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COMPARATIVE STUDY OF ADVANCED TWO DIMENSIONAL METHODS OF SLOPE STABILITY ANALYSIS

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Abstract- The analysis and design of failing slopes and highway embankments requires an in-depth understanding of the failure mechanism in order to choose the right slope stability analysis method. In the present work, the concept and theory involved in different methods of slope stability analysis of earth embankment have been discussed. The mathematical equations and the methodology for calculating the factor of safety of earth slope of any specified (chosen) slip circle by various methods has been given. By repeating the process for different slip circles, the minimum factor of safety can be calculated and critical slip circle obtained. A simple example of earth embankment has been analyzed to illustrate the methodology with the help of Microsoft Excel program for various methods and the values of FOS resulted are compared.

I. INTRODUCTION

The slope-stability analysis is usually carried out by Fellenius (Fellenius, 1936) method also called ordinary or Swedish Circle method (Terzaghi, 1962.). It ignores inter slice forces and considers only moment equilibrium and not force equilibrium conditions. Thus, it provides only moment factor of safety and not force factor of safety. It also does not provide closed force polygon in a free body force diagram for the individual slices within the failure zone of earth mass. It provides the factor of safety on lower side and thus results into a more conservative design of earth embankment. The Bishop (Bishop, 1960) simplified method takes into consideration only interslice normal force, but ignores the interslice shear force. It gives moment factor of safety only, and the force polygon does not close. The Morgenstern-Price (Morgenstern and Price, 1965) method takes into consideration both the interslice normal and shear forces, provides both moment equilibrium and force equilibrium and gives moment and force factor of safeties. In the present study a simple embankment is has been analyzed by Fellenius, Simplified Bishop's and Morgenstern-Price method and comparison has been made amongst them.

II. ANALYSIS OF EXAMPLE PROBLEM

In the following paragraph a simple embankment of earth has been taken for illustrating the various methods. The soil properties of embankment soil and foundation soil are shown in fig.

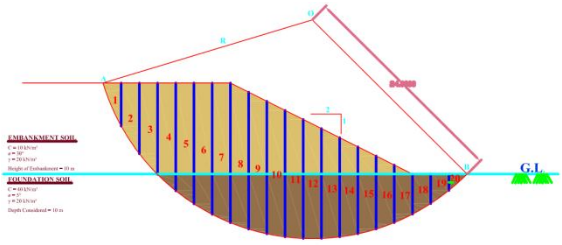

Fig.1 Earth Embankment adopted for Analysis

For the earth embankment shown in figure the twenty number of slices are made each having the width of 2m and corresponding weights of each slice is calculated by multiplying the mid height of the slice, its width and corresponding unit weight of the soil in which it exists. For the slices those are both in foundation and embankment, there weights are calculated by multiplying width by the mid height in an embankment and unit weight of embankment soil and by multiplying the width by mid height of slice in foundation and corresponding unit weight of foundation soil. An angle α of each slice is an angle made by base of that particular slice with the horizontal is calculated by drawing an embankment in AutoCAD. The given embankment is first analyzed using Fellenius method, Simplified Bishop method and Morgenstern & Price's for constant sin function method theoretical equations as discussed in earlier chapters using Excel sheets. These are iterative methods and hence seem to be very cumbersome to be used for hand calculations and hence they are analyzed using Excel sheets which make our job quite easy.

2.1 Discussion on Morgenstern-Price Method

Morgenstern and Price proposed an empirical equation for inter slice normal and shear force as shown

(Eq. 1)

Where, $f(x)$ is an assumed function that varies continuously across the slip, and ( ) is an unknown scaling factor that is solved for as part of the unknowns.

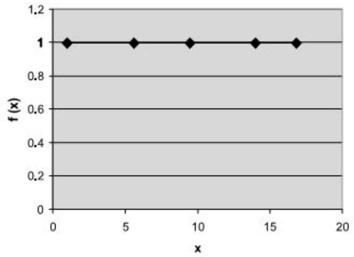

Fig. 2 Constant sine Function

2.2 What is λ?

Lambda stated in above equation is the percentage of applied function to specified function f (x). The specified function can have many shapes as shown in figure 4.1. In case of half-sine function the shape shall be as shown in figure

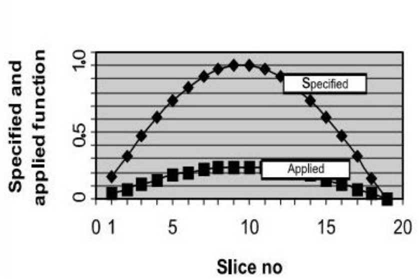

Fig. 3 Showing Specified Function and Applied Function

The value of lambda for a typical slice shall be ratio of ordinate f (x) against that slice read from applied function (lower curve in Fig. 4.2) and divided by the value of f (x) read from specified function (upper curve in Fig. 4.2) for the same slice. This is to note that the λ is same for all the slices. However, the value of f (x) itself varies from slice to slice in half sine function and hence the ratio of slice shear force to slice normal force (X/E) shall also vary from slice to slice. However, in a constant function this ratio of slice shear force to slice normal force shall not vary as the value of f (x) for all slices is (1.0) i.e. Constant. The λ in any case is constant.

2.3 Computation of Factor of Safety

The analysis has been done by various methods keeping all other features such as cross-section of the embankment, geo-technical properties of the ground soil and borrow material same, for the sole purpose of not only understanding these methods but also provide a comparative study and help selection of most appropriate method for adopting in a particular case.

To start the solution we may compute Fellenius method factor of safety obtained by putting P=Wcosα in the following equation

$$F_m = \frac{\Sigma(cl + P\tan\phi)}{\Sigma(W\sin\alpha)} \qquad (4.2)$$

F_m can be computed using above equation as all factors are known. Substituting F_m for F and W cosα for P in the equation for inter slice normal force (E_R). Right side inter slice normal force can be calculated for the first slice (E_L is zero for the first slice). By repeating, E_R for the second slice is obtained (E_R for the first slice is E_L for the second slice). This way E_R and E_L for all slices can be obtained. Choosing a function and a value for λ the inter slice shear force X_R and X_L are obtained. Knowing E_R, E_L, X_R, X_L the value of P is obtained by using equation

$$P = \frac{[W - (X_R - X_L) - \frac{1}{F}(cl\sin\alpha)}{\cos\alpha + \frac{(\sin\alpha\tan\phi')}{F}} \qquad (4.3)$$

Taking this value of P and putting it in equation 4.2 the new FOS is obtained. The new values of P and F will provide new values of E_R, E_L, X_R, X_L by the relevant equations and further new values of P and F. This process is repeated till a converged value of F is obtained. The converged value of of F is required FOS and E_R, E_L, X_R, X_L and P which produced this is required value for these factors. This appears cumbersome but very easy to apply in a Microsoft Excel program.

To obtain FOS Morgenstern-Price method (or any other method which considers inter slice shear force) λ is required to be decided. The best result is obtained when the λ is chosen such that moment FOS and force FOS become equal. This is done by plotting FOS against λ for F_m and F_f. The intersection point provides this λ the value is then read from this curve and used for analysis.

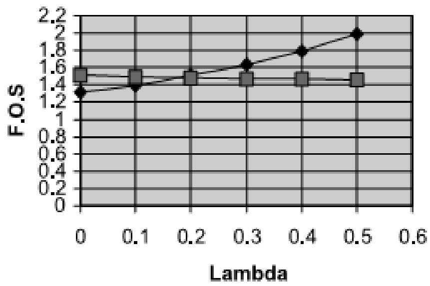

Fig. 4 Lambda vs. Factor of Safety (Constant Function)

The intersection point in case of consant sine function gives a value of λ= 0.1814. This value of λ has been used for computation.

Fellenius method of computation of FOS is used for Bishop critical circle and Morgenstern Price critical circle. Factor of safety F=1.488 and F=1.708 respectively were worked out.

In the subsequent paragraph the FOS are calculated using Bishop method, Fellenius method and Morgenstern-Price method using mathematical equations explained in the chapter 2 using Microsoft Excel program iteratively and tabulated as shown.

Table 1 FOS Calculated by Fellenius Method for Bishop's Critical Circle

FOS CALCULATED BY FELLENIUS METHOD FOR BISHOP'S CRITICAL									
Slice no.	W (kn)	α (deg)	l (m)	c (kn/m²)	ϕ (deg)	α (rad)	ϕ (rad)	w*cosα*tanϕ+cl/	w*sinα
1	108.54	67	5.138	10	30	1.169	0.524	75.865	99.912
2	254.992	57	3.634	10	30	0.995	0.524	116.520	213.854
3	359.02	49	3.027	10	30	0.855	0.524	166.260	270.956
4	439.392	42	2.684	40	5	0.733	0.087	135.928	294.011
5	503.456	36	2.47	40	5	0.628	0.087	134.423	295.924
6	554.984	30	2.309	40	5	0.524	0.087	134.410	277.492
7	596.204	25	2.206	40	5	0.436	0.087	135.530	251.967
8	608.524	19	2.126	40	5	0.332	0.087	135.366	198.116
9	592.868	14	2.065	40	5	0.244	0.087	132.940	143.428
10	569.852	10	2.038	40	5	0.175	0.087	130.618	98.954
11	539.872	5	2.007	40	5	0.087	0.087	127.329	47.053
12	502.348	0	2.000	40	5	0.000	0.087	123.950	0.000
13	459.744	-5	2.047	40	5	-0.087	0.087	121.953	-40.069
14	409.588	-10	2.035	40	5	-0.175	0.087	116.686	-71.124
15	352.46	-15	2.066	40	5	-0.262	0.087	112.434	-91.223
16	287.964	-20	2.122	40	5	-0.349	0.087	108.562	-98.489
17	215.484	-25	2.202	40	5	-0.436	0.087	105.146	-91.067
18	154.076	-30	2.318	40	5	-0.524	0.087	104.406	-77.038
19	102.324	-36	2.526	40	5	-0.628	0.087	108.262	-60.145
20	37.984	-42	2.689	40	5	-0.733	0.087	110.042	-25.416
							SUM	2436.630	1637.093

Table 2 FOS Calculated by Bishop's Method (Iteration 6)

FOS CALCULATED BY BISHOP'S METHOD (ITERATION 6)													
Slice no.	W (kn)	α (deg)	l (m)	F	c (kn/m²)	ϕ (deg)	α (rad)	ϕ (rad)	E_R= (E_L)-(c*l*cosα/F)+(P)*((sinα)-(tanϕ*cosα)/(F	E L	C*l + P *tanϕ	w*sinα	P =(((W)-((C*l*sinα))/(F)))/(cosα)+ (tanϕ*sinα)/(F)
1	108.	67	5.1	1.67	1	30	1.1	0.52	7	0	116.7	99.9	1
2	254.9	57	3.6	1.67	1	30	0.9	0.52	2	76.87	200.1	213.8	2
3	359.	49	3.0	1.67	1	30	0.8	0.52	4	249.55	247.7	270.9	3
4	439.3	42	2.6	1.67	4	5	0.7	0.08	7	436.52	151.9	294.0	5
5	503.4	36	2.4	1.67	4	5	0.6	0.08	9	709.77	147.6	295.9	5
6	554.9	30	2.3	1.67	4	5	0.5	0.08	1187.167	966.33	144.0	277.4	5
7	596.2	25	2.2	1.67	4	5	0.4	0.08	1371.169	1187.1	142.3	251.9	6
8	608.5	19	2.1	1.67	4	5	0.3	0.08	1492.804	1371.1	138.8	198.1	6
9	592.8	14	2.0	1.67	4	5	0.2	0.08	1557.757	1492.8	134.3	143.4	5
10	569.8	10	2.0	1.67	4	5	0.1	0.08	1578.647	1557.7	130.9	98.9	5
11	539.8	5	2.0	1.67	4	5	0.0	0.08	1549.500	1578.6	127.1	47.0	5
12	502.3	0	2.0	1.67	4	5	0.0	0.08	1475.301	1549.	123.9	0.00	5
13	459.7	-	2.0	1.67	4	5	-	0.08	1361.273	1475.3	122.8	-	4
14	409.5	-	2.0	1.67	4	5	-	0.08	1216.789	1361.2	118.8	-	4
15	352.	-	2.0	1.67	4	5	-	0.08	1050.332	1216.7	116.2	-	3
16	287.9	-	2.1	1.67	4	5	-	0.08	8	1050.3	113.8	-	3
17	215.4	-	2.2	1.67	4	5	-	0.08	6	872.98	111.5	-	2
18	154.0	-	2.3	1.67	4	5	-	0.08	5	698.79	111.6	-	2
19	102.3	-	25	1.67	4	5	-	0.08	3	532.64	116.5	-	1
20	37.9	-	2.6	1.67	4	5	-	0.08	2	372.08	117.5	-	1
										sum =	2734.811		

Table 3 Moment FOS Calculated by Morgenstern-Price's Method for Constant Function (Iteration 8)

Fm CALCULATED BY MORGENSTERN-PRICE'S METHOD FOR CONSTANT																
Slice n	W (kn)	P (kN)	α (deg)	l (m	c (kn/m²)	φ (deg)	α (rad)	φ (rad)	F	E_R=(E_L)-((c*l*cosα)/(F))+(P)*(sinα-	E L	X_R=0.1814	X_L=0.1814 E	P =(W-(X_R-X_L)-(c*l*sinα / F	(P *tanφ) +	w*sinα
1	108.	95.8	6	5.1	1	3	1.1	0.5	1.6	6	0.00	11.4	0.00	9	106.6	99.9
2	254.9	249.5	5	3.6	1	3	0.9	0.5	1.6	212.381	62.8	38.5	11.40	249.515	180.3	213.8
3	359.	341.8	4	3.0	1	3	0.8	0.5	1.6	379.579	212.3	68.8	38.52	341.826	227.6	270.9
4	439.3	453.0	4	2.6	4	5	0.7	0.0	1.6	616.302	379.5	111.7	68.85	453.018	146.9	294.0
5	503.4	507.9	3	2.4	4	5	0.6	0.0	1.6	844.402	616.3	153.1	111.7	507.903	143.2	295.9
6	554.9	549.4	3	2.3	4	5	0.5	0.0	1.6	1045.214	844.4	189.6	153.1	549.489	140.4	277.4
7	596.2	584.2	2	2.2	4	5	0.4	0.0	1.6	1215.352	1045.	220.4	189.6	584.276	139.3	251.9
8	608.5	592.9	1	2.1	4	5	0.3	0.0	1.6	1329.718	1215.	241.2	220.4	592.989	136.9	198.1
9	592.8	579.3	1	2.0	4	5	0.2	0.0	1.6	1391.243	1329.	252.3	241.2	579.312	133.2	143.4
1	569.8	561.0	1	2.0	4	5	0.1	0.0	1.6	1410.485	1391.	255.8	252.3	561.099	130.6	98.9
1	539.8	540.6	5	2.0	4	5	0.0	0.0	1.6	1380.443	1410.	250.4	255.8	540.637	127.5	47.0
1	502.3	516.1	0	2.0	4	5	0.0	0.0	1.6	1228.034	1304.	222.7	236.5	516.150	125.1	0.00
1	459.7	489.6	-	2.0	4	5	-	0.0	1.6	1109.826	1228.	201.3	222.7	489.658	124.7	-
1	409.5	456.9	-	2.0	4	5	-	0.0	1.6	957.823	1109.	173.7	201.3	456.915	121.3	-
1	352.	417.7	-	2.0	4	5	-	0.0	1.6	779.708	957.8	141.4	173.7	417.759	119.1	-
1	287.9	369.7	-	2.1	4	5	-	0.0	1.6	586.286	779.7	106.3	141.4	369.722	117.2	-
1	215.4	309.2	-	2.2	4	5	-	0.0	1.6	392.171	586.2	71.1	106.3	309.245	115.1	-
1	154.0	258.1	-	2.3	4	5	-	0.0	1.6	202.394	392.1	36.7	71.14	258.136	115.3	-
1	102.3	222.2	-	2.5	4	5	-	0.0	1.6	1	202.3	2.26	36.71	222.262	120.4	-
2	37.9	156.3	-	2.6	4	5	-	0.0	1.6	-146.923	12.5	-	2.26	156.399	121.2	-
														s	2692.	1637.

III. RESULTS

The Factor of Safety obtained by the various methods are tabulated as follows,

Sr. No.	Method	Factor of Safety
1	Fellenius Method	1.488
2	Simplified Bishop's method	1.671
3	Morgenster-Price's Method (Moment FOS)	1.645
4	Morgenster-Price's Method (Force FOS)	1.491

IV. CONCLUSION

The following conclusion are drawn from following study,

1. Different limit equilibrium methods of stability analysis give different critical circle and different factor of safety for same situation. Even for same slip surface different methods give different factor of safety.
2. It has been seen that the Fellenius method of slope stability analysis gives the lowest FOS amongst the all other methods and hence it requires safer design of slope hence leads to the uneconomical design.
3. The value of FOS calculated by Morgenstern-Price method is almost equal to the value obtained by Bishop's which comparatively simple method for hand calculation is. Therefore Bishop's method can be used effectively if hand calculations are need to be done.

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