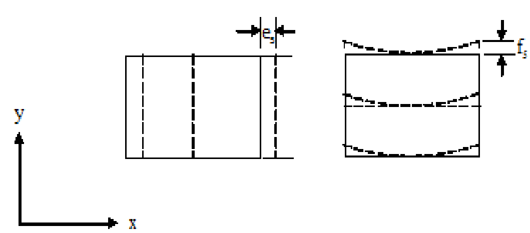
$$x = e_1 + e_2\xi + e_3\eta + e_4\xi\eta + e_5\xi^2 + e_6\eta^2 + e_7\xi^2\eta + e_8\xi\eta^2 \quad (2)$$

$$y = f_1 + f_2\xi + f_3\eta + f_4\xi\eta + f_5\xi^2 + f_6\eta^2 + f_7\xi^2\eta + f_8\xi\eta^2 \quad (3)$$

For eight node quadrilateral, additional distortion parameter emerge which are associated with the offset of the boundary nodes. For a particular boundary, this offset may be due to the curvature or when the side is straight, because the boundary node is not in the centre.

The procedure used for the four node quadrilateral for obtaining the distortion parameters from the coefficients in the simple shape polynomial is extended here to the eight node quadrilateral with curved boundaries.

In case of eight node quadrilateral the eight more parameter are there. These parameters are radius and angle for each boundary.




$x = e_5\xi^2$ $\quad$ $y = f_5\xi^2$

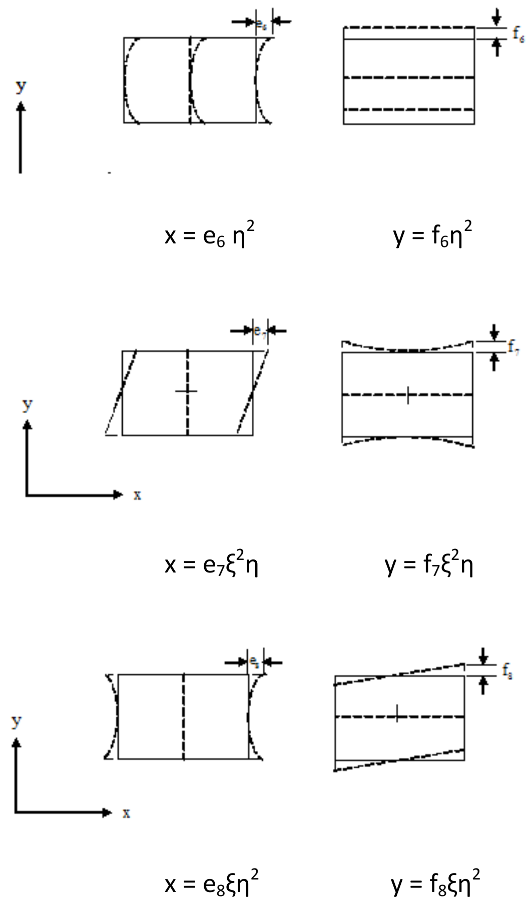



Figure 3 : Interpretation of coefficient $e_5f_5$ to $e_8f_8$

## V. TESETING OF THE ELEMENT

The objective of this paper is to present a systematic study with numerical and analytical results on the performance of the element when it is used in distorted shapes and summarize the findings that have practical consequences.

### *A. Classification of Element Distortion*

The attention is focused on the types of element distortions that occur most frequently in practice and that affect the performance of the elements. The classification of element distortions based on the shape of the element and the location of its nodes. Considering a square element with evenly spaced nodes to be the undistorted element, we may identify the following basic types of distortions:

- Aspect-ratio distortion
- Taper distortion
- Unevenly-spaced-nodes distortion
- Curved-edge distortion

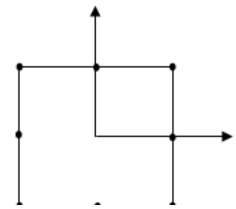


Fig. 4 : Undistorted configuration of 8 node element

These distortions are illustrated in Fig. 5 and Fig. 6. Any element distortion can generally be considered to be composed of some or all of the above types of distortions. We may also distinguish between distortions that are present in the undeformed finite element model and distortions that are created by large deformations. The latter type occurs only in non-linear analysis with large strain effects.

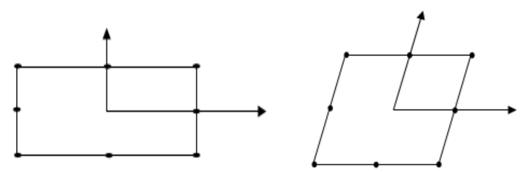


Fig. 5 : Aspect-ratio and Taper distortion

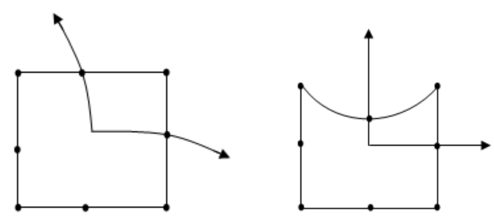


Fig. 6 : Unevenly-spaced-nodes and Curved-edge distortion

### *B. Aspect-ratio Sensitivity Test*

Aspect-ratio is given by a/b, it is the ratio between length and breadth. The test conducted on the cantilever beam, problem is shown below

Boundary conditions

$u = 0$

$u = v = 0$

E = 200x109 N/m2, υ = 0.3, P = 60 N

Fig. 7 : Aspect-ratio sensitivity test

We apply bending moment load P and tabulate the result. The calculated values agree with the theoretical values. Table below shows the result.

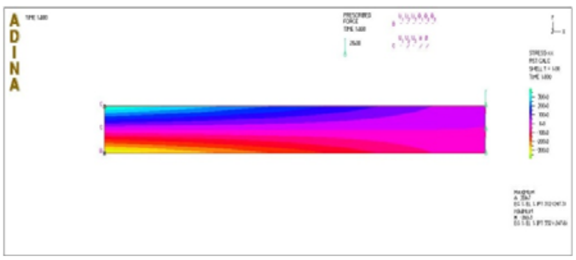


Fig. 8 : Stress plot for 8 node element with aspect-ratio 1 by 10

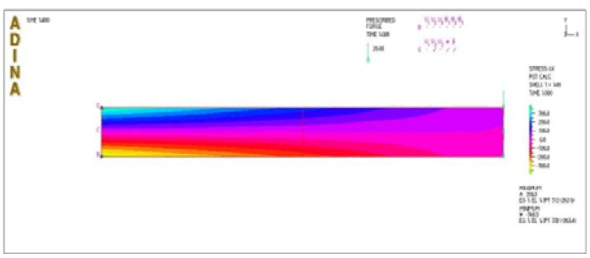


Fig. 9 : Stress plot for 8 node element with aspect-ratio 1 by 5

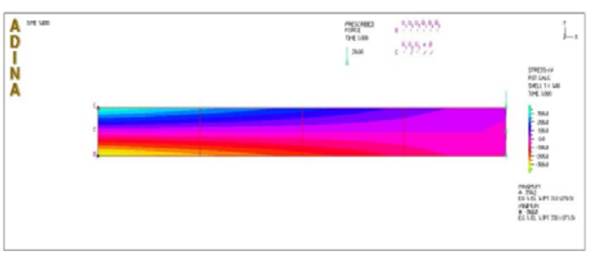


Fig. 10 : Stress plot for 8 node element with aspect-ratio 1 by 2.5

TALE 1 : ASPECT-RATIO SENSITIVITY TEST

| S. No. | Aspect-ratio | Stress $\sigma_{xx}$ for 8 node element | | Theoritical value of stress $\sigma_{xx}$ | |
|---|---|---|---|---|---|
| | | ***At upper layer*** | ***At lower layer*** | ***At upper layer*** | ***At lower layer*** |
| 1 | 1 : 10 | 359.7 | -360.3 | 360.0 | -360.0 |
| 2 | 1 : 5 | 359.5 | -360.5 | 360.0 | -360.0 |
| 3 | 1 : 2.5 | 359.2 | -360.8 | 360.0 | -360.0 |

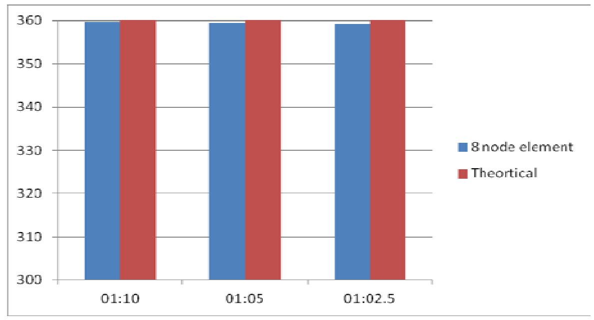



FIG. 11 : GRAPHS BETWEEN ASPECT RATIO AND STRESS AT UPPER LAYER

The effect of aspect-ratio distortions is to reduce the predictive capabilities of the overall finite element model in the direction that has fewer elements per unit length. Aspect-ratio distortions do not affect the element performance.

### *C. Taper Sensitivity Test*

It is the equal displacement of upper and lower node in opposite direction by same amount. The test is conducted on the cantilever beam problem same as for aspect ratio sensitivity test.

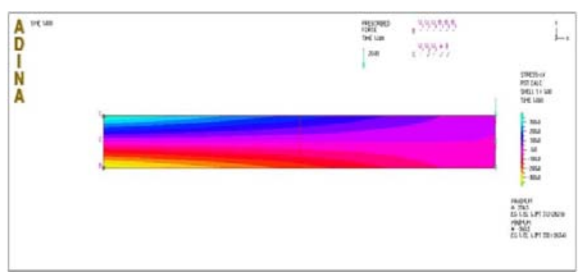


Fig. 12 : Stress plot for 8 node element without taper

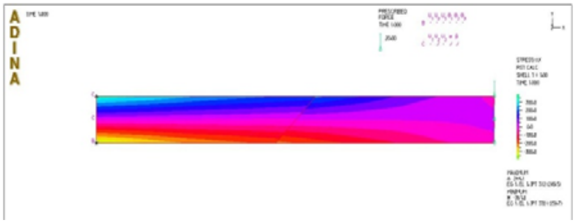



Fig. 13 : Stress plot for 8 node element with taper of 5 unit

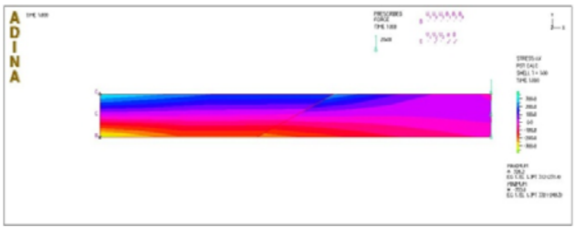



Fig. 14 : Stress plot for 8 node element with taper of 10 unit

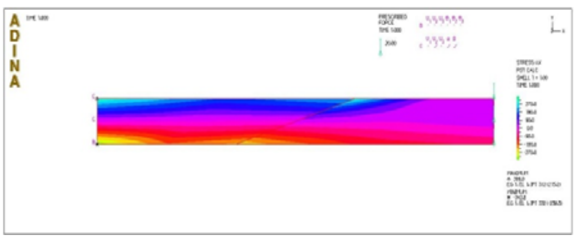



Fig. 15 : Stress plot for 8 node element with taper of 15 unit

Table II : Taper Sensitivity Test

| S. No. | Taper ($e_7$) | Stress $\sigma_{xx}$ for 8 node element | | Theoritical value of stress $\sigma_{xx}$ | |
|---|---|---|---|---|---|
| | | ***At upper layer*** | ***At lower layer*** | ***At upper layer*** | ***At lower layer*** |
| 1 | 0 | 359.5 | -360.5 | 360.0 | -360.0 |
| 2 | 5 | 344.1 | -360.0 | 360.0 | -360.0 |
| 3 | 10 | 326.2 | -353.0 | 360.0 | -360.0 |
| 4 | 15 | 309.0 | -342.8 | 360.0 | -360.0 |

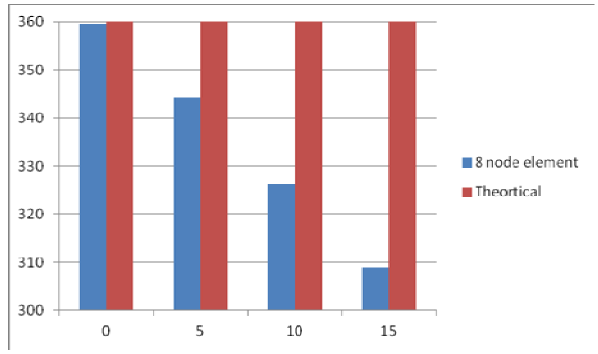



Fig. 16 : Graphs between aspect ratio and stress

The effect of taper distortions is, by increasing the taper of the element changes the value of the stress on both sides i.e. upper and lower layer by large amount. The element is sensitive to taper distortion.

*D. Unevenly-spaced-nodes distortion Test*

In this type of distortion nodes are unevenly-spaced. The test conducted on the cantileverbeam problem taken from K. J. Bathe paper is shown here.

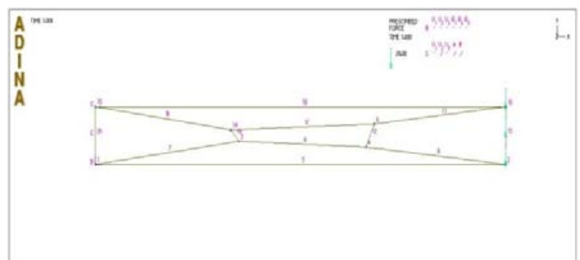



Fig. 17 : Node lables of unevenly-spaced 8 node element

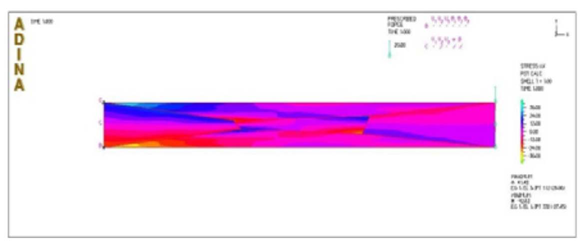



Fig. 18 : Stress plot for unevenly-spaced 8 node element

The effect of unevenly-spaced-node distortions is very large. By placing the node unevenly cause drastic change in the value of the stress on both sides i.e. upper and lower layer by large amount. Hence, the element is very sensitive to unevenly-spaced-nodes distortion.

### *E. Curved-edge distortion Test*

By offsetting the middle node we get curved boundaries. The position of the offset middle node decides the value of the distortion parameter. The figure below shows the geometry for the curved edge distortion measures. The test conducted on the cantilever beam proble stress plot for which are as below.

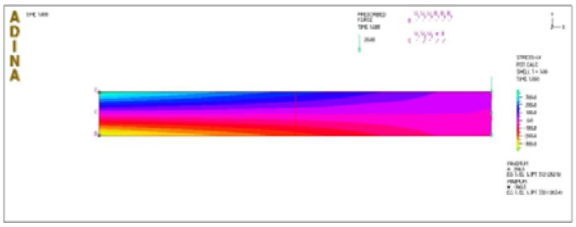


Fig. 19 : Stress plot for 8 node element without Curved-edge distortion

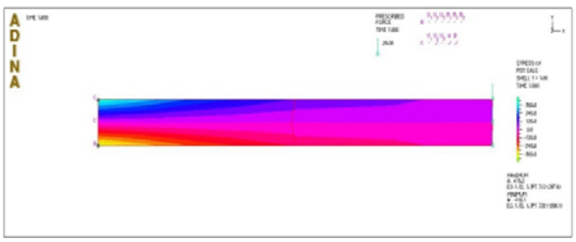


Fig. 20 : Stress plot for 8 node element with midddle node offset of 1 unit

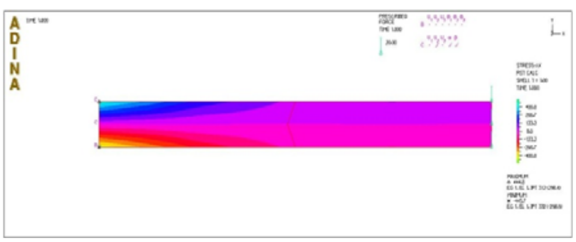


Fig. 21 : Stress plot for 8 node element with midddle node offset of 2 unit

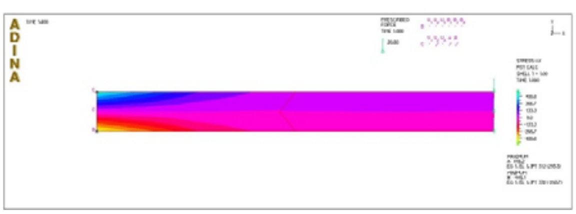


Fig. 22 : Stress plot for 8 node element with midddle node offset of 4 unit

TABLE – III : CURVED-EDGE DISTORTION SENSITIVITY TEST

| S. No. | Curve-edge ($e_6$) | Stress $\sigma_{xx}$ for 8 node element | | Theoritical value of stress $\sigma_{xx}$ | |
|---|---|---|---|---|---|
| | | ***At upper layer*** | ***At lower layer*** | ***At upper layer*** | ***At lower layer*** |
| 1 | 0 | 359.5 | -360.5 | 360.0 | -360.0 |
| 2 | 1 | 415.2 | -416.1 | 360.0 | -360.0 |
| 3 | 2 | 444.8 | -445.7 | 360.0 | -360.0 |
| 4 | 4 | 448.2 | -449.1 | 360.0 | -360.0 |

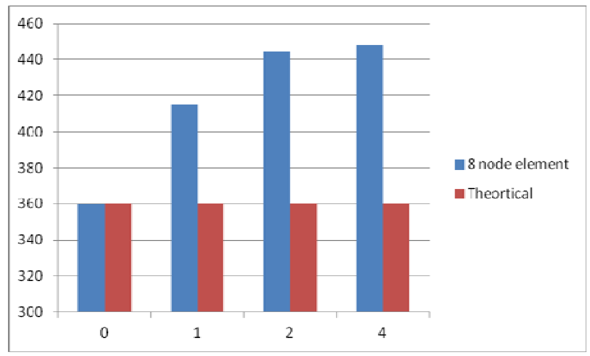



Fig. 23 : Graphs between midddle node offset and stress

## VI. CONCLUSION

In this paper we took a new look at the distortion parameters and shown that they can be expressed in terms of coefficients in simple polynomials which express the shape of the quadrilateral. After testing the element and incorporating the distortion parameters results show that there is no effect of the aspect ratio, so there is no need to incorporate the aspect ratio sensitivity into the software. But the element is senstive to taper, unevenly-spaced-nodes and curved-edge distortion, so it becomes necessary to incorporate sensitivity to these distortion parameters into the mesh generation software.

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