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Development of Empirical Mode Decomposition

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ABSTRACT:-In this paper, one of the tasks for which empirical mode decomposition is potentially useful is nonparametric signal denoising, an area for which wavelet thresholding has been the dominant technique for many years. In this paper, the wavelet thresholding principle is used in the decomposition modes resulting from applying EMD to a signal. We show that although a direct application of this principle is not feasible in the EMD case, it can be appropriately adapted by exploiting the special characteristics of the EMD decomposition modes. In the same manner, inspired by the translation invariant wavelet thresholding, a similar technique adapted to EMD is developed, leading to enhanced denoising performance.

Key words: Empirical mode decomposition, Signal denoising, Wavelet thresholding

1. INTRODUCTION:

The empirical mode decomposition method is an algorithm for the analysis of multi component signals that breaks them down into a number of amplitude and frequency modulated zero-mean signals, termed intrinsic mode functions. EMD expresses the signal as an expansion of basis functions that are signal-dependent and are estimated via an iterative procedure called sifting. Although many attempts have been made to improve the understanding of the way EMD operates or to enhance its performance, EMD still lacks a sound mathematical theory and is essentially

described by an algorithm. it has found a vast number of diverse applications to name

a few biomedical, watermarking, and audio processing. Apart from the specific

applications of EMD listed above, a more generalized task in which EMD can prove useful is signal denoising. In this paper, inspired by standard wavelet

thresholding and translation invariant thresholding, a number of EMD-based denoising techniques are developed and tested in different signal scenarios and white Gaussian noise. It is shown that although the main principles between wavelet and EMD thresholding are the same, in the case of EMD, the thresholding operation has to be properly adapted in order to be consistent with the special characteristics of the signal modes resulting from EMD.

II. EMD:

Adaptively decomposes a multicomponent signal into a number L of the so-called IMFs

$$h^l(t), 1 \leq l \leq L$$

$$x(t) = \sum_{l=1}^L h^l(t) + d(t)$$

Where $d(t)$ is a reminder that is a non-zero-mean slowly varying function with only few extrema. Each one of the IMFs, say, the i^{th} one $h^i(t)$, is estimated with the aid of an iterative process, called sifting, applied to the residual multicomponent signal

$$x^l(t) = \begin{cases} x(t) & l = 1 \\ x(t) - \sum_{i=1}^{l-1} h^i(t) & l \geq 2 \end{cases}$$

The sifting process is as follows the $(n+1)^{\text{th}}$ sifting iteration, the temporary IMF estimate $h_n(t)$ is improving according to the following steps. 1) Find the local maxima and minima of $h_n(t)$.

2) Interpolate, $h_n(t)$ estimated in the first step in order to form an upper and a lower envelope.

3) Compute the mean of the two envelopes.

4) Obtain the refined estimate $h_{n+1}^{(1)}(t)$ of the IMF by subtracting $h_n^{(1)}(t)$

5) proceed from step 1) again unless a stopping criterion has been fulfilled. Which is actually the

corresponding IMF, i.e., $h^i(t) = x^i(t) - m^i(t)$
Each IMF

III. SIGNAL DENOISING:-

Digital signal denoising can be described as follows. Having a sampled noisy signal $x(t)$ given by $x(t) = \bar{x}(t) + n(t) \quad t = 1, 2, \dots, N$ Where $\bar{x}(t)$ is the noiseless signal and $n(t)$ are independent random variables Gaussian distributed $N(0,1)$, produce an estimate $\hat{x}(t)$ of $\bar{x}(t)$ signal. The novelty of this paper lies in the introduction of new nonparametric thresholding techniques applied to the decomposition modes resulting from EMD instead of the wavelet components. As will be seen, thresholding in EMD is not a straightforward application of the concepts used in wavelet thresholding.

Wavelet Based Denoising: Employing a chosen orthonormal wavelet basis, an orthogonal $N \times N$ Matrix W is the discrete wavelet transform (DWT) $c = Wx$ where $x = [x(1), x(2), x(3), \dots, x(N)]$ and $c = [c(1), c(2), \dots, c(N)]$ contains the resultant wavelet coefficients. Using major thresholding operators—hard and soft, the estimated denoised signal is given by $\hat{x} = W^T \hat{c}$ where $\hat{c} = [\rho_T(c_1), \rho_T(c_2), \dots, \rho_T(c_N)]$ and W^T With respect to the threshold selection, the universal Threshold $T = \sigma\sqrt{2 \ln N}$. Such a threshold guarantees with high probability

Conventional EMD Denoising: The EMD as a denoising tool emerged from the need to know whether a specific IMF contains useful information or primarily noise. Then, the noise-only IMF energies can be approximated according to $E_k = \frac{E_1^2}{\beta} \rho^{-k}, k = 2, 3, 4, \dots$ where E_1 is the energy of the first IMF and depend mainly on the number of sifting iterations used.

IV. IMF Thresholding-Based Denoising:-

In wavelet thresholding a generalized reconstruction of the denoised signal is given by

$$x(t) = \sum_{k=M_1}^{M_2} h^k(t) + \sum_{k=M_2+1}^L h^k(t)$$

Where the parameters M_1 and M_2 gives us flexibility. In the study of thresholds, multiples of the IMF-

dependent universal thresholds, i.e. $T_k = C\sqrt{E_k 2 \ln N}$ where C is a constant, are used.

Thresholding Adapted to EMD Characteristics: This newly developed EMD hard thresholding, as EMD interval thresholding (EMD-IT), translates to

$$\tilde{h}^i(z_j^{(i)}) = \begin{cases} h^i(z_j^{(i)}), & h^i(z_j^{(i)}) > T_i \\ 0, & h^i(z_j^{(i)}) \leq T_i \end{cases}$$

For $j = 1, 2, \dots, N_z^{(i)}$, where $h^i(z_j^{(i)})$ indicates the samples from instant $z_j^{(i)}$ to $z_{j+1}^{(i)}$ of the i th IMF.

Iterative EMD Interval-Thresholding: This EMD is summarized in the following steps.

- 1) Perform an EMD expansion of the signal
- 2) perform a partial reconstruction using the last $L - 1$ IMFs only,

$$x_p(t) = \sum_{i=2}^L h^i(t)$$

- 3) Randomly alter the sample positions of the first IMF $\tilde{h}_a^{(1)}(t) = ALTER(h^{(1)}(t))$

- 4) Construct a different noisy version of the original signal $x_a(t) = x_p(t) + \tilde{h}_a^{(1)}(t)$

- 5) Perform EMD on the new altered noisy signal $x_a(t)$.

- 6) Perform the EMD-IT denoising on the IMFs $x_a(t)$ of to obtain a denoised version of $\tilde{x}_1(t)$ of x .

- 7) Iterate $k-1$ times between steps 3)–6), where k is the number of averaging iterations in order to obtain k denoised versions of x , i.e. $\tilde{x}_1, \tilde{x}_2, \tilde{x}_3, \dots, \tilde{x}_k$.

- 8) Average the resulted denoised signals

$$\hat{x}(t) = (1/K) \sum_{k=1}^K \tilde{x}_k(t)$$

Clear Iterative EMD Interval Thresholding: When the noise is relatively low, denoising can be achieved with a variant called clear iterative interval-thresholding (EMD-CIIT).EMD-IIT has to be replaced with the following four steps.

- 1) Perform an EMD expansion of the original noisy signal.
- 2) Perform a thresholding operation to the first IMF of $x(t)$ to obtain a denoised version of $\tilde{h}^1(t)$ of $h^{(1)}(t)$

3. Compute the actual noise signal that existed in $h^1(t)$

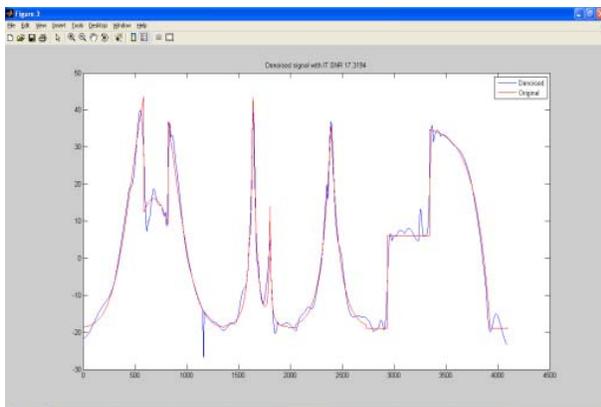
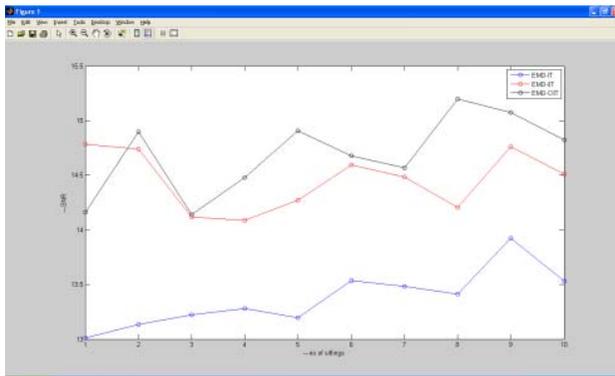
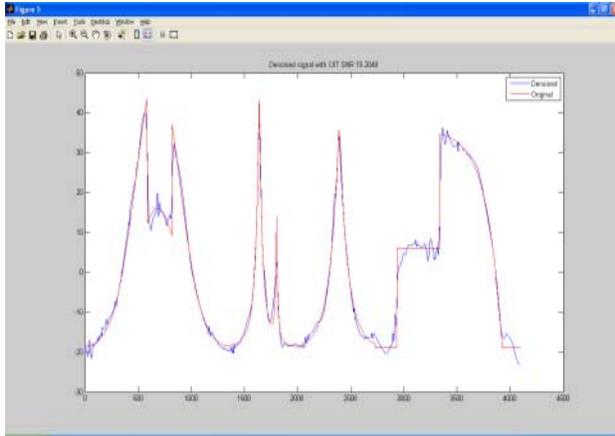
$$h_n^{(1)}(t) = \tilde{h}^1(t) - h^1(t).$$

$$x_p(t) = \sum_{i=2}^L h^{(i)}(t) + \tilde{h}^1(t).$$

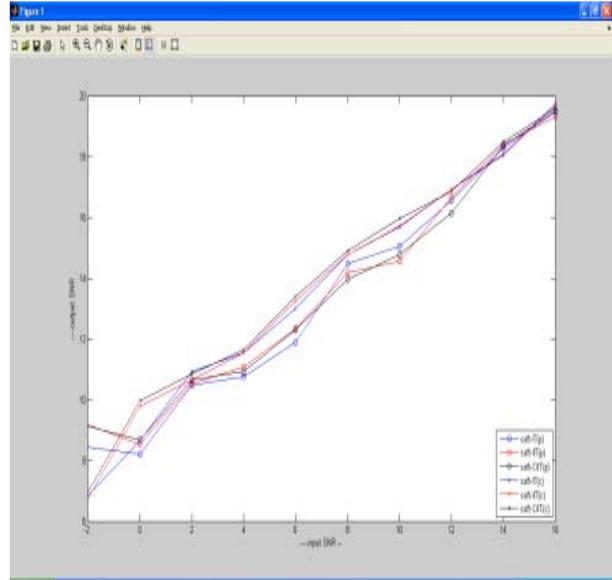
5) Randomly alter the sample positions of the noise-only part of the first IMF

$$h_n^{(1)}(t) = \text{ALTER}(h_n^{(1)}(t))$$

V. RESULTS



4) Perform a partial reconstruction using the last L-1 IMFs plus the information signal contained in the first IMF



The effect SNR performance adapting IMF with respect to the sifting iterations is studied in the above figure by taking various signals. According to irregularities e.g., the piece-regular signal case, the best performance is achieved with a relatively low number of sifting iterations. These results have been evaluated with other regular and irregular signals. The balanced tradeoff between number of sifting and performance is realized with about eight sifting iterations. Secondly, it is apparent that the sifting-dependent IMF curves do not offer significant advantages over fixed curves since the performance difference never exceeds 0.2 dB. In addition, the sifting-dependent curves can even lead to slight performance deterioration in the case of EMD-CIT when the signal has intense irregularities and a small number of sifting iterations are used.

VI. CONCLUSION

In this paper, the principles of wavelet thresholding were appropriately modified to develop denoising methods suited for thresholding EMD modes. Presented denoising in the cases when the signal SNR is low and/or the sampling frequency is high and enhanced performance compared to wavelet. These results suggest further efforts for improvement of EMD based denoising when denoising the signals with moderate to high SNR

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