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Evaluation of Two Terminal Reliability of Fault-tolerant Multistage Interconnection Networks

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Abstract: This paper iOntroduces a new method based on multi-decomposition for predicting the two terminal reliability of fault-tolerant multistage interconnection networks. The method is well supported by an efficient algorithm which runs polynomially. The method is well illustrated by taking a network consists of eight nodes and twelve links as an example. The proposed method is found to be simple, general and efficient and thus is as such applicable to all types of fault-tolerant multistage interconnection networks. The results show this method provides a greater accurate probability when applied on fault-tolerant multistage interconnection networks. Reliability of two important MINs are evaluated by using the proposed method.

Key words: *Probabilistic graph, Reliability, Fault-tolerant, Interconnection network*

1. INTRODUCTION:

Almost all of the interconnection networks can be broadly classified into two groups: static networks and dynamic networks. Static networks are formed of point-to-point direct connections which will not change during program execution. On the other hand, dynamic networks are implemented by switched channels, which are dynamically configured to match the communication demand in use programs. There are three well known types of dynamic interconnection networks: Crossbar networks, bus networks and multistage interconnection networks. Out of these networks multistage interconnection networks have attracted great interest recently. Multistage Interconnection Network (MIN) is a low cost network, which interconnects N inputs with N outputs and has $\log_2 N$ switching stages. Each stage consists of 2x2 switching elements with or without loop dependent upon the regular or irregular class of network [1] and [2]. Some of the important examples of fault tolerant multistage interconnection networks are Extra Stage Cube, Extra Stage Shuffle Exchange Network, and Multipath Chained Baseline Network (MPN) etc [3]. The main advantages associated with these networks are high bandwidth, low diameter, constant degree switches for which they have been used for various commercial machines including super computers.

With the increase in size and complexity of the interconnection systems, their reliability becomes extremely important. There are many reliability measures of interest, out of which the node-pair (two terminal) reliability is an important performance measure. Two terminal reliability addresses the probability that a given source-destination pair has at least one fault free path between them. In this context, Blake and Trivedi [4] have developed closed form reliability expressions for two selected multistage interconnection networks (MINs). Subsequently, in a later paper, they have proposed a 2-level hierarchical model in which each sub-system is modeled as a Markov chain and the system reliability of a MIN is modeled as a series system of 'Markov' components. Their methods are good enough for MINs of smaller sizes. Colbourn et al. [5] proposed an efficient method to compute the source-to-terminal reliability of MINs. However, their method computes only the bounds on reliability, and does not provide the exact solution. Gunwan [11] proposed an analytical technique to find the reliability of Extra Stage Shuffle Exchange Networks. For gamma networks the reliability has been estimated by using the redundant paths [12]. Reviewing the literature reveals that some other important methods that find the reliability or its related measures [6], [7], [8], [9] and [10].

This motivates our study to propose a simple, general and efficient method based on multi-decomposition of networks for predicting the exact value of terminal reliability of MINs.

2. ARCHITECTURAL DETAILS

Extra Stage Cube Network

The Extra Stage Cube (ESC) network is a fault–tolerant network. It derives its topology from the generalized cube multistage interconnection network (MIN). The Generalized cube is an N×N multistage inter-connection network, with $N = 2^n$, $n = \log_2 N$ stages; each stage consisting of N links connected to N /2 switches. The extra stage cube is formed from the generalized cube by adding an extra stage to the input side of the network along with multiplexers and demultiplexers at the input and output stages respectively. In an 8 × 8 ESC, the stage n is connected like stage 0 and the links that differ in the low order bits are paired (Fig. 1).

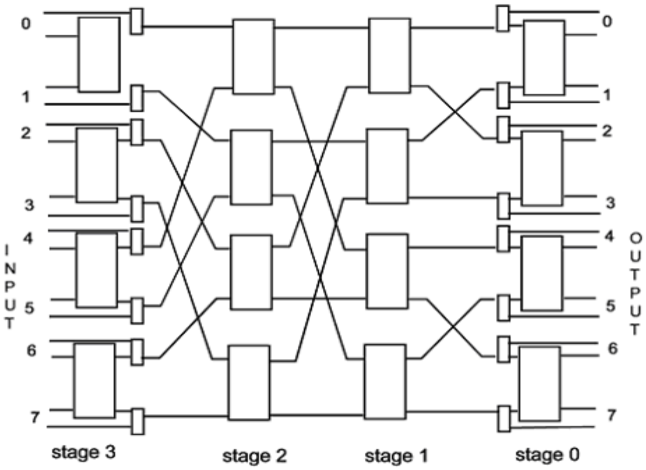


Figure 1: Extra Stage Cube Network

Extra Stage Shuffle Exchange Network

An $N \times N$ Extra Stage Shuffle Exchange Network (ESEN) is a fault–tolerant version of the $N \times N$ Shuffle Exchange Network (SEN) with an additional stage (Fig.2). The additional stage is added in order to increase the fault-tolerance of the Shuffle exchange network. The first stage (labeled stage 0) is the additional stage and requires implementation of a different control strategy. The basic idea to add a stage to Shuffle Exchange Network is to allow two simultaneous paths for communication between each source and each destination. An Extra Stage Shuffle Exchange (ESEN) is the only network in a large class of topological equivalent multistage interconnection networks that includes the Omega, Indirect Binary *n*-cube, Baseline and Generalized cube. The switching element of an ESEN can either transmit the inputs straight through itself or as a cross connection. The Extra Stage Shuffle Exchange multistage interconnection network has $N = 2^n$ inputs, termed as source (*s*), and 2^n outputs termed as destinations (*t*). It also has *n+1*stages, where each stage has $N/2$ switching elements. The network complexity, defined as the total number of switching elements in the MIN, is $(N/2)(\log_2 N)$. The position of switching element *i* in stage *j* is represented by $SEij$. The Fig.2 illustrates an ESEN of size (8 × 8).

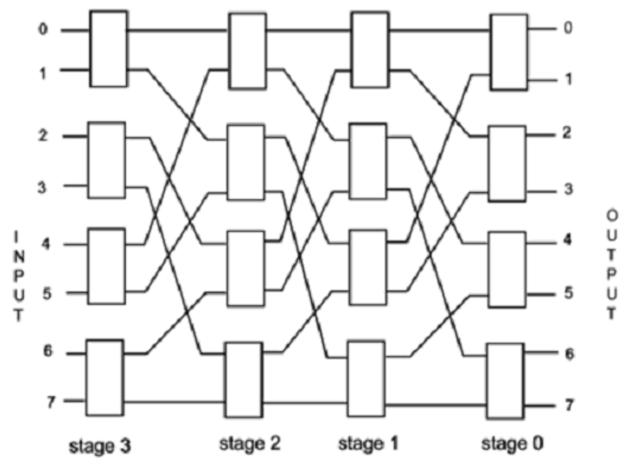


Figure 2: Extra Stage Shuffle Exchange Network

3. PROPOSED METHOD:

3.1 Notations

TR	*terminal reliability of the network*
N	*number of nodes in the network*
C_i	*common nodes of the decomposed sub graph (i-1) and i*
s	*source node*
t	*destination node*
m	*number of sub graphs*
P_i	*paths of sub graph i*
p	*cardinality of* P_i *i.e.* $\lvert P_i \rvert$
λ	*link failure rate*
t	*mission time*
G	*probabilistic graph or Reliability Logic Graph*
V	*set of vertices*
E	*set of edges*
$Z, \overline{Z}$	*indicator variable for successful and unsuccessful operation of component Z; Z=1 and* $\overline{Z}$ *=0 if x is good, and* Z *=0 and* $\overline{Z}$ *=1 if x is failed.*
S	*indicator variable for success of the system in connecting its source node* x_s *and sink node* x_t*.*
S_{dis}	*disjoint sum-of-product*
K	*minimal cut through which the graph is decomposed.*
X_i	*indicator variable for the union of success events corresponding to all the paths in Sub graph I from the source node s to node* x_j*, where* $x_i \in C$

Y_i *indicator variable for the union of success events corresponding to all the paths in Sub graph I from node* x_i *to sink node t, where* $x_{ij} \in C$.

3.2 Brief description about the Multi-decomposition Process

In Reliability Logic Graphs (RLG), MINs are represented as directed graphs. The inter change boxes or switching elements (SEs) can be set to one of the four legitimate states: (i) Straight (ii) Lower broadcast (iii) Exchange (iv) Upper broadcast. We consider the internal connectivity of the SEs and retain the complete permutation capability of the MINs. The parallel and cross connection of a switch is determined by the logic level applied at its control line.

In the proposed graph model for MINs, each node represents a link and edges represent switches. Here, only two edges of the graph are used for each state of SE and the other two are utilized in the complementary operating mode. The I/O stages of the ESC networks use SEs with multiplexers and demultiplexers. Enabling and disabling in stages n and O is accomplished with a demultimplexer (DEMUX) at each SE input and a multiplexer (MUX) at its output. The DEMUX and MUX of the ESC have been represented by parallel dotted edges from X_1 to Y_1 and from X_2 to Y_2 respectively in the graphs, as they serve as by-pass to SEs. The family of permutations that could be passed in a conflict-free manner varies from one MIN to other. The full connectivity requires that the outgoing edges in a graph model are always directed to new nodes so that a connection between any inputs to any one of the outputs could be established.

Multiple Decomposition of the RLG

For the purpose of reliability evaluation, a MIN is modeled as a multistage directed graph, denoted by $G\{V,E\}$. For $h= (n+1)$, $V= V_1 U V_2 U \ldots\ldots V_h$ is the disjoint union of h sets of vertices, each set being a stage of N vertices. Similarly, $E = E_1\ U\ E_2\ U \ldots E_{h-1}$, represents the disjoint union of $(h-1)$ sets of edges, each edge connecting the vertex V_{g+1}. Without loss of generality, we will use the terms vertex and node interchangeably throughout this paper.

The multistage graph $G\{V,E\}$ having *n* stages is decomposed into *n* sub-graphs by taking K^I minimal cuts through the common nodes between two consecutive stages. The first sub graph *A(1)* contains all the source nodes V_s for $s=1,2,\ldots..,N$ and nodes which are common to 1st sub graph and the 2nd sub graph. The nth sub graph *A(n)* contains all the destination nodes V_t, *for* $t=1,2,\ldots.N$, and the nodes common to both the sub graph *A(n)* and *A(n-1)*. All the intermediate sub graphs are designated as *A (j+1)*.

The process is illustrated by taking Extra stage cube network as example (Refer Fig. 3(a), (b)).

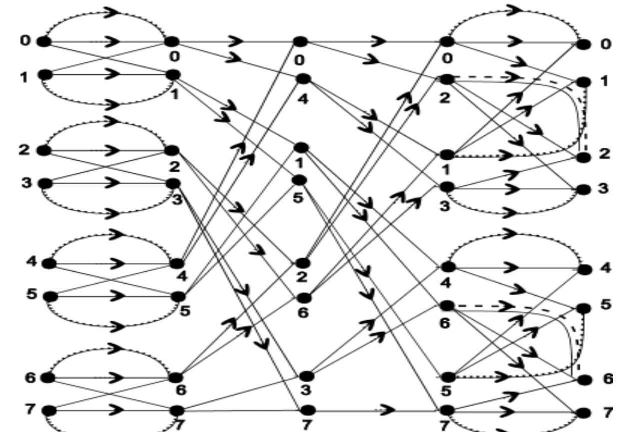


Fig. 3(a): Reliability logic graph of Extra stage cube network

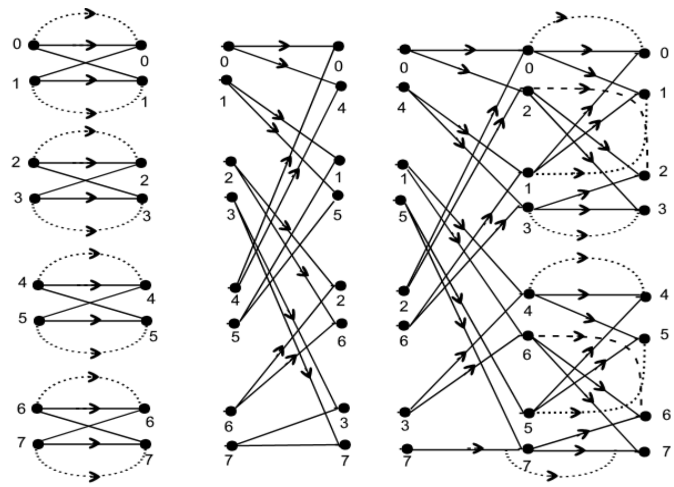


Fig. 3 (b): Multiple decomposition of ESC in Fig. 3(a)

3.3 Proposed Algorithm (TREMD)

(Multiple_decomposition)

Step 1: Convert the Multistage Interconnection Network into its equivalent Probabilistic Graph, G(N,E)

Step 2: Find the minimal cut sets C_1, C_2 *and* C_3 *of the graph G*

Step 3: Decompose the graph G into three subgraphs G_1, G_2, *and* G_3 *such that*

$G_1 \cap G_2 \cap G_3 = \phi$ *through minimal cut-sets C_1, C_2 and C_3*

Step 4: *Enumerate all the paths ($P_{1,i,j}$) of sub graph G_1 from source node s to n_i , $n_i \in C_1$, $i \in C_1$, $j \geq 1$;*

for i=1 to $|C_1|$

$p_{1,i}$= cardinality of $P_{1,i,j}$;

X= ϕ ;

for i= 1 to $|C_1|$

for j = 1 to $p_{1,i}$

$W_i = W_i \cup P_{1,i,j}$;

Step 5: *Enumerate all the paths ($P_{2,i,j}$) of sub graph G_2 from source node*

n_i to n_j , $n_i \in C_1$ & $n_j \in C_2$, $i \in C_2$, $j \geq 1$;

for i=1 to $|C_2|$

$p_{2,i}$= cardinality of $P_{2,i,j}$;

X= ϕ ;

for i= 1 to $|C_2|$

for j = 1 to $p_{2,i}$

$X_i = X_i \cup P_{2,i,j}$;

Step 6: *Enumerate all the paths ($P_{3,i,j}$) of sub graph G_3 from source node*

n_j to n_t , $n_j \in N_{C_2}$, $i \in C_3$, $j \geq 1$;

for i=1 to $|C_3|$

p_3= cardinality of $P_{3,i,j}$;

Y= ϕ ;

for i= 1 to $|C_3|$

for j= 1 to $p_{3,i}$

$Y_i = Y_i \cup P_{3,i,j}$;

Step 7: *Enumerate all the paths ($P_{4,i,j}$) of sub graph G_3 from source node n_j to destination nodes $n_i \in N_{C_3}, N_j$, $j \in O, j \geq 1$;*

p_4= cardinality of $P_{4,i,j}$;

Z= ϕ ;

for i= 1 to $|C_3|$

for j= 1 to $p_{4,i}$

$Z_i = Z_i \cup P_{4,i,j}$;

Step 8: *Express the system success*

$$S_1 = (W_1)_{dis}(X_1)_{dis} \cup ((\overline{W}_1 W_2)_{dis}(X_2)_{dis} \cup (W_1 W_2)_{dis}(\overline{X}_1 X_2)_{dis}) \cup ((\overline{W}_1 \overline{W}_2 W_{i3})_{dis}(X_3)_{dis} \cup (\overline{W}_1 W_2 W_3)_{dis}(\overline{X}_2 X_3)_{dis} \cup (W_1 \overline{W}_2 W_3)_{dis}(\overline{X}_1 X_3)_{dis} \cup (W_1 W_2 W_3)_{dis}(\overline{X}_1 \overline{X}_2 X_3)_{dis} \cdots \;;$$

$$S_2 = (\overline{X}_1 Y_1)_{dis} \cap (\overline{X}_2 Y_2)_{dis} \cap (\overline{X}_3 Y_3)_{dis} \cdots \;;$$

$$S_3 = (\overline{Y}_1 Z_1)_{dis} \cap (\overline{Y}_2 Z_2)_{dis} \cap (\overline{Y}_3 Z_3)_{dis} \cdots \;;$$

$$S = S_1 \cup S_2 \cup S_3;$$

Step 9: $TR = S_{\{b_i, \bar{b}_i, \cup, \cap\} \to \{p_i, q_i, +, .\}}$;

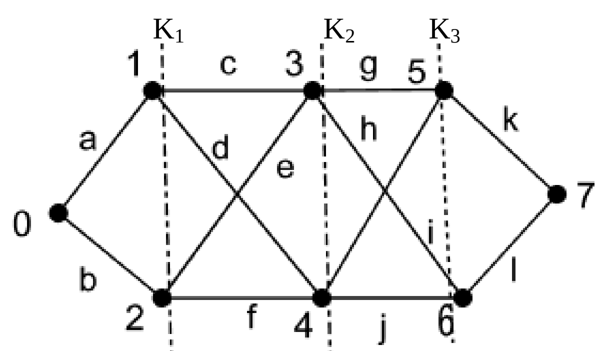
Step 10: Replace all indicator variables by their probabilities; the logical sum and product operators by their arithmetic counterparts and evaluate the TR.

In what follows, we illustrate the applicability of the proposed algorithm through a simple example.

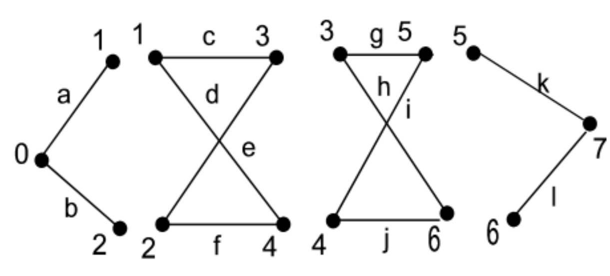
3.4 Illustration

The proposed algorithm is illustrated through the following example.

Example: Let us consider the network in Fig. 4. (a) for its reliability evaluation.



(a)



(b) (c) (d) (e)

Figure 4: An example network with 8 nodes

The given network in the Figure 4(a) is decomposed into four sub graphs as shown in Figs.4 (b), 4(c), 4(d) and 4(e) through the cuts

$K_1 = \{a,b\}, K_2 = \{c,d,e,f\}$ and $K_3 = \{g,h,i,j\}$.

The common node sets C_1 = {1, 2}, C_2 = {3, 4} and C_3 = {5, 6}. Hence, the system success is expressed as

$$S_1 = be \cup \bar{b}ac \cup ab\bar{e}c$$

$$S_2 = \bar{c}\bar{d}(g + \bar{g}h) \cap \bar{e}\bar{f}(i + \bar{i}j)$$

$$S_3 = \bar{g}\bar{h}k \cap \bar{i}\bar{j}l$$

$$S = S_1 \cup S_2 \cup S_3$$

Then replacing all indicator variables by their probabilities and logical sum and product operator by their arithmetic counterparts the two terminal reliability (TR) of the network in Fig. 4(a) is obtained as

$$TR = p_b p_e + p_a q_b p_c + p_a p_b p_c q_e + q_c q_d q_e q_f (p_g + p_h q_g)(p_i + p_j q_i) + q_g q_h p_k q_i q_j p_l$$

With success probability of links *p=0.9*, the value of the terminal reliability of the example network is found to be TR=0.96

4. RESULTS AND DISCUSSIONS

The terminal reliability of two important fault-tolerant multistage interconnection networks viz. Extra Stage Cube Network (ESC), Extra Stage Shuffle Exchange Network (ESEN) are evaluated using the proposed multi decomposition method for different link failure rates and are plotted against the mission time. The Fig.5 and Fig.6 illustrates the results.

The Fig.5 provides a comparative picture of the terminal reliability of Extra Stage Cube Network (ESC) under different link failure rates. Under low link failure rate such as λ=0.0001, the reliability is 73% at mission time t=1000 hours. However at the same mission time, the reliability of Extra Stage Cube Network becomes less than 40% under high link failure rate such as λ=0.0005. For high and moderate link failure rate, the reliability becomes zero at mission time 10000 hours but, however it is only 20% for low link failure rate.

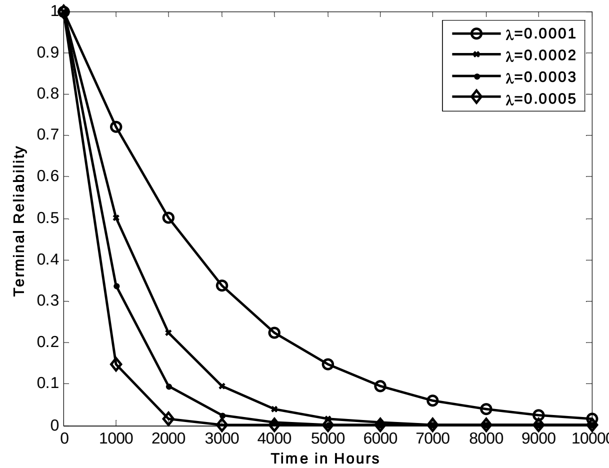


Fig. 5: Terminal Reliability of Extra Stage Cube Network

The Extra Stage Shuffle Exchange Network (ESEN) provides its terminal reliability value of slightly less than 70% at mission time 1000 hours and at λ=0.0001 (Fig.6). The reliability degrades to almost 30% corresponding to the following mission times and λ:

up to *t=2000 hrs,* λ=0.0001, up to *t=1000 hrs,* λ=0.0002

up to *t=1000 hrs,* λ=0.0003, up to *t=3000 hrs,* λ=0.0001

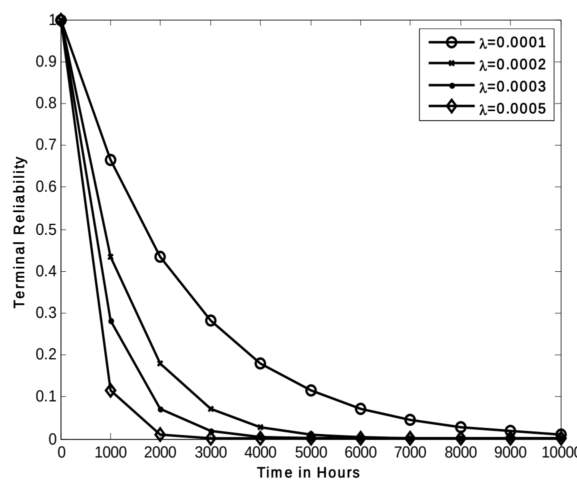


Fig.6: Terminal Reliability of ESSE Network

5. CONCLUSION

In this paper a new method based on multi-decomposition for predicting the exact two terminal reliability of fault-tolerant multistage interconnection network is proposed. The approach assumes links of the network to be imperfect and switching elements to be perfect. The detailed mathematical model of the method is presented. The proposed method is well supported by an efficient algorithm. The complexity of the algorithm is found to be polynomial in nature. The method is well illustrated through a simple example. As the proposed method is found to be simple, general and efficient and thus applicable to all types of fault-tolerant multistage interconnection networks.

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