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FORMULATION OF MATHEMATICAL MODEL FOR PRODUCTION TURNOVER OF BIODIESEL PLANT BASED ON DIMENSIONAL ANALYSIS AND MULTIPLE REGRESSION

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Abstract--- This paper presents the approach for the mathematical modeling of production turnover for the set up of new Biodiesel plant based on the dimensional analysis and multiple regression. Presented production turnover mathematical model is derived based on the generated design data. Design data is generated from the estimated design data. Estimation of design data is carried out based on the assumed plant layouts of different capacities. Dimensional analysis is used to make the independent and dependent variables dimensionless and to get dimensionless equation. Later, multiple regression analysis is applied to this dimensionless equation to obtain the index values based on the least square method. The mathematical model of production turnover is formulated using these obtained index values. Finally, the formulated model is evaluated on the basis of correlation and root mean square error between the computed values by model and the estimated values.

Keywords*- Biodiesel Plant Design, Dimensional Analysis, Multiple Regression, Mathematical Model*

I. INTRODUCTION

The most important reason for interest in Biodiesel production in India is that the India’s climatic conditions are conducive for production of wide range of oil seeds such as: soybean, groundnut, safflower, mustard, castor and sunflower etc. which are easily available. Main issue in the Biodiesel production is the design of the plant and is one of the important challenges. The scope of research exists in this typical area for the mechanical and industrial engineering researchers.

This paper presents an approach for mathematical modeling of production turnover for the set up of new Biodiesel plant. Plant design is the main issue for any manufacturing plant. Presented work is based on the assumed Biodiesel plant design. Concerned plant includes many resources and the optimal use of these resources with certain constraints is the focal issue. Biodiesel plant layouts are prepared in AUTOCAD based on certain assumptions. Based on the plant layouts, various design data is estimated and further the design data is generated which later used in formulation of mathematical model of production turnover. This paper mainly describes the approach for formulation of mathematical model which is based on the dimensional analysis and multiple regression.

This paper is organized as follows: section II discusses the Biodiesel plant design key issues and the related work in brief. Suggested approach and the motivation behind it are summarized in section III. Discussion on estimation and generation of the design data is presented in section IV. Derivation of mathematical model of production turnover for the set up of new Biodiesel plant based on dimensional analysis and multiple regression is presented in section V. Section VI discusses the conclusion and future scope followed by references.

II. BIODIESEL PLANT DESIGN AND RELATED WORK

Various issues have to be considered in designing of any Biodiesel plant. Here, some of the important issues are identified and briefly discussed.

Purity of the feedstock: Biodiesel processing and quality are closely related. The processes used to refine the feedstock and convert it to Biodiesel determine whether the fuel will meet the applicable specifications as per standard or not [1]. *Type of production process:* Different methodologies or processes [2], generally, used for production of Biodiesel are: Direct use / blending, Micro-emulsion, Pyrolysis and Transesterification. *Capacity of the plant:* The first decision point for the design of a production unit is its capacity. Based on capacity of the plant, design of various equipments involved in production process can be specified, as well as other requirement such as total cost, land, power consumption, raw materials, and man-hours etc. can be estimated. *Production cost analysis:* Production cost depends upon the prices of raw materials, the method of production, and utilization of by-products etc. The unit production cost can be obtained by taking into account the cost of oil, methanol, utilities and operating labor and all other costs directly related to production. *Equipment cost analysis:* Capital cost estimation is one of the most critical elements in plant valuation, capital budgeting, feasibility studies and finance decisions. Investment (capital cost) for plant and equipment is important in establishing Biodiesel production capabilities. Cost estimation is used for proposal preparation, feasibility analysis and evaluation [3]. *Land acquisition and infrastructure cost analysis:* Land requirement and infrastructure cost are directly related to production capacity. Initial capital costs of the plants differ primarily based on feedstock needs and output. The physical requirements of a plant consist of a production facility, a tank yard allowing storage of manufactured Biodiesel and feedstock, offices, and loading/unloading facilities which adds the cost to initial capital cost. *Maintenance cost:* To operate a safe plant is the first step, but maintaining a safe plant at safe working environment to handle any emergency is a full time job. For various plant capacities, the contributions of maintenance cost overhead become higher with bigger capacities. The maintenance cost of the plant is associated with the breakdown(s) or failure(s) of equipment(s). *Operating profit:* Profitability of a Biodiesel plant can be used as the basic parameter for comparison of various plants. In general, the operating profit is related to the capacity of the plant. The capacity of the plant directly affects the operating profit and therefore the operating profit can be the base for comparison of various plant capacities. *Feasibility Analysis:* The proximity of feedstock is a crucial component for the feasibility analysis of the Biodiesel plant. Apart from this, the others like equipment cost, Biodiesel selling price, human resources involved etc. also drive the feasibility analysis. The feasibility analysis of a plant can provide useful conclusions with respect to the unit production cost and various other technical and economical parameters. *Performance evaluation parameters:* The cost of the plant installation according to the capacity of production; the profit as per the capacity of the plant; quality of the Biodiesel; maintenance cost required as per the maintenance schedule; risk analysis etc. are the general parameters can be used for the evaluation of any Biodiesel plant design.

The design and feasibility analysis of Biodiesel production plant is difficult to standardize the total investment and production cost, since its main characteristics (feedstock, final products, the equipment items cost, land acquisition) are subject to market price fluctuations. Also, the cost of conventional diesel fuel, which is directly related to the price of crude oil, is subject to similar fluctuations, creating uncertainty in targets for Biodiesel production cost/selling price [4]. As discussed previously, the important issues in the designing of any plant are the equipments, land availability, investment amount, capacity requirement etc. Therefore, the scope of optimization in plant designing exists with the imposed constraint of certain issues. One can design the Biodiesel plant with the consideration of all above mentioned important issues or can considered only selected issues.

Prominent work related to plant design is reported in [4-11], where most of the parameters, already discussed in previous section, are used in formulation of the problem. Skarlis et al. [4] carried out the feasibility analysis where they focused on the profit analysis, while Hass et al. [5] carried out the simulation of computer model to estimate the capital and operating costs based on changes in feedstock costs. Van Kasteren and Nisworo [6] have described the process model where they presented the conceptual design to estimate the cost of Biodiesel production. Al-Zuhair et al. [7] have designed and installed a pilot plant for Biodiesel production from waste/used vegetable oil using enzymatic approach. Kapilakam and Peugtong [8] have carried out the simulation for optimality where optimal operating condition for the Biodiesel production is studied and identified. Marchetti and Errazu [9] have discussed the work on simulation of Biodiesel plant to produce the conceptual design and simulate each technology. Economic analysis of Biodiesel production from vegetable oils is carried out by Apostolakou et al. [10]. Myint and El-Halwagi [11] carried out work of optimization of Biodiesel production from Soybean oil. Abbasi and Diwekar [12] discussed the stochastic modeling approach for Biodiesel production plant efficiency. Zhang et al. [13] carried out the economic analysis of Biodiesel production from waste cooking oil. Yet no work is reported in literature related to mathematical modeling of Biodiesel plant design.

III. MOTIVATION AND SUGGESTED APPROACH

A. Motivation

After analyzing the available literature, it is found that the plant design is an open issue where the research scope exists. The gaps and the observations identified in literature are summarized as follows.

(a) No concrete results are reported in literature regarding the launching/installation of new Biodiesel plant with variety of objectives.
(b) Existing approaches which are suggested in the literature for the plant design move around the cost parameters and the chemical processes with the fixed plant capacity.
(c) No approach exists as per our knowledge which can address the problem of design of Biodiesel plant with variety of capacities.
(d) No approach exists as per our knowledge which can address the problem of design of Biodiesel plant with cost and capacity perspective.
(e) No mathematical model based approach exists for designing of Biodiesel plant.
(f) The capacity of plant, capital cost, operating cost, profit analysis and storage tank material are the important basic issues to be considered in designing of any Biodiesel plant.
(g) Any plant design move around the parameters related to these basic issues.

The above mentioned gaps and observations are the key motivation for the development of the mathematical model based approach for the Biodiesel manufacturing plant design.

B. Basic Idea and Suggested Approach

The cost related parameters can be used to evaluate the design of the Biodiesel plant with respect to the economics. These performance evaluation parameters are- Capacity (Production turnover); maintenance cost; operating profit. One can develop the approach to focus on these parameters when go for designing of the Biodiesel production plant. Biodiesel plant design basically concerned with the resource management, i.e. how optimally one can use the resources? Model based approach can be developed, where the relativity amongst the various resources can be used in model development. The cost evaluation of each individual resource can be the base in forming the relativity amongst the various resources.

An approximate model can be developed using the design data, where the design data can be related to- Feedstock cost, Equipment specification, Tank design, Land specification, Power consumption, Man hours, Production turnover, Maintenance cost, and Operating profit. Various symbols or the variables can be used for the identification of the above mentioned data parameters, and based on this, the mathematical model can be formulated using the generated data and later the developed model can be used to answer the specific questions concerned to the Biodiesel production plant design.

The suggested approach for design of Biodiesel manufacturing plant with cost and capacity perspective consists of the following steps.

Step-1: Identification of input and output parameters.

Step-2: Estimation of the design data for various capacities (discussed in section IV).

Step-3: Generation of the design data for various capacities (discussed in section IV).

Step-4: Formulation of the mathematical model for the plant manufacturing Biodiesel (discussed in section V).

C. *Input and Output Parameters (Step-1)*

In presented work, Biodiesel plant design related input and output parameters are identified with reference to the base parameters discussed in section II and following assumptions.

- Method of Biodiesel production is alkali catalytic methanol transesterification.
- Quality of raw material is as per the standard required to produce Biodiesel.
- Plant is operated for one batch per day.
- Plant is having integrated crushing (seeds) plant.
- Construction and site preparation cost is not considered in this study.
- Specifications of process equipments can accommodate all types of oil (raw material) for biodiesel production.
- Quality of Biodiesel is as per the standard (EN 14214/IS 15607 biodiesel fuel standard).
- Plat layout design is developed at our own.

Presented work concern with the following input and output parameters. Input parameters and output parameters of the Biodiesel plant design are identified as the inputs and responses of the Biodiesel plant and the same nomenclature is used hereafter in the remaining text of this paper.

Inputs: Equipment Cost, Power Consumption, Water Requirement, Total Factory Area (Land Area), Oil Seeds, Methanol, Catalyst (KOH), Man-hours.

Responses: Production Turnover, Maintenance Cost, and Operating Profit

IV. DATA ESTIMATON AND GENERATION

A. *Design Data Estimation (Step-2)*

Various inputs and responses with their unit of measure and the estimation based on are summarized in Table I. The data related to the inputs and response (Production turnover) is generated based on the plant capacity and the estimated data of inputs respectively.

TABLE I. SUMMARY OF INPUTS AND RESPONSES

Specification	**Unit**	**Parameter**	**Estimation Based On**
Equipment Cost in Lacs	`	Input	Design and supplier Quotations
Power	*HP*	Input	Power rating of equipments
Water	*Litre*	Input	Estimated as per process requirement
Total Factory Area	m^2	Input	Layout of plant plotted in AutoCAD
Oil Seeds	*Kg*	Input	Capacity of Biodiesel plant
Methanol	*Litre*	Input	Capacity of Biodiesel plant
Catalyst KOH	*Kg*	Input	Capacity of Biodiesel plant
Man-hours	*Hours*	Input	Human resource required for operation of plant
Production Turnover (Kg. converted in Rupees)	`	Response	Expected output at each stage of production
Maintenance Cost in Lacs	`	Response	Expected failure causes and preventive maintenance schedule for each equipment
Operating Profit in Lacs	`	Response	Production cost and revenue generated

Estimation of all the inputs for the capacities (1, 2, 3, 5, 7, 9, and 10 ton) is carried out by referring the Biodiesel production plant layout designs which are prepared in AutoCAD for all the capacities with certain assumptions and the response (Production turnover) is also evaluated independently. Basic requirements of the Biodiesel production plant may get changed as per the desired capacity. Estimation of some of the input parameters is also changed due to the varying specification requirements for different capacities of Biodiesel production plant. Shape of the oil tank for all capacity is same. Number of washing tanks and drier tanks considered for all capacities are 3 and 2 respectively, while single tank is considered for other type of tanks. Estimation of expected production turnover is based on production process and minimum output at each stage of the process. Capacity of oil expeller varies due to specification and number of units used for various capacities of Biodiesel production plant. Chemical composition as per the standard reaction for estimated oil, methanol and catalyst and expected quantities of Glycerol and Biodiesel for all capacity are given in Table II. Summary of estimated values of all inputs and response (Production turnover) for 1, 2, 3, 5, 7, 9, and 10 *ton* capacity Biodiesel plants is given in Table III.

TABLE II. CHEMICAL COMPOSITION AS PER STANDARD FOR ESTIMATED OIL, METHANOL AND CATALYST FOR 1, 2, 3, 5, 7, 9, AND 10 TON CAPACITY BIODIESEL PRODUCTION PLANT

Capacity of plant *(ton)*	**Oil *(kg)***	**Methanol *(kg)***	**Catalyst (KOH) *(kg)***	**Glycerol *(kg)***	**Biodiesel *(kg)***
1	919.9	101.19	13.8	101.19	919.9
2	1839.8	202.38	27.6	202.38	1839.8
3	2760	303.6	41.4	303.6	2760
5	4599.8	505.98	69	505.98	4599.8
7	6439.9	708.39	96.6	708.39	6439.9
9	8280	910.8	124.2	910.8	8280
10	9199.9	1011.99	138	1011.99	9199.9

TABLE III. SUMMARY OF ALL INPUTS AND RESPONSES ESTIMATION FOR 1, 2, 3, 5, 7, 9, AND 10 TON CAPACITY BIODIESEL PLANTS

Capacity of Plant *(ton)*	**Inputs**								**Response**
	Equipment Cost (₹ *In Lacs*)	**Power (*HP*)**	**Water (*Litre*)**	**Total Factory Area (m^2)**	**Oil Seeds (*kg*)**	**Meth-anol (*Litre*)**	**Cata-lyst KOH (*kg*)**	**Man-hours (*Hours*)**	**Prod. Turnover (₹*In Lacs*)**
1	14.58419	59.88	525	125	3333	139	15.00	80	0.34204
2	23.89304	104.60	1050	152	6666	278	30.00	92	0.68408
3	33.35492	162.26	1500	253	10000	471	45.00	92	1.02624
5	50.2963	273.57	2550	464	16666	695	75.00	104	1.71032
7	66.51348	380.85	3750	728	23333	973	105.00	116	2.39452
9	81.78912	492.16	4500	1044	30000	1251	135.00	136	3.07872
10	89.62276	545.80	5250	1141	33333	1390	150.00	148	3.42076

B. Design Data Generation (Step-3)

Design data is generated using the estimated values of the inputs and response (Production turnover). Intermediate values of the inputs and response for various capacities are generated by using the MATLAB tool. This generated design data for inputs and response is used later for development of mathematical model.

Previously estimated design data values given in Table III are used to generate design data. The flow of the approach to generate the design data consist of following two steps: *Step-1:* Data fitting- For each inputs and response, form the vector between the two consecutive capacity values. *Step-2:* Finding intermediate data- Increment the lowest value by 0.1 to the highest value of the two consecutive capacities which formed the vector to get the corresponding intermediate values of the inputs and response by referring the vector formed in step 1. Generated design data of inputs and response (Production turnover) for few of the intermediate values of capacity is given in Table IV.

TABLE IV. GENERATED DESIGN DATA FOR ALL INPUTS AND RESPONSE (PRODUCTION TURNOVER)

Capacity of Plant *(ton)*	**Inputs**								**Responses**		
	Equipment Cost (₹ *In Lacs*)	**Power (*HP*)**	**Water (*Litre*)**	**Total Factory Area (m^2)**	**Oil Seeds (*kg*)**	**Methanol (*Litre*)**	**Catalyst KOH (*kg*)**	**Man-hours (*Hours*)**	**Production Turnover (₹*In Lacs*)**	**Maintenance Cost (₹*In Lacs*)**	**Operating Profit (₹*In Lacs*)**
1.1	15.51508	64.352	577.5	127.7	3666.3	152.9	16.5	81.2	0.376244	2.029249	44.25739
1.2	16.44596	68.824	630	130.4	3999.6	166.8	18	82.4	0.410448	2.070658	48.65693
1.3	17.37685	73.296	682.5	133.1	4332.9	180.7	19.5	83.6	0.444652	2.112067	53.05648
1.4	18.30773	77.768	735	135.8	4666.2	194.6	21	84.8	0.478856	2.153476	57.45602
2.6	29.57017	139.196	1320	212.6	8666.4	393.8	39	92	0.889376	2.939296	104.8866
2.7	30.51636	144.962	1365	222.7	8999.8	413.1	40.5	92	0.923592	3.028857	108.3921
2.8	31.46254	150.728	1410	232.8	9333.2	432.4	42	92	0.957808	3.118418	111.8977
2.9	32.40873	156.494	1455	242.9	9666.6	451.7	43.5	92	0.992024	3.207979	115.4033
3.2	35.04906	173.391	1605	274.1	10666.6	493.4	48	93.2	1.094648	3.450082	128.4454
3.3	35.89613	178.9565	1657.5	284.65	10999.9	504.6	49.5	93.8	1.128852	3.526353	133.2137
3.4	36.7432	184.522	1710	295.2	11333.2	515.8	51	94.4	1.163056	3.602624	137.982
4.1	42.67268	223.4805	2077.5	369.05	13666.3	594.2	61.5	98.6	1.402484	4.136521	171.3602
4.2	43.51975	229.046	2130	379.6	13999.6	605.4	63	99.2	1.436688	4.212792	176.1285
4.3	44.36682	234.6115	2182.5	390.15	14332.9	616.6	64.5	99.8	1.470892	4.289063	180.8968

5.5	54.3506	300.39	2850	530	18332.75	764.5	82.5	107	1.88137	5.082503	236.3547
5.6	55.16145	305.754	2910	543.2	18666.1	778.4	84	107.6	1.91558	5.134411	240.7706
5.7	55.97231	311.118	2970	556.4	18999.45	792.3	85.5	108.2	1.94979	5.18632	245.1866
6.3	60.83747	343.302	3330	635.6	20999.55	875.7	94.5	111.8	2.15505	5.497771	271.6822
6.4	61.64833	348.666	3390	648.8	21332.9	889.6	96	112.4	2.18926	5.549679	276.0981
6.5	62.45919	354.03	3450	662	21666.25	903.5	97.5	113	2.22347	5.601588	280.5141
6.6	63.27004	359.394	3510	675.2	21999.6	917.4	99	113.6	2.25768	5.653496	284.93
7.5	70.33239	408.6775	3937.5	807	24999.75	1042.5	112.5	121	2.56557	6.12832	324.7629
7.6	71.09617	414.243	3975	822.8	25333.1	1056.4	114	122	2.59978	6.181758	329.1967
7.7	71.85995	419.8085	4012.5	838.6	25666.45	1070.3	115.5	123	2.63399	6.235196	333.6305
8.2	75.67886	447.636	4200	917.6	27333.2	1139.8	123	128	2.80504	6.502386	355.7996
8.3	76.44265	453.2015	4237.5	933.4	27666.55	1153.7	124.5	129	2.83925	6.555824	360.2334
8.4	77.20643	458.767	4275	949.2	27999.9	1167.6	126	130	2.87346	6.609262	364.6672
9.1	82.57248	497.524	4575	1053.7	30333.3	1264.9	136.5	137.2	3.112924	7.046523	395.5673
9.2	83.35585	502.888	4650	1063.4	30666.6	1278.8	138	138.4	3.147128	7.163156	399.8645
9.3	84.13921	508.252	4725	1073.1	30999.9	1292.7	139.5	139.6	3.181332	7.279789	404.1617
9.4	84.92258	513.616	4800	1082.8	31333.2	1306.6	141	140.8	3.215536	7.396422	408.4588
9.5	85.70594	518.98	4875	1092.5	31666.5	1320.5	142.5	142	3.24974	7.513055	412.756
9.6	86.4893	524.344	4950	1102.2	31999.8	1334.4	144	143.2	3.283944	7.629688	417.0532

V. FORMULATION OF MODEL FOR PRODUCTION TURNOVER

To start Biodiesel production plant, one will have to decide what should be the capacity of plant in order to get maximum production turnover, operating profit and minimum maintenance cost. These issues can be addressed if quantitative relationship between the inputs and responses of the plant is formulated in terms of mathematical model. Relationship amongst the inputs and response (Production turnover) is established first, by doing the dimensional analysis of independent and dependent variables and followed by formulating multiple-linear-regression model. Formulated mathematical model is based on the generated designed data. In this paper, the mathematical model is formulated for production turnover.

A. Dimensional Analysis

Formulation of dimensional equation is the first step to formulate the model of Biodiesel production plant. The variables to be predicted are called the responses or dependent variables and the variables predicting the responses are called the inputs or independent variables. The functional relationship amongst the inputs and response (Production turnover) affecting the Biodiesel production plant is formulated using dimensional analysis. Following are the two methods for dimensional analysis:

- Buckingham's π – theorem
- Rayleigh's method

Above two methods provides the same results, in most of the cases but having slightly different approach of formulation. Rayleigh's method of dimensional analysis is used in this work and it expresses a functional relationship of inputs and response (Production turnover) in the form of an exponential equation.

The method involves the following steps:

- Identification of the inputs those are likely to influence the response.
- If X is a variable that depends upon input variables $X_1, X_2, X_3, \ldots\ldots, X_n$, then the functional equation can be written as $X = F\left(X_1, X_2, X_3, \ldots\ldots, X_n\right)$.
- Write the above equation in the form where C is a dimensionless constant and $a, b, c, \ldots\ldots, m$ are arbitrary exponents.
- Express each of the quantities in the equation in some fundamental units in which the solution is required.
- By using dimensional homogeneity, obtain a set of simultaneous equations involving the exponents $a, b, c, \ldots\ldots, m$.
- Solve these equations to obtain the value of exponents $a, b, c, \ldots\ldots, m$.
- Substitute the values of exponents in the main equation, and form the non-dimensional parameters by grouping the inputs with like exponents.

Dimensional equation so obtained can be formulated into model using multiple-linear-regression analysis. Multiple-linear-regression analysis is a statistical tool that utilizes the relation between two or more quantitative variables so that one variable can predict from another. By using this methodology the dimensional equation and model is formulated for the production turnover. The formulated model is evaluated on the basis of correlation and root mean square error between the computed values by model and the estimated values.

B. Variables with Symbols and Dimensions

Various inputs like equipments cost, power, water, factory area, oil seeds, methanol, catalyst and man-hours that affects the production turnover of Biodiesel production plant. Dimensional analysis is used to reduce the complexity of phenomenon of Biodiesel production at initial stage and to deduce various inputs in non-dimensional form. Inputs involved in Biodiesel production plant is expressed dimensionally in terms of three fundamental quantities i.e. mass [M], length [L], time [T] and cost parameter represented as rupee [`] in dimensional form in presented work. List of various inputs (dependent variables) and responses (independent variables) with their symbol and dimensions is given in Table V.

TABLE V. LIST OF VARIABLE WITH SYMBOL AND DIMENSIONS (NV-NAME OF VARIABLE, TV-TYPE OF VARIABLE (Ind-Independent, Dep-Dependent), SL-SYMBOL, DM-DIMENSIONS, UM-UNIT OF MEASUREMENT)

Sr. No.	NV	TV	SL	DM	UM
1	Equipments Cost	Ind	E_c	`	`
2	Power HP	Ind	P	$M^1 L^2 T^{-3}$	HP
3	Water	Ind	W	L^3	m^3
4	Factory Area	Ind	A	L^2	m^2
5	Oil Seeds	Ind	O_s	M^1	Kg
6	Methanol	Ind	M_e	L^3	m^3
7	Catalyst (NaOH/ KOH)	Ind	C_a	M^1	Kg
8	Man-hours	Ind	M_h	T^1	Hrs
9	Gravitational Acceleration	Ind	G	$L^1 T^{-2}$	m/s^2
10	Production Turnover	Dep	P_t	`	`
11	Maintenance Cost	Dep	M_c	`	`
12	Operating Profit	Dep	O_p	`	`

C. Formulation of Dimensional Equation for the Response Production Turnover

Formulation of dimensional equation for the response production turnover is based on the inputs identified for Biodiesel production plant. The production turnover as a function of inputs is represented in equation (1).

$$P_t = f\left(E_c, P, W, A, O_s, M_e, C_a, M_h, g\right) \qquad (1)$$

As per the methodology (dimensional analysis, as previously discussed), it is assumed that the relationship between these quantities exist and which is written as given in equation (2).

$$P_t = f\left[(E_c)^a, (P)^b, (W)^c, (A)^d, (O_s)^e, (M_e)^f, (C_a)^g, (M_h)^h, (g)^i\right] \qquad (2)$$

where $(a, b, c, d, e, f, g, h, i)$ are arbitrarily powers.

Equation (2) can be rewritten in the dimensional form using the dimensions from Table V and is of the form shown in equation (3).

$$` = f\begin{bmatrix} (`)^a, (M^1L^2T^{-3})^b, (L^3)^c, (L^2)^d, (M^1)^e, \\ (L^3)^f, (M^1)^g, (T^1)^h, (L^1T^{-2})^i \end{bmatrix} \qquad (3)$$

If equation (3) is to be dimensionally homogeneous (the dimensions of all terms are the same), the following relationship amongst the exponent must exist:

$$For\ M: \quad 0 = b + e + g \qquad (4)$$

$$For\ L: \quad 0 = 2b + 3c + 2d + 3f + i \qquad (5)$$

$$For\ T: \quad 0 = -3b + h - 2i \qquad (6)$$

$$For\ `: \quad 1 = a \qquad (7)$$

Simplifying equations (4), (5), and (6) to get the values of *b, h,* and *d.* Substituting the values of exponents *a, b, h* and *d* from equations (7), (8), (9), and (10) respectively in (2) and rewrite the equation (2) as equation (11).

$$b = -e - g \qquad (8)$$

$$h = -3e - 3g + 2i \qquad (9)$$

$$d = e + g - (3/2)c - (3/2)f - (1/2)i \qquad (10)$$

$$P_t = f\begin{bmatrix} (E_c)^1, (P)^{-e-g}, (W)^c, (A)^{e+g-(3/2)c-(3/2)f-(1/2)i}, \\ (O_s)^e, (M_e)^f, (C_a)^g, (M_h)^{-3e-3g+2i}, (g)^i \end{bmatrix} \qquad (11)$$

Now, collecting the terms with like exponents to get the dimensionless groups and is given in equation (12).

$$P_t/E_c = f\begin{bmatrix} \left(W/A^{(3/2)}\right)^c, \left(AO_s/M_h^{(3)}P\right)^e, \left(M_e/A^{(3/2)}\right)^f, \\ \left(AC_a/M_h^{(3)}P\right)^g, \left(gM_h^{(2)}/A^{(1/2)}\right)^i \end{bmatrix} \qquad (12)$$

Equation (12) represents the groups of non-dimensional parameter for the response (production turnover). Nine inputs for the Biodiesel production plant are reduced to five dimensionless groups. Each group is represented as π term and is given in Table VI.

TABLE VI. NON-DIMENSIONAL GROUPS AS π TERMS

Variable term	Dimensionless groups	π terms
Inputs	$W/A^{(3/2)}$	π_1
	$AO_s/M_h^{(3)}P$	π_2
	$M_e/A^{(3/2)}$	π_3
	$AC_a/M_h^{(3)}P$	π_4
	$gM_h^{(2)}/A^{(1/2)}$	π_5
Response	P_t/E_c	π_6

D. Formulation of Model for Production Turnover

Formulation of dimensionless groups for production turnover is given in Equation (12) and is represented using π terms (as given in Table VI) to get equation (13) as follows:

$$\pi_6 = f\left[(\pi_1)^{a_1}, (\pi_2)^{a_2}, (\pi_3)^{a_3}, (\pi_4)^{a_4}, (\pi_5)^{a_5}\right] \qquad (13)$$

Where, the π terms are calculated from generated design data (Table IV) for Biodiesel production plant. Equation (13) represents the relationship of the response term π_6 with the dimensionless group terms $\pi_1, \pi_2, \pi_3, \pi_4,$ and π_5. Therefore, the multiple-regression model for production turnover as a function of various inputs is written as equation (14).

$$\pi_6 = a_0 \times (\pi_1)^{a_1} \times (\pi_2)^{a_2} \times (\pi_3)^{a_3} \times (\pi_4)^{a_4} \times (\pi_5)^{a_5} \quad (14)$$

where, a_0, a_1, a_2, a_3, a_4, and a_5 are the constant exponent or called as an index of respective π terms or regression coefficients. Equation (14) represents a nonlinear relationship between inputs and response. The logarithmic transformation of response provides log-linear form, which is commonly used in linear regression analysis. Equation (14) is simplified by taking *log* of both the sides, and is expressed in equation (15).

$$[\log \pi_6] = \begin{bmatrix} \log a_0 + a_1 \log \pi_1 + a_2 \log \pi_2 + \\ a_3 \log \pi_3 + a_4 \log \pi_4 + a_5 \log \pi_5 \end{bmatrix} \quad (15)$$

Equation (15) is multiple-linear-regression model with five regressor variables and is linear function of unknown parameters a_0, a_1, a_2, a_3, a_4, and a_5. Equation (15) is written in general form of multiple-regression as equation (16).

$$Y = k + a_1 X_1 + a_2 X_2 + a_3 X_3 + a_4 X_4 + a_5 X_5 \quad (16)$$

where $Y = \log \pi_6$, $k = \log a_0$, $X_1 = \log \pi_1$, $X_2 = \log \pi_2$, $X_3 = \log \pi_3$, $X_4 = \log \pi_4$, and $X_5 = \log \pi_5$. X_1, X_2, X_3, X_4, and X_5 are calculated by using the π terms values and is given in Table VII. Constants k, a_1, a_2, a_3, a_4, and a_5 are calculated from the generated designed data using least squares method. Therefore, above equation (16) is solved for k, a_1, a_2, a_3, a_4, and a_5 using least square method and the calculations are worked out by using the equation (17), where n is total number of data samples. Equation (17) is solved using MATLAB.

$$\begin{bmatrix} \sum Y \\ \sum Y \times X_1 \\ \sum Y \times X_2 \\ \sum Y \times X_3 \\ \sum Y \times X_4 \\ \sum Y \times X_5 \end{bmatrix} = \begin{bmatrix} n & \sum X_1 & \sum X_2 & \sum X_3 & \sum X_4 & \sum X_5 \\ \sum X_1 & \sum X_1 \times X_1 & \sum X_1 \times X_2 & \sum X_1 \times X_3 & \sum X_1 \times X_4 & \sum X_1 \times X_5 \\ \sum X_2 & \sum X_2 \times X_1 & \sum X_2 \times X_2 & \sum X_2 \times X_3 & \sum X_2 \times X_4 & \sum X_2 \times X_5 \\ \sum X_3 & \sum X_3 \times X_1 & \sum X_3 \times X_2 & \sum X_3 \times X_3 & \sum X_3 \times X_4 & \sum X_3 \times X_5 \\ \sum X_4 & \sum X_4 \times X_1 & \sum X_4 \times X_2 & \sum X_4 \times X_3 & \sum X_4 \times X_4 & \sum X_4 \times X_5 \\ \sum X_5 & \sum X_5 \times X_1 & \sum X_5 \times X_2 & \sum X_5 \times X_3 & \sum X_5 \times X_4 & \sum X_5 \times X_5 \end{bmatrix} \times \begin{bmatrix} k \\ a_1 \\ a_2 \\ a_3 \\ a_4 \\ a_5 \end{bmatrix} \quad (17)$$

TABLE VII. LOG EVALUATION OF π TERMS

Capacity	$\log \pi_1$	$\log \pi_2$	$\log \pi_3$	$\log \pi_4$	$\log \pi_5$	$\log \pi_6$	Capacity	$\log \pi_1$	$\log \pi_2$	$\log \pi_3$	$\log \pi_4$	$\log \pi_5$	$\log \pi_6$
1	-0.97907	-4.29848	-2.30800	-1.51737	8.63330	-3.75277	5.5	-1.45424	-3.63425	-2.77009	2.37227	8.49262	-3.36346
1.1	-0.91582	-4.29849	-2.24474	-1.37333	8.65239	-3.71933	5.6	-1.47031	-3.62610	-2.78898	2.41581	8.49150	-3.36024
1.2	-0.86019	-4.30175	-2.18911	-1.24223	8.67127	-3.69059	5.7	-1.48591	-3.61846	-2.80729	2.45823	8.49062	-3.35714
1.3	-0.81089	-4.30754	-2.13981	-1.12211	8.68994	-3.66560	5.8	-1.50108	-3.61131	-2.82507	2.49957	8.48996	-3.35413
1.4	-0.76690	-4.31533	-2.09583	-1.01145	8.70840	-3.64368	5.9	-1.51584	-3.60461	-2.84234	2.53989	8.48950	-3.35121
1.5	-0.72744	-4.32472	-2.05637	-0.90901	8.72666	-3.62428	6	-1.53020	-3.59834	-2.85913	2.57921	8.48924	-3.34838
1.6	-0.69186	-4.33540	-2.02079	-0.81379	8.74472	-3.60700	6.1	-1.54419	-3.59248	-2.87546	2.61759	8.48917	-3.34564
1.7	-0.65965	-4.34712	-1.98858	-0.72494	8.76259	-3.59149	6.2	-1.55783	-3.58702	-2.89135	2.65505	8.48927	-3.34298
1.8	-0.63038	-4.35970	-1.95930	-0.64176	8.78026	-3.57750	6.3	-1.57113	-3.58192	-2.90683	2.69165	8.48954	-3.34039
1.9	-0.60368	-4.37299	-1.93261	-0.56367	8.79774	-3.56482	6.4	-1.58410	-3.57718	-2.92191	2.72740	8.48997	-3.33788
2	-0.57928	-4.38685	-1.90820	-0.49014	8.81504	-3.55327	6.5	-1.59677	-3.57277	-2.93662	2.76234	8.49055	-3.33544
2.1	-0.63381	-4.32737	-1.93758	-0.32336	8.78287	-3.54330	6.6	-1.60914	-3.56868	-2.95097	2.79650	8.49126	-3.33307
2.2	-0.68420	-4.27132	-1.96535	-0.16547	8.75265	-3.53415	6.7	-1.62123	-3.56490	-2.96497	2.82991	8.49212	-3.33077
2.3	-0.73098	-4.21832	-1.99167	-0.01557	8.72415	-3.52572	6.8	-1.63306	-3.56141	-2.97865	2.86259	8.49310	-3.32853
2.4	-0.77460	-4.16804	-2.01668	0.12713	8.69719	-3.51793	6.9	-1.64462	-3.55820	-2.99201	2.89458	8.49421	-3.32635
2.5	-0.81541	-4.12022	-2.04052	0.26329	8.67161	-3.51071	7	-1.65594	-3.55525	-3.00507	2.92590	8.49543	-3.32422
2.6	-0.85373	-4.07463	-2.06328	0.39349	8.64727	-3.50400	7.1	-1.67820	-3.55985	-3.02309	2.95031	8.50186	-3.32145
2.7	-0.88983	-4.03105	-2.08505	0.51823	8.62407	-3.49775	7.2	-1.69987	-3.56468	-3.04063	2.97408	8.50838	-3.31876
2.8	-0.92392	-3.98933	-2.10592	0.63796	8.60189	-3.49191	7.3	-1.72100	-3.56971	-3.05772	2.99725	8.51496	-3.31612
2.9	-0.95621	-3.94930	-2.12596	0.75306	8.58065	-3.48644	7.4	-1.74159	-3.57494	-3.07437	3.01982	8.52161	-3.31356
3	-0.98686	-3.91084	-2.14523	0.86389	8.56028	-3.48130	7.5	-1.76168	-3.58035	-3.09061	3.04183	8.52832	-3.31105
3.1	-1.01374	-3.89042	-2.18301	0.95175	8.55286	-3.47360	7.6	-1.78129	-3.58594	-3.10645	3.06330	8.53509	-3.30861
3.2	-1.03936	-3.87143	-2.21892	1.03600	8.54615	-3.46632	7.7	-1.80043	-3.59168	-3.12191	3.08425	8.54190	-3.30622

3.3	-1.06382	-3.85374	-2.25312	1.11688	8.54010	-3.45943	7.8	-1.81913	-3.59757	-3.13700	3.10470	8.54877	-3.30389
3.4	-1.08723	-3.83725	-2.28576	1.19462	8.53466	-3.45290	7.9	-1.83740	-3.60360	-3.15175	3.12466	8.55567	-3.30161
3.5	-1.10966	-3.82187	-2.31695	1.26943	8.52977	-3.44671	8	-1.85525	-3.60977	-3.16616	3.14416	8.56261	-3.29938
3.6	-1.13120	-3.80752	-2.34681	1.34150	8.52540	-3.44083	8.1	-1.87272	-3.61605	-3.18025	3.16322	8.56958	-3.29721
3.7	-1.15190	-3.79414	-2.37543	1.41099	8.52151	-3.43523	8.2	-1.88980	-3.62245	-3.19403	3.18184	8.57658	-3.29508
3.8	-1.17182	-3.78164	-2.40290	1.47805	8.51806	-3.42990	8.3	-1.90652	-3.62896	-3.20752	3.20004	8.58361	-3.29300
3.9	-1.19103	-3.76997	-2.42930	1.54284	8.51503	-3.42481	8.4	-1.92289	-3.63557	-3.22072	3.21784	8.59066	-3.29097
4	-1.20957	-3.75908	-2.45471	1.60548	8.51237	-3.41995	8.5	-1.93892	-3.64227	-3.23365	3.23525	8.59773	-3.28897
4.1	-1.22748	-3.74892	-2.47918	1.66608	8.51008	-3.41531	8.6	-1.95462	-3.64906	-3.24632	3.25229	8.60482	-3.28703
4.2	-1.24480	-3.73943	-2.50279	1.72476	8.50812	-3.41087	8.7	-1.97001	-3.65594	-3.25873	3.26897	8.61193	-3.28512
4.3	-1.26157	-3.73059	-2.52558	1.78162	8.50647	-3.40662	8.8	-1.98509	-3.66289	-3.27089	3.28529	8.61904	-3.28325
4.4	-1.27782	-3.72235	-2.54760	1.83676	8.50512	-3.40255	8.9	-1.99988	-3.66991	-3.28282	3.30127	8.62617	-3.28142
4.5	-1.29359	-3.71467	-2.56890	1.89025	8.50404	-3.39864	9	-2.01439	-3.67700	-3.29452	3.31693	8.63330	-3.27963
4.6	-1.30889	-3.70753	-2.58952	1.94219	8.50322	-3.39488	9.1	-2.01173	-3.69390	-3.29735	3.32171	8.64625	-3.27811
4.7	-1.32376	-3.70089	-2.60949	1.99264	8.50265	-3.39128	9.2	-2.00922	-3.71066	-3.30016	3.32640	8.65909	-3.27663
4.8	-1.33822	-3.69472	-2.62887	2.04167	8.50230	-3.38781	9.3	-2.00684	-3.72727	-3.30297	3.33101	8.67181	-3.27517
4.9	-1.35228	-3.68901	-2.64766	2.08936	8.50217	-3.38447	9.4	-2.00459	-3.74376	-3.30577	3.33552	8.68443	-3.27375
5	-1.36598	-3.68372	-2.66591	2.13576	8.50224	-3.38125	9.5	-2.00246	-3.76011	-3.30857	3.33995	8.69694	-3.27235
5.1	-1.38480	-3.67254	-2.68819	2.18577	8.49972	-3.37744	9.6	-2.00045	-3.77633	-3.31136	3.34430	8.70936	-3.27098
5.2	-1.40300	-3.66204	-2.70970	2.23436	8.49752	-3.37376	9.7	-1.99856	-3.79242	-3.31414	3.34857	8.72166	-3.26963
5.3	-1.42062	-3.65218	-2.73049	2.28160	8.49561	-3.37021	9.8	-1.99677	-3.80838	-3.31691	3.35276	8.73387	-3.26831
5.4	-1.43769	-3.64293	-2.75061	2.32755	8.49399	-3.36677	9.9	-1.99509	-3.82421	-3.31968	3.35687	8.74598	-3.26702

E. Implementation Resuts and Discussion

After solving equation (17), the obtained results for k, a_1, a_2, a_3, a_4, and a_5 are -6.42163, 0.06093, 0.23261, 0.04088, 0.08671, and 0.45905 respectively. As $k = \log a_0$, hence a_0 = 0.00163. Now, the values of a_0, a_1, a_2, a_3, a_4, and a_5 are available, therefore, the multiple-regression model for production turnover as a function of various inputs is written as equation (18).

$$\pi_6 = 0.00163(\pi_1)^{0.06093}(\pi_2)^{0.23261}(\pi_3)^{0.04088}(\pi_4)^{0.08671}(\pi_5)^{0.45905} \quad (18)$$

VI. CONCLUSION AND FUTURE SCOPE

The important issue at the formulation of mathematical model is that one should keep in mind the production process flow of the Biodiesel plant. As per the process flow and the specifications of the desired Biodiesel production plant design, the dependency and independency of the resources can be evaluated and accordingly the relation of the resources can be established. Once the relational model is prepared, the objectives of the desired mathematical model can be identified, which then make the concern problem as the single or multi-objective problem. Later it can be solved by the classical on non-classical methods. Different models can be possible according to the desires of individual, perspectives of the engineer involved in designing of the plant.

In presented work, an approach for mathematical model of production turnover of Biodiesel production plant is discussed. Presented work deals with the dimensional analysis and multiple regression analysis. Formulated mathematical model provides the detail of dependency of the production turnover on the inputs. As, it is identified that there are five groups on which the production turnover depends, out of which some typical group have the dominant role in deciding the quantity of production turnover. From equation (18), it is clear that the fifth group plays the very important role in evaluation of production turnover, as the power of the fifth group is largest one. The generated design data is used for the mathematical model formulation for the plant manufacturing Biodiesel. All this work is carried out with certain assumptions. If the data range is other than the range which is used in mathematical model formation, then, the curve for the dependent variable deviates for the other data range. This is happening because the model is developed using multiple linear regression analysis. Therefore, presented model is suitable for the range of the identified capacities. Mathematical formulation can be carried out by using different existing mechanisms and/or techniques. This paper provides the new direction of work for the researchers to optimize the design of any plant by generating design data which then be used for the formulation of mathematical model.

In future, different mathematical model can be formulated, such as, maintenance cost model and operating profit model. Presented work deals with the multiple linear regression analysis, but one can go for the same problem as the multiple-nonlinear regression problem. Presented model can be enhanced or evaluated against the other models which can be developed by considering other techniques of regression. Presented model can be evaluated by using the neural network. For this, one has to go for the formation neural network based on the generated data for the typical model, here for instance, production turnover model. Reliability of the presented model can be evaluated based on the evaluation of the actual values of the group terms and the evaluated values of the group terms using the proposed mathematical model. Correlation of these two values can be evaluated, based on which the reliability of the presented model can be predicted.

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