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THERMAL RESPONSE OF ISOTROPIC PLATES USING HYPERBOLIC SHEAR DEFORMATION THEORY

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Abstract- In this paper, a hyperbolic shear deformation theory taking into account transverse shear deformation effects, is presented for the bending analysis of thick isotropic plates subjected to linear thermal load. The displacement field of the theory contains three variables. The hyperbolic sine and cosine function is used in the displacement field in terms of thickness coordinate to represent the effect of shear deformation. The most important feature of the theory is that the transverse shear stresses can be obtained directly from the use of constitutive relations, satisfying the stress free boundary conditions at top and bottom surfaces of the plate. Hence, the theory eliminates the need of shear correction factor. Governing differential equations and boundary conditions of the theory are obtained using the principle of virtual work. Results obtained for bending analysis of isotropic plates subjected to linear thermal load are compared with those of other higher order theories, lower order theories to validate the accuracy of the present theory.

Keywords- *Shear deformation, isotropic, shear correction factor, transverse shear stress, thermal load*

I. INTRODUCTION

Thick beams and plates, either isotropic or an isotropic, basically form two-and three-dimensional problems of elasticity theory. Reduction of these problems to the corresponding one- and two-dimensional approximate problems for their analysis has always been the main objective of research workers. As a result, numerous refined theories of beams and plates have been formulated in last two decades which approximate the three dimensional solutions with reasonable accuracy.

The shear deformation effects are more pronounced in the thick plates when subjected to transverse loads than in the thin plates under similar loading. The shear deformation effects are more significant in the thick plates. These effects are neglected in Classical Plate Theory (CPT). In order to describe the correct bending behavior of thick plates including shear deformation effects and the associated cross sectional warping, shear deformation theories are required. This can be accomplished by selection of proper kinematic and constitutive models.\

A. Various Shear Deformation Theories

Shear deformation theories can be classified into two major classes on the basis of assumed fields:
(1) Stress based theories
(2) Displacement based theories.

The stress based theories are derived from assumed stress field of axial stresses, which are assumed to vary linearly over the thickness of the plate. The transverse normal and shear stresses are then derived from the equilibrium equations of three dimensional problems in the theory of elasticity. The governing equations of the theory are derived using a stationary variational theorem.

The displacement based theories are as follows:

A. Classical plate theory (CPT)
B. First order shear deformation theory (FSDT)
C. Second order shear deformation theory (SSDT)
D. Higher order shear deformation theory (HSDT)
E. Parabolic shear deformation theory (PSDT)
F. Trigonometric shear deformation theory (TSDT)
G. Hyperbolic shear deformation theory (HYDT)
H. Exponential shear deformation theory (ESDT)

A. Classical Plate Theory (CPT)

Well-known classical plate theory (CPT) is based on the Kirchhoff hypothesis that straight lines normal to the undeformed midplane remain straight and normal to the deformed midplane and do not undergo thickness stretching (i.e., inextensible). The displacement field of the theory is as:

$$u = -z\frac{\partial w}{\partial x},\ v = -z\frac{\partial w}{\partial y},\ w = w(x, y) \qquad (1)$$

Where u, v and w are the displacement components in the x, y and z directions respectively.

B. First-order Shear Deformation Theories (FSDT):

To take into account the effect of shear deformation FSDT has been developed based on the hypothesis that the straight line normals to the mid-surface before deformation remains straight but not necessarily normal to the mid-surface after deformation. In FSDT transverse shear strain

distribution is assumed to be constant through the thickness and thus shear correction factors are required to take into account appropriate strain energy due to shear deformation.
Mindlin [1] and Ressiner [2,3] works on this theory. The displacement field of the theory is as:

$$u = z\phi,\ v = z\psi,\ w = w_0(x, y) \quad (2)$$

where, w_0 is the unknown function of position (x, y) to be determined and ϕ, ψ are the rotations of a transverse normal about the y and x axes, respectively.

C. Second-order Shear Deformation Theories (SSDT):

The second order shear deformation theories are given by Naghdi [4], Pister and Westmann [5]. The displacement field of the theory is as:

$$u = z\phi_x + z^2\psi_x, v = z\phi_y + z^2\psi_y, w = w_0 + z\phi_z + z^2\psi_z \quad (3)$$

where, $w_o, \phi_x, \phi_y, \phi_z, \psi_x, \psi_y, \psi_z$ are the unknown functions of position (x, y) to be determined.

D. Higher-order Shear Deformation Theory (HSDT):

In order to remove the deficiencies in CPT and FSDT, higher order shear deformation theories are developed to obtain the improved global response. In these theories the displacement field is expanded up to the third power of thickness coordinate of beams to have the parabolic variation of transverse shear stresses. Teregulov [6] presented a general method of formulating refined theories of plates and shells, which is based on the expansion of displacements, stresses, and strains in terms of thickness coordinate. The displacement field of the third order shear deformation theory is as:

$$u = z\phi_x + z^2\psi_x + z^3\xi_x,\ v = z\phi_y + z^2\psi_y + z^3\xi_y$$
$$w = w_0 + z\phi_z + z^2\psi_z + z^3\xi_z \quad (4)$$

where, $w_o, \phi_x, \phi_y, \phi_z, \psi_x, \psi_y, \psi_z, \xi_x, \xi_y, \xi_z$ are the unknown functions of position (x, y) to be determined.

E. Parabolic Shear Deformation Theory:

The third order parabolic shear deformation theories for the bending analysis of thick plates are developed by Reddy [7]. The displacement field of Reddy's third order shear deformation theory is as:

$$u = z\left[\phi_x - \frac{4}{3}\left(\frac{z}{h}\right)^2\left(\phi_x + \frac{\partial w}{\partial x}\right)\right], v = z\left[\phi_y - \frac{4}{3}\left(\frac{z}{h}\right)^2\left(\phi_y + \frac{\partial w}{\partial y}\right)\right], \quad (5)$$
$$w = w(x, y)$$

where, w, ϕ_x and ϕ_y are the unknown functions of position (x, y) to be determined.

F. Trigonometric Shear Deformation Theory:

There exists an another class of refined shear deformation theories wherein use of trigonometric function is made to take into account shear deformation effects. Levy [8] developed a refined theory for thick isotropic plate. The displacement field of the theory is as:

$$u = \sum_{n=0}^{N} u_n z^{2n+1} + \sum_{n=0}^{N} \sin\frac{(2n+1)\pi z}{h}\phi_x \quad (6)$$
$$v = \sum_{n=0}^{N} v_n z^{2n+1} + \sum_{n=0}^{N} \sin\frac{(2n+1)\pi z}{h}\phi_y, w = \sum_{n=0}^{N} w_n z^{2n}$$

where, u_n, v_n and w_n are the unknown functions of position (x, y) to be determined and are ϕ_x and ϕ_y are the rotations of a transverse normal about the x and y axes, respectively.

G. Hyperbolic Shear deformation theory:

Soladetos [9] develops a hyperbolic function theory for analysis of thick laminated plates, wherein use of hyperbolic function is made to take into account shear deformation effects. The displacement field is given in eq. (8) where, the hyperbolic function in terms of thickness coordinate in both the displacements u and v is associated with the transverse shear stress distribution through the thickness of plate and the functions $\phi(x, y)$ and $\psi(x, y)$ are the unknown functions associated with the shear slopes.

H. Exponential Shear deformation theory

Akavci [10] develops an exponential shear deformation theory for analysis of thick laminated plates on elastic foundation, wherein use of exponential function is made to take into account shear deformation effects. The displacement field is given as follows:

$$u = -z\frac{\partial w}{\partial x} + \left[z\exp\left(-2\left\{\frac{z}{h}\right\}^2\right)\right]\phi(x, y) \quad (7)$$
$$v = -z\frac{\partial w}{\partial y} + \left[z\exp\left(-2\left\{\frac{z}{h}\right\}^2\right)\right]\psi(x, y),\ w = w(x, y)$$

where, the functions $\phi(x, y)$ and $\psi(x, y)$ are the unknown functions associated with the shear slope.

II. STUDY OF THERMAL LOADING ON THICK PLATE

Various researchers like Ghugal and Kulkarni [11], Zhen and Wanji [12], Zhen and Cheng [13], Matsunaga [14, 15], Ali et. al. [16], Zenkour [17], Wang et al. [18], Nguyen and Caron [19] and Maenghyo Cho [20] studied the behavior of thick plates subjected to thermal load. This paper presents the bending response of thick isotropic plate under linearly thermal load using hyperbolic shear deformation theory. The principal of virtual work is used for deriving the governing equation and the boundary conditions. A simply supported plate is considered in the illustrative examples.

III. DEVELOPMENT OF THEORY

Consider the plate occupies in O – x – y - z Cartesian coordinate system the region:

$$0 \le x \le a; \quad 0 \le y \le b; \quad -h/2 \le z \le h/2$$

where, x, y, z are Cartesian coordinates, a and b are the edge lengths in the x and y directions respectively, and h is the thickness of the plate. The plate is made up of homogeneous isotropic material and obeys generalized Hooke's law.

A. The Displacement Field:

The displacement field of the present theory can be expressed as follows:

$$u = -z\frac{\partial w}{\partial x} + \left[z\cos h\left(\frac{1}{2}\right) - h\sinh\left(\frac{z}{h}\right)\right]\phi(x,y) \qquad (8)$$

$$v = -z\frac{\partial w}{\partial y} + \left[z\cos h\left(\frac{1}{2}\right) - h\sinh\left(\frac{z}{h}\right)\right]\psi(x,y),\; w = w(x,y)$$

Here u and v are the inplane displacement components in the x and y directions respectively, and w is the transverse displacement in the z direction. The hyperbolic function in terms of thickness coordinate in both the displacements u and v is associated with the transverse shear stress distribution through the thickness of plate and the functions $\phi(x,y)$ and $\psi(x,y)$ are the unknown functions associated with the shear slopes.

B. Normal Strain: $\varepsilon_x = \dfrac{\partial u}{\partial x},\ \varepsilon_y = \dfrac{\partial v}{\partial y}$ (9)

C. Shear Strain

$$\gamma_{xy} = \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y},\ \gamma_{zx} = \frac{\partial u}{\partial z} + \frac{\partial w}{\partial x},\ \gamma_{yz} = \frac{\partial v}{\partial z} + \frac{\partial w}{\partial y} \qquad (10)$$

D. Stress-Strain Relationships

$$\begin{Bmatrix}\sigma_x\\ \sigma_y\\ \tau_{xy}\end{Bmatrix} = \begin{bmatrix}Q_{11} & Q_{12} & 0\\ Q_{12} & Q_{22} & 0\\ 0 & 0 & Q_{66}\end{bmatrix}\begin{Bmatrix}\varepsilon_x - \alpha_x T\\ \varepsilon_y - \alpha_y T\\ \gamma_{xy}\end{Bmatrix} \text{ and } \begin{Bmatrix}\tau_{yz}\\ \tau_{zx}\end{Bmatrix} = \begin{bmatrix}Q_{44} & 0\\ 0 & Q_{55}\end{bmatrix}\begin{Bmatrix}\gamma_{yz}\\ \gamma_{zx}\end{Bmatrix} \qquad (11)$$

For a linearly elastic isotropic material, stresses $(\sigma_x$ and $\sigma_y)$ are related to strains $(\varepsilon_x$ and $\varepsilon_y)$ and shear stresses $(\tau_{xy}, \tau_{yz}, \tau_{zx})$ are related to shear strains $(\gamma_{xy}, \gamma_{yz}, \gamma_{zx})$

T is a thermal load which consists of linear temperature distribution through the thickness of plate.

E. Governing Equations and Boundary Conditions:

Using the expressions for strains and stresses (10) through (12) and using the principle of virtual work [13], variationally consistent governing differential equations and boundary conditions for the plate under consideration can be obtained. The principle of virtual work when applied to the plate leads to:

$$\int_{-h/2}^{h/2}\int_0^a\int_0^b \begin{bmatrix}\sigma_x\delta\varepsilon_x + \sigma_y\delta\varepsilon_y + \tau_{xy}\delta\gamma_{xy}\\ +\tau_{xz}\delta\gamma_{xz} + \tau_{yz}\delta\gamma_{yz}\end{bmatrix} dx\,dy\,dz = \int_0^a\int_0^b q\,\delta w\,dx\,dy \qquad (12)$$

where, symbol δ denotes the variation operator. The governing differential equations obtained are as follows:

δw:
$$\left(D_{11}\frac{\partial^4 w}{\partial x^4} + (2D_{12}+4D_{66})\frac{\partial^4 w}{\partial x^2\partial y^2} + D_{22}\frac{\partial^4 w}{\partial y^4}\right) - \left(S_{11}\frac{\partial^3\phi}{\partial x^3} + S_{22}\frac{\partial^3\psi}{\partial y^3}\right) - (S_{12}+2S_{66})\left(\frac{\partial^3\phi}{\partial x\partial y^2} + \frac{\partial^3\psi}{\partial x^2\partial y}\right)$$
$$+(TD_{11}+TTD_{12})\frac{\partial^2 T_1}{\partial x^2} + (TD_{12}+TTD_{22})\frac{\partial^2 T_1}{\partial y^2} = 0$$

$$\delta\phi: S_{11}\frac{\partial^3 w}{\partial x^3} + (S_{12}+2S_{66})\frac{\partial^3 w}{\partial x\partial y^2} - \left(SS_{11}\frac{\partial^2\phi}{\partial x^2} + SS_{66}\frac{\partial^2\phi}{\partial y^2}\right)$$
$$+C_{55}\phi - (SS_{12}+SS_{66})\frac{\partial^2\psi}{\partial x\partial y} + (TS_{11}+TTS_{12})\frac{\partial T_1}{\partial x} = 0$$

$$\delta\psi: S_{22}\frac{\partial^3 w}{\partial y^3} + (S_{12}+2S_{66})\frac{\partial^3 w}{\partial x^2\partial y} - \left(SS_{66}\frac{\partial^2\psi}{\partial x^2} + SS_{12}\frac{\partial^2\psi}{\partial y^2}\right)$$
$$+C_{44}\psi - (SS_{12}+SS_{66})\frac{\partial^2\phi}{\partial x\partial y} + (TS_{12}+TTS_{22})\frac{\partial T_1}{\partial y} = 0$$

(13)

The associated consistent boundary conditions obtained are as below: Along the edge x = 0 and x = a

$$-D_{22}\frac{\partial^3 w}{\partial y^3} - (D_{12}+4D_{66})\frac{\partial^3 w}{\partial y\partial x^2} + \left(2S_{66}\frac{\partial^2\psi}{\partial x^2} + S_{22}\frac{\partial^2\psi}{\partial y^2}\right)$$
$$+(S_{12}+2S_{66})\frac{\partial^2\phi}{\partial x\partial y} - (TD_{12}+TTD_{22})\frac{\partial T_1}{\partial y} = 0$$

or w is prescribed

$$\left(D_{12}\frac{\partial^2 w}{\partial x^2} + D_{22}\frac{\partial^2 w}{\partial y^2}\right) - S_{12}\frac{d\phi}{dx} - S_{22}\frac{d\psi}{dy} +$$
$$(TD_{12}+TTD_{22})T_1 = 0$$

or $\dfrac{\partial w}{\partial y}$ is prescribed

$$SS_{66}\left(\frac{d\phi}{\partial y} + \frac{d\psi}{\partial x}\right) - 2S_{66}\frac{\partial^2 w}{\partial y\partial x} = 0$$ or ϕ is prescribed

$$-S_{12}\frac{\partial^2 w}{\partial x^2} + SS_{12}\frac{\partial\phi}{\partial x} - S_{22}\frac{\partial^2 w}{\partial y^2} + SS_{22}\frac{\partial\psi}{\partial y}$$
$$-(TS_{12}+TTS_{22})T_1 = 0$$

or ψ is prescribed (14)

Along the edge y = 0 and y = b:

$$-D_{11}\frac{\partial^3 w}{\partial x^3} - (D_{12}+4D_{66})\frac{\partial^3 w}{\partial x\partial y^2} + \left(2S_{66}\frac{\partial^2\phi}{\partial y^2} + S_{11}\frac{\partial^2\phi}{\partial x^2}\right)$$
$$+(S_{12}+2S_{66})\frac{\partial^2\psi}{\partial x\partial y} - (TD_{11}+TTD_{12})\frac{\partial T_1}{\partial x} = 0$$

or w is prescribed

$$\left(D_{11}\frac{\partial^2 w}{\partial x^2} + D_{12}\frac{\partial^2 w}{\partial y^2}\right) - S_{11}\frac{d\phi}{dx} - S_{12}\frac{d\psi}{dy}$$

$$+(TD_{12}+TTD_{11})T_1=0$$

or $\dfrac{\partial w}{\partial x}$ is prescribed

$$-\left(S_{11}\frac{\partial^2 w}{\partial x^2}+S_{12}\frac{\partial^2 w}{\partial y^2}\right)+SS_{11}\frac{d\phi}{dx}+SS_{12}\frac{d\psi}{dy}$$

$$-(TS_{11}+TTS_{12})T_1=0$$

or ϕ is prescribed

$$SS_{66}\left(\frac{d\psi}{dx}+\frac{d\phi}{dy}\right)-2S_{66}\frac{\partial^2 w}{\partial x\partial y}=0$$ or ψ is prescribed (15)

IV. ILLUSTRATIVE EXAMPLES

A. The Closed form Solution:

The following is the solution form for *w(x, y), ϕ(x, y),* and *ψ(x, y)* satisfying the boundary conditions given by the equations through perfectly for a plate with all the edges simply supported:

$$w(x,y)=\sum_{m=1}^{\infty}\sum_{n=1}^{\infty} w_{mn}\sin\frac{m\pi x}{a}\sin\frac{n\pi y}{b}$$
$$\phi(x,y)=\sum_{m=1}^{\infty}\sum_{n=1}^{\infty} \phi_{mn}\cos\frac{m\pi x}{a}\sin\frac{n\pi y}{b} \quad (16)$$
$$\psi(x,y)=\sum_{m=1}^{\infty}\sum_{n=1}^{\infty} \psi_{mn}\sin\frac{m\pi x}{a}\cos\frac{n\pi y}{b}$$
$$T_1(x,y)=\sum_{m=1}^{\infty}\sum_{n=1}^{\infty} T_{mn}\sin\frac{m\pi x}{a}\sin\frac{n\pi y}{b}$$

where, w_{mn}, ϕ_{mn}, ψ_{mn} and T_{mn} are coefficients, which can be easily evaluated after substitution of Eq. (16) in the set of three governing differential equations (14) and solving the resulting simultaneous equations (see Appendix A). Having obtained the values of w_{mn}, ϕ_{mn}, ψ_{mn} and T_{mn} one can then calculate all the displacement and stress components within the plate. For linear thermal load $T_{mn}=T_0$ and $\alpha_x=\alpha_y=\alpha_c$

B. Illustrative Example:

A plate of length a, width b, and thickness h is considered. The plate has simply supported boundary conditions at edges x = 0, a and y = 0, b. The plate subjected to sinusoidal thermal load as given below

$$T(x,y,z)=\frac{2}{h}zT_1(x,y)$$

The following isotropic material properties are used,

$$E=380\,GPa,\ \mu=0.3,\ \alpha_x=\alpha_y=7.4\times10^{-6}$$

V. RESULTS AND DISCUSSION

Displacements and stresses are obtained for isotropic plates under linear thermal load and presented in the following normalized forms for the purpose of discussion.

$$\bar{u}=\frac{u}{\alpha_c T_0 h},\quad \bar{v}=\frac{v}{\alpha_c T_0 h},\quad \bar{w}=\frac{w}{\alpha_c T_0 h}$$

$$\bar{\sigma}_x=\frac{\sigma_x}{\alpha_c T_0 E_C},\quad \bar{\sigma}_y=\frac{\sigma_y}{\alpha_c T_0 E_C},\quad \bar{\tau}_{xy}=\frac{\tau_{xy}}{\alpha_c T_0 E_C}$$

TABLE I COMPARISON OF INPLANE DISPLACEMENT FOR THE ISOTROPIC PLATE SUBJECTED TO SINUSOIDAL THERMAL LOAD.

a/h	Source	Model	$\bar{u}$
5	Present	HYDT	1.0345
	Ghugal and Kulkarni [11]	TSDT	1.0345
10	Present	HYDT	2.0690
	Ghugal and Kulkarni [11]	TSDT	2.0690
	Mindlin [1]	FSDT	2.0690
	Kirchoff	CPT	2.0690

TABLE II COMPARISON OF TRANSVERSE DISPLACEMENTS FOR THE ISOTROPIC PLATE SUBJECTED TO SINUSOIDAL THERMAL LOAD.

a/h	Source	Model	$\bar{w}$
5	Present	HYDT	3.2930
	Ghugal and Kulkarni[11]	TSDT	3.2930
10	Present	HYDT	13.1719
	Ghugal and Kulkarni [11]	TSDT	13.1719
	Mindlin [1]	FSDT	13.1717
	Kirchoff	CPT	13.1718
	Matsunga[14,15]	HSDT	13.1100

TABLE III COMPARISON OF INPLANE NORMAL STRESSES FOR THE ISOTROPIC PLATE SUBJECTED TO SINUSOIDAL THERMAL LOAD.

a/h	Source	Model	$\bar{\sigma}_x$	$\bar{\sigma}_y$
5	Present	HYDT	0.500	0.500
	Ghugal and Kulkarni [11]	TSDT	0.500	0.500
10	Present	HYDT	0.500	0.500
	Ghugal and Kulkarni [11]	TSDT	0.500	0.500
	Mindlin [1]	FSDT	0.500	0.500
	Kirchoff	CPT	0.500	0.500
	Matsunga [14,15]	HSDT	---	--

TABLE IV COMPARISON OF INPLANE AND TRANSVERSE SHEAR STRESS FOR THE ISOTROPIC PLATE SUBJECTED TO SINUSOIDAL THERMAL LOAD.

a/h	Source	Model	$\bar{\tau}_{xy}$	$\bar{\tau}_{yz}$
5	Present	HYDT	0.50	0.50

	Ghugal and Kulkarni [11]	TSDT	0.50	0.50
10	Present	HYDT	0.50	0.50
	Ghugal and Kulkarni [11]	TSDT	0.50	0.50
	Mindlin [1]	FSDT	0.50	0.50
	Kirchoff	CPT	0.50	0.50
	Matsunga [14,15]	HSDT	---	---

C. Discussion of Result

The results obtained for displacement and stresses for simply supported isotropic plate subjected to linear thermal load are presented in Tables I through IV. Through thickness variation of displacement and stresses for aspect ratio 10 are shown in Figures 1 through 3.

From Tables and Figures, it is observed that, the results obtained by present theory for inplane displacements, inplane normal stresses, inplane shear stress and transverse shear stress, are identical with those obtained by other theories. The transverse displacement (w) obtained by present theory is identical with that obtained by Ghugal and Kulkarni's TSDT theory whereas Mindlin's FSDT and Kirchhoff's CPT underestimates the same.

D. Conclusions

Thermal response of isotropic plate under linear temperature distribution through the thickness of plate has been studied by using hyperbolic shear deformation theory. From the numerical results and discussion following conclusions are drawn.

1. The present theory is variationally consistent and does not require shear correction factor.
2. Present theory gives accurate prediction of thermal response of isotropic plate respect of displacement and stresses.
3. Inplane displacements and normal stresses obtained by present theory and other higher order theories are identical.
4. Transverse displacements obtained by present theory are in excellent agreement with those of other higher order theories.
5. Transverse shear stresses are zero in case of isotropic plate subjected to linear thermal load.

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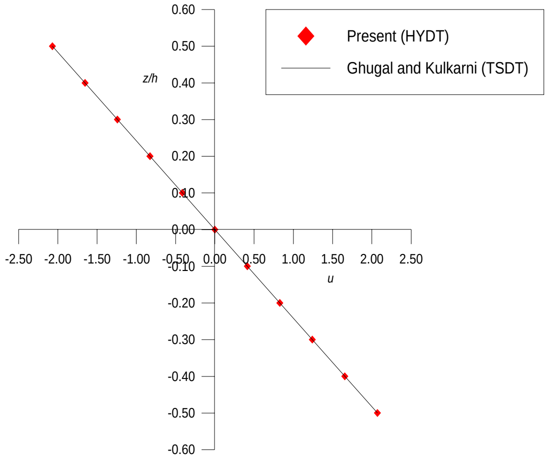
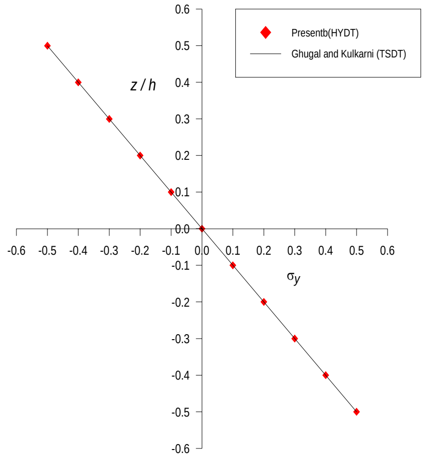
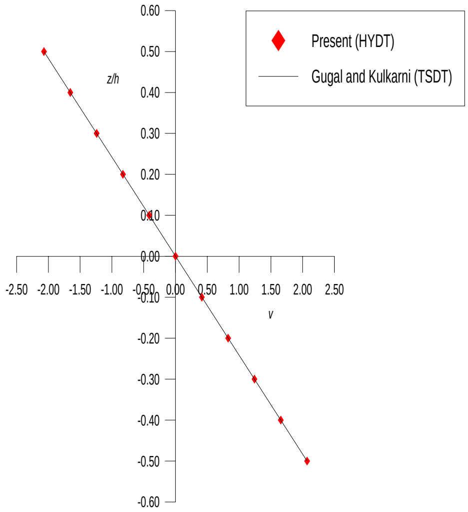
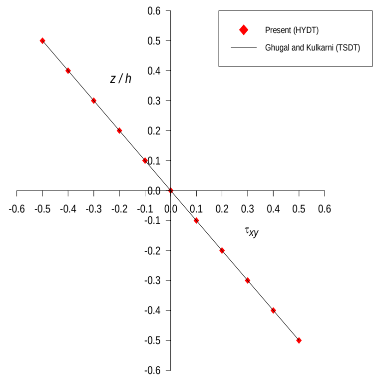
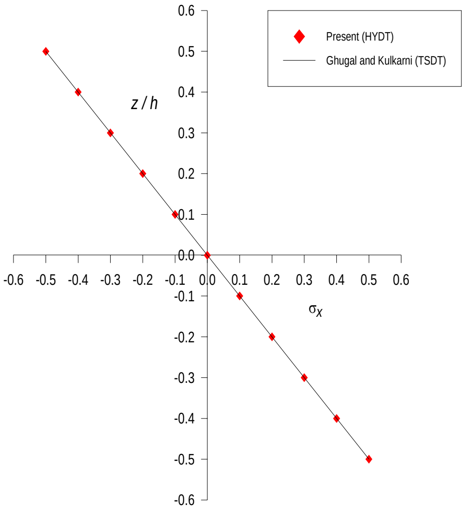








Through thicness distribution of inplane n normal stresses Figure 3.Through thicness distribution of inplane shear stresss of shisotropic plate for aspct ratio 10. of isotropic plate for aspect ratio 10.



Through thicness distribution of inplane displacements of isotropic plate for aspct ratio 10. Figure 2. Through thicness distribution of inplane normal stresses of shisotropic plate for aspct ratio 10.

VI. APPENDIX

The coefficients appearing in the governing differential equations and boundary conditions are as follows:

$$f(z)=\left[z\cos h\left(\frac{1}{2}\right)-z\sin h\left(\frac{z}{h}\right)\right]$$

$$A=\int_{-h/2}^{+h/2} Z^2\,dz,\; B=\int_{-h/2}^{+h/2} Zf(z)\,dz,\; C=\int_{-h/2}^{+h/2}\left[f(z)\right]^2 dz$$

$$D=\int_{-h/2}^{+h/2}\left[\frac{\partial\left[f(z)\right]}{\partial Z}\right]^2 dz$$

$D_{11}=AQ_{11}$; $D_{12}=AQ_{12}$; $D_{22}=AQ_{22}$; $D_{66}=AQ_{66}$;

$S_{11}=BQ_{11}$; $S_{12}=BQ_{12}$; $S_{22}=BQ_{22}$; $S_{66}=BQ_{66}$;

$SS_{11}=CQ_{11}$; $SS_{12}=CQ_{12}$; $SS_{22}=CQ_{22}$; $SS_{66}=CQ_{66}$;

$TD_{11}=AQ_{11}\alpha_x$; $TD_{12}=AQ_{12}\alpha_x$; $TD_{22}=AQ_{22}\alpha_x$;

$TS_{11}=BQ_{11}\alpha_x$; $TS_{12}=BQ_{12}\alpha_x$; $TS_{22}=BQ_{22}\alpha_x$;

$TTD_{11}=AQ_{11}\alpha_y$; $TTD_{12}=AQ_{12}\alpha_y$; $TTD_{22}=AQ_{22}\alpha_y$;

$TTS_{11}=BQ_{11}\alpha_y$; $TTS_{12}=BQ_{12}\alpha_y$; $TTS_{22}=BQ_{22}\alpha_y$;

$C_{55}=DQ_{55}$; $C_{44}=DQ_{44}$

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