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Basanta Kumar Panigrahi

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Locational Marginal Pricing (LMP) in Deregulated Electricity Market

Basanta Kumar Panigrahi
Department Of Electrical Engineering.
IIT, Roorkee, Roorkee, INDIA
E-mail: basanta1983@gmail.com

ABSTRACT—The deregulation of electricity markets has a very large impact on almost all the power systems around the world. Competitive markets are complex systems with many participants who buy and sell electricity. Much of the complexity arises from the limitations of the underlying transmission systems and the fact that supply and demand must be in balance at all times. Generally the Locational marginal pricing (LMP) is obtained by solving a linear programming formulation. Network losses are considered through the preset loss factor based on historical operational information. This usually brings error in the calculated loss under different new scenarios. In this paper a new iterative LMP calculation method is proposed to overcome the aforementioned drawbacks associated with the traditional LMP calculations. At each iteration, a linear programming problem for market clearing is solved first. Losses on branches are considered as fictitious nodal demand at their terminal buses. Secondly the AC power flow calculated according to the dispatch results. Loss factors and fictitious nodal demand are then updated according to the AC power flow solution. The effectiveness of the proposed method is illustrated on PJM 5 bus

Keywords-AC power flow, active power loss, congestion, locational marginal pricing, power market, reference bus

NOMENCLATURE

B_{ij} -Susceptance between bus i and j .
 C -Generation bid price vector.
 θ_{ij} -Voltage phase angle between bus i and j .
 D -Loss distribution factor.
 e -Vector whose elements are 1.
 G_{ij}, R_{ij} -Conductance and resistance between bus i and j .
 k_{ij} -Tap ratio of the branch between bus i and j .
 LF -Loss factor
 N_l -Total number of branches
 $offset$ -System loss linearization offset.
 P_G, P_L -Energy supply vector and demand vector
 P_{Gmin}, P_{Gmax} -Lower and upper limits of generator output
 P_{ij} -Branch active power flow from bus i to j .
 P_{Loss} -System losses variable
 $P_{fnd,i}$ -Fictitious nodal demand at bus i .
 Q -Reactive power injection vector

S - Sensitivity matrix of nodal voltages to nodal power injection.

T - Sensitivity matrix of line flows to nodal power injection.

T_{max} -Maximum allowable line flow.

λ, μ - Shadow price for the energy balance and branch flow constraints equation

I. INTRODUCTION

Electricity Supply Industry throughout the world, is restructuring for better utilization of resources and providing quality service and choice to the consumer at competitive prices. Restructuring of the power industry abolishing the monopoly in the generation and trading sectors, thereby, introducing competition at various levels wherever it is possible. Electricity sector restructuring, also popularly known as deregulation, is expected to draw private investment, increase efficiency, promote technical growth and improve customer satisfaction as different parties compete with each other to win their market share and remain in business. Competitive electricity markets are complex systems with many participants who buy and sell electricity. Much of the complexity arises from the limitations of the underlying transmission systems and the fact that supply and demand must be in balance at all times. When the producers and consumers of electrical energy desire to produce and consume in amounts that would cause the transmission system to operate at or beyond one or more transfer limits, the system is said to be congested.

The LMP (Locational Marginal Pricing) at a location is defined as the marginal cost to supply an additional increment of power to the location without violating any system security limits. This price reflects not only the marginal cost of energy production, but also its delivery. Because of the effects of both transmission losses and transmission system congestions, LMP can vary significantly from one location to another.

The rest of this paper is organized as follows. Section II discuss about the literature review. In Section III, the proposed LMP calculation model and the detail calculation procedure are described. An example presented in Section IV. Finally, concluding remark are given in Section V.

II. LITERATURE REVIEW

G. Hamoud [1] states that in a deregulated environment, the number of bilateral transactions will grow rapidly and there-fore, new methods and tools will be required to help system operators evaluate their impacts on the operation of

the systems. It states that a transaction is divided into two types feasible and unfeasible. It states that bilateral transaction is deemed to be unfeasible if it violates any of system operating constraints. In order to accommodate an unfeasible transaction, the system economic dispatch may be altered and, in this case, there will be a penalty (additional fuel cost) associated with the transaction.

A methodology based on non linear optimal power flow (OPF) model proposed in [2] to break down LMP into a variety of parts corresponding to different factors, such as generations, transmission congestion, voltage limitations and other constraints. Nevertheless, LMP calculations are usually based on a linear programming model and DC power flows[3] for computational efficiency and stability. Research in [4] shows that's the results of the DC approximations are close to the full AC solutions. As such, LMP is usually decomposed into three components: marginal energy price (MEP), marginal loss price (MLP), and marginal congestion price (MCP), which is carefully analyzed in [5]. However, it is proved in [6] that such decomposition is not unique and there is a rather large level of arbitrariness in any decomposition.

Because of the inherent nonlinearity of transmission losses, there is a great desire to improve the accuracy in loss calculation and pricing [7].Based on the distributed-slack power flow formulation, the LMP calculation formulation. and the three component decomposition are proposed in [8]. The idea is widely accepted and employed in American power markets [9], although the difference of the loss components between any node pair depends explicitly on the selected reference buses and participation factors.

In order to consider network losses in the DC power flow model, [10] proposed a fictitious nodal demand(FND)model to offset the effect of active power losses .However the major drawback with the approach is that the LMP results are dependent on the choice of reference bus.[11] presented a new model to balance the consumed losses in the DC models by introducing loss distribution factors.However,the loss factors and loss distribution factors in[11] must be preset and the results of LMP and its components are heavily dependent on these preset values.

II. PROBLEM FORMULATION

A. Proposed LMP Model

In real power market security checks are performed after the market clearing and LMP calculations. In the security check Step, reactive power may also be considered .The influence of reactive power can be taking into account if the full AC instead of DC power flow constraints is considered in the procedure of LMP calculations. Non linear programming model is not employed because of the problem in solution robustness algorithm and the difficulties in the software development for practical implementation.

The following model is proposed for the iterative LMP calculations

$$\text{Min} \quad C^T P_G$$

$$\text{St.} \quad e^T (P_G - P_L) = P_{Loss} \quad (1)$$

$$P_{Loss} = (LF^{AC})^T (P_G - P_L) + \text{offset}^{AC} \quad (2)$$

$$T ((P_G - P_L - D^{AC} \cdot P_{Loss}) \leq T_{max} \quad (3)$$

$$P_{Gmin} \leq P_G \leq P_{Gmax} \quad (4)$$

Where LF^{AC} , offset^{AC} , and D^{AC} are not preset before solving this model. Instead they are obtained from the solution of the full AC power flow.

B. Loss and Loss Sensitivity Vector

At first, offset^{AC} and all elements of LF^{AC} , D^{AC} can be set equal to 0. Solving the proposed model gives a generator dispatch P_G . Then the full AC power flow can be solved. Choosing power injection into the system as the positive direction of power, the active power flow of a branch can be calculated by

$$P_{ij} = k_{ij} G_{ij} V_i^2 - V_i V_j (G_{ij} \cos \theta_{ij} + B_{ij} \sin \theta_{ij}) \quad (5)$$

The total system loss is an accumulation of the losses of all branches

$$P_{loss}^{AC} = \sum_{l=1}^N (P_{ij} + P_{ji}) \quad (6)$$

$$= G_{ij} (t_{ij} V_i^2 + t_{ji} V_j^2 - 2V_i V_j \cos \theta_{ij})$$

Where node i and j are the ends of branch l .

Now, it is not difficult to build the following equation, representing the sensitivity of the system total loss to the nodal power injections:

$$\Delta P_{loss}^{AC} = \left[\frac{\partial P_{loss}^{AC}}{\partial \theta} \quad \frac{\partial P_{loss}^{AC}}{\partial V} \right] S \left[\frac{\Delta P}{\Delta Q} \right] \quad (7)$$

$$= \left[\frac{\partial P_{loss}^{AC}}{\partial \theta} \quad \frac{\partial P_{loss}^{AC}}{\partial V} \right] \begin{bmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{bmatrix} \begin{bmatrix} \Delta P \\ \Delta Q \end{bmatrix}$$

Sensitivity matrix S is the inverse of the jacobian matrix of the AC power flow equations. For the proposed model, the sensitivities of active power injections to the system loss are desired, which is given by the following equation:

$$LF^{AC} = \frac{\partial P_{loss}^{AC}}{\partial \theta} S_{11} + \frac{\partial P_{loss}^{AC}}{\partial V} S_{21} \quad (8)$$

Then the loss offset can be easily obtained by

$$\text{offset}^{AC} = P_{loss}^{AC} - (LF^{AC})^T (P_G - P_L) \quad (9)$$

C. Nodal Fictitious Demands and Loss Distribution Factors

By analogy with the approximation idea of the DC power flow model, the following approximate formula can be derived:

$$\begin{aligned} P_{ij} &\approx G_{ij} - (G_{ij} \cos \theta_{ij} + B_{ij} \sin \theta_{ij}) \\ &= G_{ij} (1 - \cos \theta_{ij}) - B_{ij} \sin \theta_{ij} \\ &\approx 0.5 G_{ij} \theta_{ij}^2 - B_{ij} \theta_{ij} \end{aligned} \quad (10)$$

For DC power flow, P_{ij} equals $-B_{ij} \theta_{ij}$. So $0.5 G_{ij} \theta_{ij}^2$ can be considered as the approximate active power loss at one

terminal of a branch. The approximate loss at the other terminal also equals $0.5G_{ij} \theta_{ij}^2$. The total loss of a branch equals $R_{ij} I_{ij}^2$. If the loss $0.5G_{ij} \theta_{ij}^2$ is represented as a fictitious nodal demand at each end of a branch, branch losses can be approximately considered

For the proposed method, the full AC power flow is solved. Accurate branch flows and branch losses can be obtained. According to the above discussion, the loss of each branch can be divided into two equal halves and attached to the two nodes of the branch. The fictitious nodal demand is then defined as follows:

$$P_{fnd,i} = 0.5 \sum_{j \in i} (P_{ij} + P_{ji}) \quad (11)$$

Where $J \in i$ means node i and j are the two ends of a branch. Now, each element of D^{AC} can be determined as the ratio of the corresponding fictitious nodal demands to P_{loss}^{AC} and calculated as

$$D_i^{AC} = P_{fnd,i} / P_{loss}^{AC} \quad (12)$$

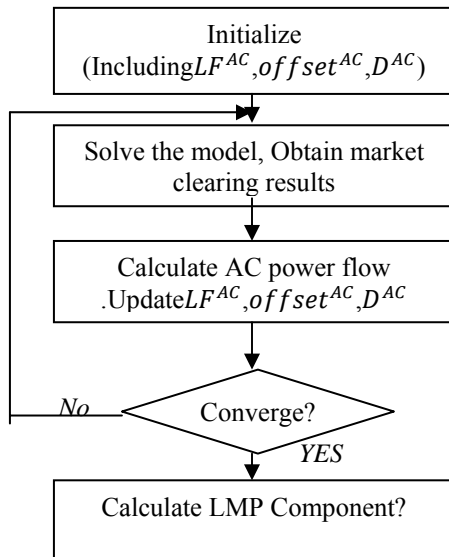
D. Calculation Procedure and Convergence

The basic procedure of the proposed LMP calculation method is shown in the Fig. 2. The criterion for checking convergence is given as follows.

Compare the clearing results for each generator with the results of the previous iteration. If the difference in the output of each generator is smaller than a predefined tolerance, then stop the iteration. Otherwise, continue the iteration.

If P_G converges after the $(i+1)$ th iteration, the solutions of AC power flow obtained at the i th and the $(i+1)$ th iteration are almost the same. Thus LF^{AC} , D^{AC} and P_{loss} all converge.

Although LF^{AC} , $offset^{AC}$, and D^{AC} are not preset in the proposed model, it can be proved that the primary solution and the result of each LMP and its congestion component are independent of the selection of reference bus



IV. AN EXAMPLE

It has been applied to several test system. If not particularly pointed out, the initial values of LF^{AC} , P_{loss} and D^{AC} are all zeros.

A. PJM Five-Bus System

The diagram of the slightly modified PJM five-bus system is shown in Fig.3. Table I lists the line impedance and power flow limits. Generator bids and upper power limits are given in Table II.

The voltage magnitude of reference bus and PV buses are all set equal to 1.0.p.u. Reactive power demands at buses B,C and D are all 100MVar. The convergence criterion is the maximum power output error of all units between two successive iterations is lower than 0.01 MW.

1) *Results of the Proposed Method:* The results obtained by the proposed method with reference at bus A, C, D and E are listed in Table III-respectively. Taking the convergence criterion into account, one can see that

(a) the generation dispatch results are the same in all four tables (the tiny difference is relate to the convergence critertion)

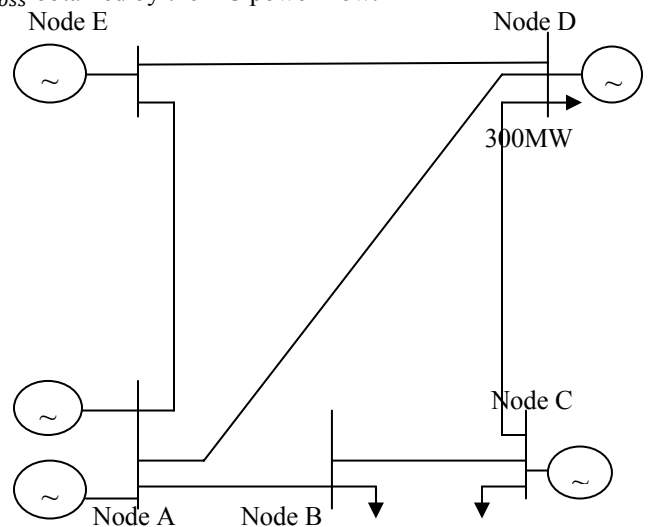
(b) for different reference buses fictitious nodal demands are the same in spite of different loss factors.

(c) $LMP^{Energy+LMP^{Loss}}$ and $LMP^{Congestion}$, LMP at each bus are the same for all four cases.

The initial outputs for all generators (P_G) are set to 0. The proposed method converges after three iterations.

B. Dicussions on Convergence of the Proposed Method

For the zero initial values of LF^{AC} , $offset^{AC}$, and D^{AC} the generator dispatch results are the same at the first iteration for any choice of slack bus. At the second iteration, the clearing results for (P_G) are slightly different for different slack bus. At convergences, the clearing results for (P_G) are almost the same because P_{loss} is almost equal to the power loss P_{loss}^{AC} obtained by the AC power flow.



300MW 300MW
PJM five-bus diagram

E	5.785	-0.0025	22.1375	-0.0560	-5.6991	16.3825	
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Table I
Line Impedance and Power Flow Limits

Per Unit	A-B	A-D	A-E	B-C	C-D	D-E
R	0.281	0.304	0.064	0.108	0.297	0.297
X	2.81	3.04	0.64	1.08	2.97	2.97
Limit	999	999	999	999	999	240

Table-IV (Clearing results of the proposed method with reference at C):

Bus	Bus Gen(P.U)	Loss Factor	LMP Energy	LMP Loss	LMP Congestion	Total LMP
A	2.10	-0.0184	22.1184	-0.4080	-6.5769	15.1336
B	0	-0.0145	22.1184	-0.3197	10.5481	32.3468
C	0	0	22.1184	0	7.3035	29.4220
D	1.1207	-0.0057	22.1184	-0.1271	-1.6190	20.3723
E	5.785	-0.0025	22.1184	-0.0559	-5.6979	16.3646

Table II
Bid Prices and Economic Maximum of Generators

Unit	Alta	Park City	Solitude	Sundance	Brighton
Bus	A	A	C	D	E
$P_{Gmax}(MW)$	110	100	520	200	600
Bid Price(\$/MW)	14	15	30	35	10

Table-V (Clearing results of the proposed method with reference at D):

Bus	Bus Gen(P.U)	Loss Factor	LMP Energy	LMP Loss	LMP Congestion	Total LMP
A	2.10	-0.0184	22.3640	-0.4125	-6.5769	15.3746
B	0	-0.0145	22.3640	-0.3233	10.5481	32.5888
C	0	-0.0057	22.3640	-0.1285	7.3035	29.5390
D	1.1272	0	22.3640	0	-1.6191	20.7450
E	5.785	-0.0025	22.3640	-0.0566	-5.6980	16.6095

It is possible that the AC power flow fails to converge with a dispatch results obtained from the proposed linear programming model. But this is not a problem that brings only with the proposed method.

All DC power flow based LMP solvers have the same problem because AC power flows should be run in order to check the feasibility and network security of the dispatch results. The following measures can improve the convergence and stability of the proposed method.

Table-III (Clearing results of the proposed method with reference at A):

Bus	Bus Gen(P.U)	Loss Factor	LMP Energy	LMP Loss	LMP Congestion	Total LMP
A	2.10	0	22.1375	0	-6.5781	15.5595
B	0	-0.0184	22.1375	-0.4083	10.5469	32.2762
C	0	-0.0145	22.1375	-0.3200	7.3024	29.1200
D	1.388	-0.0057	22.1375	-0.1271	-1.6202	20.3902

Table-VI (Clearing results of the proposed method with reference at E):

Bus	Bus Gen(P.U)	Loss Factor	LMP Energy	LMP Loss	LMP Congestion	Total LMP
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A	2.10	-0.0184	22.0419	-0.4066	-6.5770	15.0583
B	0	-0.0145	22.0419	-0.3186	10.5480	32.2711
C	0	-0.0057	22.0419	-0.1266	7.3034	29.2189
D	1.1542	-0.0025	22.0419	-0.0557	-1.6192	20.3669
E	5.785	0	22.0419	0	-5.6981	16.3438

The calculations of the proposed method converge after 3 ~ 4 iterations. Test results also indicate that appropriate initial values for loss factor, loss offset, and nodal loss distribution factor would further improve the convergent speed.

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- (1)Set good initial values of $offset^{AC}$ and D^{AC} from historic data and offline analysis
- (2)Set good initial values of LF^{AC} from historic data and offline analysis.
- (3)Use distributed references buses. During the iterative process, there is some mismatch between the sum of dispatched generation powers and the sum of power demand and losses. Comparing to single slack bus, the amount of power that each distributed reference bus is responsible for balance is smaller. This will generally reduce the possibility of AC power flow divergence if the participation factors of reference buses are chosen according to the system operation conditions (not chosen randomly).

V.CONCLUSION

In this paper, a new method to calculate LMP iteratively is presented to tackle the main drawbacks with the DC power flow based LMP calculations that would lead to inaccuracies in loss calculations and dependency on the choice of reference bus. For easy implementation and consistency with the widely used LMP model, a linear programming problem is formulated and solved at each iteration.

It is proved that the market clearing results are independent of the selection of reference bus, more importantly, the congestion component of LMP is also reference bus independent .This is a desirable property for providing consistent and accuracy congestion information for market participant, which is crucial for effective congestion management.

Test on a PJM five bus system shows that the proposed method can obtain reference bus independent congestion component of LMP without presetting the loss factor, loss offset, and loss distribution factor. The improvement and contribution of the proposed method are demonstrated through comparing the resulting LMP and their components.