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## FREE INTERACTOR MATRIX METHOD FOR CONTROL PERFORMANCE ASSESSMENT OF MULTI-VARIATE SYSTEMS

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# FREE INTERACTOR MATRIX METHOD FOR CONTROL PERFORMANCE ASSESSMENT OF MULTI-VARIATE SYSTEMS

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**Abstract**—In this paper, an alternative method for the assessment of multi-variate control loop performance with consider twocircumstances. First, known time delays between each pair of inputs and outputs, and second, without relying on any a priori knowledge about the process model or timedelays. The performance of the control loop is calculated from data driven autoregressive moving average (ARMA) and prediction error model. It is clear that the limited data in scalar measure used for performance assessment results tends to steady-state as time tends to infinity, but large number of samples gives risen in scalar measures and tends to infinity as time samples tends to infinity and therefore it becomes difficult to calculate the performance index. In this paper, the later problem is solved by considering initial part of scalar measures with steady value for next-to-next time samples to calculate the control-loop performance index which would be utilized to decide healthy working of the control loop. Simulation example is included to show the performance index of multi-variate control loop.

**Keywords**- *Interactor matrices; Performance assessment; Performance Index; Multi-variate systems; Moving average; prediction error*

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## I. INTRODUCTION

Control performance assessment (CPA) techniques provide an indication of how current controller performance compares with what would be considered to be ideal. The ideal performance is typically referred to as a 'benchmark'. There are two fundamental requirements for any CPA algorithm. The first is that it should be able to detect any change in the performance of a control system and the second is that it should be able to identify the potential improvement that can be made to the performance of the control system if it were to be re-tuned or re-designed [1]. The performance of a control system relates to its ability to deal with the deviations between controlled variables and their set-points (or desired values). These deviations can be quantified by a single number, the performance index (indicator/potential/metric). Traditional performance measures (such as rise time, settling time, overshoot, offset from set-point, integral error criteria, etc.) have been used by [2,3,4,5]. In the case of frequentdeterministicdisturbances. The most widespread criterion considered for CPA is the variance (or, equivalently, the standard deviation), particularly for regulatory control. The performance of a control loop might be deemed unacceptable if the variance of the controlled variable exceeds some critical values, because of its direct relationship to process performance, product quality, and profit. In line with [6,7,8] the control performance indices (CPIs) should be scaled to lie within [0,1], where values close to 1 mean better/tighter control:

$$\eta = \frac{J_{des}}{J_{act}} \quad (1)$$

Where  $J_{des}$  is any ideal, optimal or desired/expected value for a given performance criterion (typically the variance), and  $J_{act}$  the actual value extracted from measured data. This definition is chosen for its practical acceptance and is not in line with[33], who use the reverse index. However, all indices (of the same category) are equivalent, i.e., they can easily be transformed into each other.

Several method exposures for CPA like Linear Quadratic Gaussian (LQG)[9], Model Prediction Control (MPC)[10,11]user-specified(US)[12,13,14]Minimum Variance Control (MVC) [15,16,17,18,19].Among a number of approaches for control performance monitoring, minimum variance control (MVC) - benchmark remains the most popular benchmark. One of the reasons for the suitability of MVC benchmark to assess performance of control loops in the industry is that it is non-intrusive and routine closed-loop operating data are sufficient for the calculation of this benchmark [20,21,22]. However, this convenience holds only in the univariate case where the time delay is the only a priori knowledge that needs to be available. For multi-variate processes, this simplicity is lost and the time delay is no longer a simple technical concept. An interactor matrix is needed for multi-variate process, and its calculation is beyond the knowledge of the time delay between each pair of inputs and outputs. The earlier work in this area is Huang[30,34] and Harris[29]. Both approaches

require an explicit knowledge of the interactor matrix [23].

**II. INTRACTOR MATRIX**

Consider the following multi-variate process

$$Y_t = TU_t + Na_t. \tag{2}$$

Where  $T$  and  $N$  are proper (causal), rational transfer function matrices in the backshift operator  $q^{-1}$ ;  $Y_t, U_t$  and  $a_t$  are output, input and noise vectors of appropriate dimensions.  $a_t$  is further assumed to be white noise with zero mean and  $Var(a_t) = \Sigma a$ .  $N$  is rational realization of disturbance spectrum with the standard assumptions [35] that  $N(q^{-1} = 0) = I$  and  $N$  is minimum phase, both of which are true through an appropriate realization of the disturbance spectrum. For every  $n \times m$  proper, rational polynomial transfer function matrix  $T$ , there exists non-singular,  $n \times n$  (non-unique) polynomial matrix  $D$ , such that  $|D| = q^r, D^T D = I$  and

$$\lim_{q^{-1} \rightarrow 0} DT = \lim_{q^{-1} \rightarrow 0} \tilde{T} = K. \tag{3}$$

where  $\tilde{T}$  is a full rank constant matrix, the integer  $r$  is defined as the number of infinite zeros of  $T$ , and  $\tilde{T}$  is the delay-free transfer function (factor) matrix of  $T$  which contains only finite zeros. The matrix  $D$  is known as the unitary interactor matrix, an equivalent form of the conventional lower triangular interactor matrix and can be written as

$$D = D_0 q^d + D_1 q^{d-1} + \dots + D_{d-1} q. \tag{4}$$

Where  $d$  is denoted as the order of the interactor matrix and is unique for a given transfer function matrix [24,25,26], and  $D_i$  are coefficient matrices. The interactor matrix  $D$  can be one of the three forms described in the sequel. If  $D$  is of the form:  $D = I + D_1 q^{-1} + \dots + D_{d-1} q^{-(d-1)}$  then the transfer function matrix  $T$  is regarded as having a simple interactor matrix. If  $D$  is a diagonal matrix, i.e.,  $D = \text{diag}\{d_1, d_2, \dots, d_n\}$ , then  $D$  is regarded as having a diagonal interactor matrix. Otherwise,  $D$  is considered to have a general interactor matrix.

The computation of the interactor matrix needs a complete process model or at least the first few Markov parameters of the process model [21], which is beyond the knowledge of time delays between each pair of the inputs and outputs. This requirement of process model information has been the main difficulty to the application of the multi-variate control performance assessment technique.

If the pair-wise time delays are unknown or the interactor matrix has been determined to be non-diagonal, then it is not possible to estimate minimum variance from closed loop routine operating data. We shall consider an alternative method for the assessment of multi-variate control loop performance without relying on any a priori knowledge of the interactor matrices [23].

**III. ASSESSMENT OF MULTI-VARIATE CONTROL PERFORMANCE WITH KNOWN PAIR-WISE TIME DELAYS**

It has been shown in [20,29] that the first  $d$  terms of the following moving average expansion of the interactor filtered multi-variate closed-loop output are feedback control invariant, where  $d$  is the order of the interactor matrix.

$$\tilde{Y}_t = q^{-d} = \tilde{F}_0 a_t + \tilde{F}_1 a_{t-1} + \dots + \tilde{F}_{d-1} a_{t-(d-1)} + \dots \tag{5}$$

The first  $d$  terms represent the closed-loop output of the minimum variance feedback control is implemented, where the minimum variance is in the sense of minimizing the trace of the covariance of  $\tilde{Y}_t$ . Due to the property of the unitary interactor matrix, the trace of the covariance of  $\tilde{Y}_t$  is the same as that of  $Y_t$ . If the interactor matrix is known, then Eq. (5) can be easily obtained through time series analysis of  $Y_t$  followed by the filtering of  $\tilde{Y}_t$  and then the moving average expansion, and the minimum variance term can be calculated, which can be used as a benchmark for multi-variable control performance assessment.

The problem in practical application is the interactor matrix as discussed in the last section, calculation of which, except for the diagonal interactor matrix, needs a priori knowledge of the process model. In particular, an experiment and identification effort has to be undertaken in order to calculate the interactor matrix.

Unlike univariate control performance assessment, for multi-variate control performance assessment, knowing pair-wise time delays is not sufficient for calculating minimum variance unless the interactor matrix has a simple or diagonal structure. However, if the time delays between each pair of inputs and outputs are indeed known, we should search for a possible simple or diagonal structure of the interactor matrix, which can directly lead to the computation of the multi-variate minimum variance. Both the simple and the diagonal interactor matrices can be calculated from the time delays between each pair of inputs and outputs of the process. One may surprisingly find that the simple and diagonal interactor matrices are not uncommon, particularly in industrial process, where the sparse structure of the transfer function matrix is often observed. The sparse structure also facilitates the determination of the interactor structure.

Consider a multi-variable transfer function matrix of dimension  $n \cdot m$  given by

$$T = \begin{bmatrix} T_{11} q^{-d_{11}} & T_{12} q^{-d_{12}} & \dots & T_{1m} q^{-d_{1m}} \\ T_{21} q^{-d_{21}} & T_{22} q^{-d_{22}} & \dots & T_{2m} q^{-d_{2m}} \\ \vdots & \vdots & \ddots & \vdots \\ T_{n1} q^{-d_{n1}} & T_{n2} q^{-d_{n2}} & \dots & T_{nm} q^{-d_{nm}} \end{bmatrix} \tag{6}$$

where  $\psi$  is a scalar transfer function from the  $i$ th input to the  $j$ th output. Define a delay matrix

$$\psi = \begin{bmatrix} t_{11}q^{-d_{11}} & t_{12}q^{-d_{12}} & \dots & t_{1m}q^{-d_{1m}} \\ t_{21}q^{-d_{21}} & t_{22}q^{-d_{22}} & \dots & t_{2m}q^{-d_{2m}} \\ \vdots & \vdots & \ddots & \vdots \\ t_{n1}q^{-d_{n1}} & t_{n2}q^{-d_{n2}} & \dots & t_{nm}q^{-d_{nm}} \end{bmatrix} \quad (7)$$

where  $d_{ij}$ 's are time delays that are assumed known;  $t_{ij}$  is the first non-zero impulse response coefficient from the  $j$ th input to the  $i$ th output, which is typically unknown. From  $W$ , we can obtain a diagonal matrix

$$\theta = \begin{bmatrix} q^{d_1} & \square & \square & \square \\ \square & q^{d_2} & \square & \square \\ \square & \square & \ddots & \square \\ \square & \square & \square & q^{d_n} \end{bmatrix} \quad (8)$$

where  $d_1 = \min \{d_{ij}; j = 1, \dots, m\}$ .

**Example 2:** Consider four processes discussed in [29]. The transfer functions matrices are given in Table 1. With sampling interval  $T$ , the four continuous-time transfer function matrices can be transferred to discrete-time transfer function matrices (by assuming zero-order hold). The time delay matrices  $W$  are summarized in the first row of Table 2. The  $H$  matrices are obtained and summarized in the second row. The multiplications  $\theta W$  are listed in the third row, and their determinants are shown in the fourth row. The fifth row shows the conditions for the determinants to be zero. It is not

difficult to find out that Wood–Berry and Wardle–Wood both have the diagonal interactor matrices; Ogunnaike and Ray has the simple interactor matrix structure

unless the first non-zero impulse responses of the four sub-transfer functions satisfy the condition, which is not the case; Vinante–Luyben does not have the simple or diagonal interactor matrix.

#### IV. ASSESSMENT OF MULTI-VARIATE CONTROL FREE INTRACTOR MATRIX

There are several interactor matrix-free methods in the literature, mainly based on closed-loop impulse response [21,27,28] and variance of multi-step prediction errors [27,29,37]. Earlier work in using interactor-free approach may be traced back to [31,32].

Consider a closed-loop multi-variate process represented by moving average model. It is a time series model of close loop transfer function of process when order of auto regressive part of ARMA model is zero. This is obtained by MATLAB2011a software.

The output model should come to constant and for this we can change the order of moving average part and the delay order. So, prediction error comes out from moving average and from that the covariance matrix is obtained.

Table 1 Four classical multi-variable processes

PROCESS	$g_{11}(s)$	$g_{12}(s)$	$g_{21}(s)$	$g_{22}(s)$
Wood and Berry (WB)	$\frac{12.8e^s}{16.7s + 1}$	$\frac{-18.9e^{-3s}}{21s + 1}$	$\frac{6.6e^{-7s}}{10.9s + 1}$	$\frac{-19.4e^{-3s}}{14.4s + 1}$
Vinante and Luyben (VL)	$\frac{-2.2e^s}{7s + 1}$	$\frac{1.3e^{-0.3s}}{7s + 1}$	$\frac{-2.8e^{-1.8s}}{9.5s + 1}$	$\frac{4.3e^{0.35s}}{9.2s + 1}$
Wardle and Wood (WW)	$\frac{0.126e^{-6s}}{60s + 1}$	$\frac{-0.101e^{-12s}}{(48s + 1)(45s + 1)}$	$\frac{0.094e^{-8s}}{38s + 1}$	$\frac{-0.12e^{-8s}}{35s + 1}$
Ogunnaike and Ray (OR)	$\frac{22.89e^{-0.2s}}{4.572s + 1}$	$\frac{-11.64e^{-0.4s}}{1.807s + 1}$	$\frac{4.689e^{-0.2s}}{2.174s + 1}$	$\frac{5.8e^{-0.4s}}{1.801s + 1}$

Table 2 Determination of interactor structure for four classical multivariable processes

	WB	VL	WW	OR
$\psi$	$\begin{bmatrix} t_{11}q^{-2} & t_{12}q^{-4} \\ t_{21}q^{-8} & t_{22}q^{-4} \end{bmatrix}$	$\begin{bmatrix} t_{11}q^{-2} & t_{12}q^{-1} \\ t_{21}q^{-2} & t_{22}q^{-1} \end{bmatrix}$	$\begin{bmatrix} t_{11}q^{-3} & t_{12}q^{-14} \\ t_{21}q^{-9} & t_{22}q^{-6} \end{bmatrix}$	$\begin{bmatrix} t_{11}q^{-1} & t_{12}q^{-1} \\ t_{21}q^{-1} & t_{22}q^{-1} \end{bmatrix}$
$\theta$	$\begin{bmatrix} q^2 & \\ & q^4 \end{bmatrix}$	$\begin{bmatrix} q & \\ & q \end{bmatrix}$	$\begin{bmatrix} q^7 & \\ & q^0 \end{bmatrix}$	$\begin{bmatrix} q & \\ & q \end{bmatrix}$
$K = \lim_{q \rightarrow 1} \theta \psi$	$\begin{bmatrix} t_{11} & 0 \\ 0 & t_{22} \end{bmatrix}$	$\begin{bmatrix} 0 & t_{12} \\ 0 & t_{22} \end{bmatrix}$	$\begin{bmatrix} t_{11} & 0 \\ t_{21} & t_{22} \end{bmatrix}$	$\begin{bmatrix} t_{11} & t_{12} \\ t_{21} & t_{22} \end{bmatrix}$
$\det(K)$	$t_{11}t_{22}$	0	$t_{11}t_{22}$	$t_{11}t_{22} = t_{12}t_{21}$
Cond.	None	Any $t_{ij}$	None	$\frac{t_{11}}{t_{12}} = \frac{t_{21}}{t_{22}}$

The size of covariance matrix depends on how much sample is going to be studied, for each point we bring out smaller covariance matrix for example in the fourth point we take first term of general matrix and scalar measure of *i.e.* the sum of original diameter terms of covariance matrix, this value after some limited number of samples tends to be fixed so we assume this as the final value to calculate that is obtained from (9).

$$p_i = \frac{s_{\infty} - s_i}{s_{\infty}} \quad (9)$$

In the flowing shows an algorithm of this intractor matrix-free method.

First find the closed-loop multi-variate process; define the data in iddata form, (input, delay), then define the  $T$ ,  $N$  and  $Q$  which are proper (causal), rational transfer function matrices in the backshift operator  $q^{-1}$ .

Find the closed-loop multi-variate process represented by;

$$CLTF = \frac{N}{(I+TQ)} \quad (10)$$

Find the output regarding to input by LSIM syntax The closed-loop multi-variate process represented by a moving average form;

$$Y_t = F_0 a_t + F_1 a_{t-1} + \dots + F_{i-1} a_{t-(i-1)} + F_i a_{t-i} + \dots \quad (11)$$

This is obtain in Matlab-software by syntax as well as IDENT-Toolbox, In IDENT toolbox after dataimport, in time domain we should select the linear parametric models and change the order which the part of auto regressive be zero by select  $an=0$  and estimate the model.

Since at is white noise, the optimal  $i$ th step prediction is given by;

$$y_{t|t-i} = F_i a_{t-i} + F_{i+1} a_{t-i-1} + \dots \quad (12)$$

The prediction error  $e_{t|t-i} = Y_t - Y_{t|t-i}$  is given by;

$$e_{t|t-i} = F_0 a_t + F_1 a_{t-1} + \dots + F_{i-1} a_{t-(i-1)} \quad (13)$$

Prediction error represented by flowing syntax; (14)

When

$E$  = prediction model

$Pe$  =syntax for finding prediction of MA model and DATA

$ARX$  =moving average model

$DATA$  = two input and output data, in IDDATA form.

The covariance of the prediction error can be calculated as;

$$COV(e_{t|t-i}) = F_0 F_0^T + F_1 F_1^T + \dots + F_{i-1} F_{i-1}^T \quad (15)$$

and its scalar measure ( $s_i$ )

$$s_i \triangleq tr[COV(e_{t|t-i})] = tr(F_0 F_0^T + F_1 F_1^T + \dots + F_{i-1} F_{i-1}^T) \quad (16)$$

### V. SIMULATIONEXAMPLE

**Example 2:** Consider a multi-variable process with the open-loop transfer function matrix  $T$  and disturbance transfer function matrix  $N$  given by;

$$T = \begin{bmatrix} \frac{q^{-1}}{1 - 0.4q^{-1}} & \frac{0.5q^{-2}}{1 - 0.1q^{-1}} \\ \frac{0.3q^{-1}}{1 - 0.4q^{-1}} & \frac{q^{-2}}{1 - 0.8q^{-1}} \end{bmatrix}$$

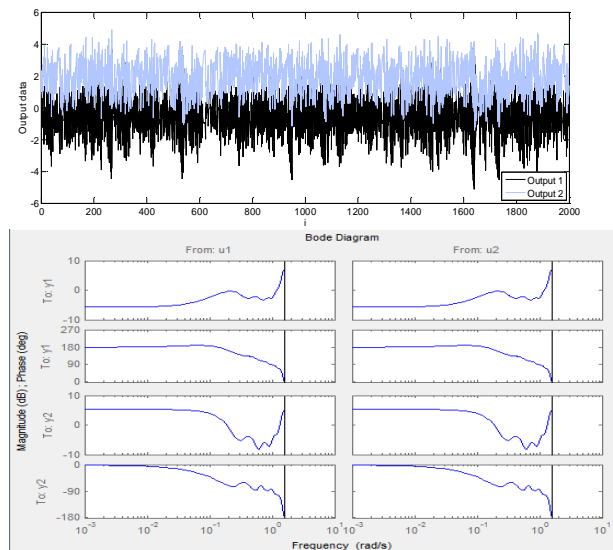
$$N = \begin{bmatrix} 1 & -1 \\ \frac{1}{1 - 0.5q^{-1}} & \frac{1}{1 - 0.6q^{-1}} \\ q^{-1} & 1 \\ \frac{1}{1 - 0.7q^{-1}} & \frac{1}{1 - 0.8q^{-1}} \end{bmatrix}$$

The white noise excitation is a two-dimensional normally distributed white noise sequence with

Consider that the following multi-loop controller is implemented in the process:

$$Q = \begin{bmatrix} k \frac{0.5 - 0.20q^{-1}}{1 - 0.5q^{-1}} & 0 \\ 0 & \frac{0.25 - 0.200q^{-1}}{(1 - 0.5q^{-1})(1 + 0.5q^{-1})} \end{bmatrix}$$

In this example, three controller gains are considered, respectively. As per the study on 2000 samples the output data and ARX model are plotted in the below figure (Fig. 1).



**Fig. 1. output data and ARX model**

In Fig.2. is calculated and plotted for three different range of samples. It indicates that the closed-loop settling time increases with the increase in controller gain. Also it shows, for first few samples the value is constant, but for large number of samples final value ( ) increases gradually with increase in .



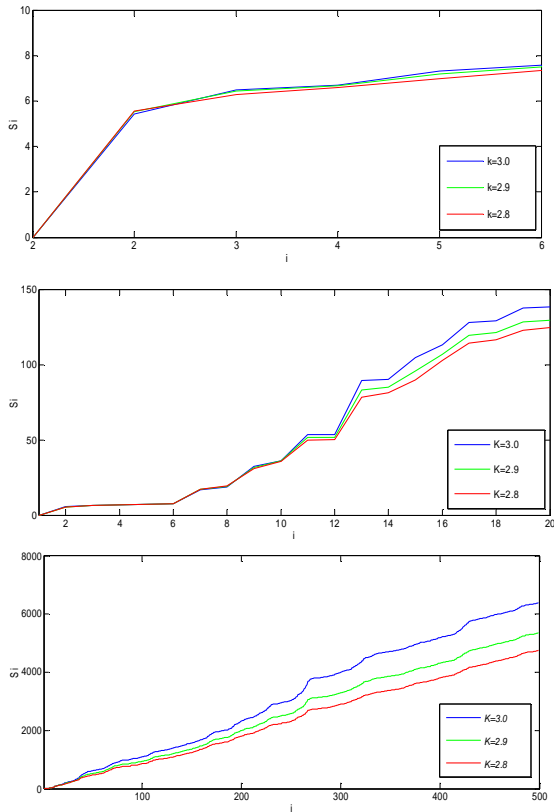


Fig. 2. Plot for first 6,20 and 500samples

The closed-loop potential  $p_i$  define as

$$P_i = \frac{S_{\infty} - S_i}{S_{\infty}} \quad (17)$$

Since  $S_i$  is monotonically increasing with an increase in  $i$ ,  $P_i$  is monotonically decreasing.  $S_0 = tr[cov(Y_t - Y_{t|t})] = 0$ ,  $P_0 = 1$ . Therefore,  $P_i$  starts from 1 at  $i = 0$  and monotonically decreases to 0 and  $0 \ll P_i \ll 1$ . Unlike the impulse response or variance of prediction error,  $P_i$  is dimensionless and facilitate the comparison of control performance. Due to the monotonically decreasing nature of the potentials, the area below the potential plot well reflects the rate of its decaying. Therefore, it is possible to define a scalar index to monitor the change of the closed-loop potential. This index is called relative closed-loop potential index and can be calculated as

$$\eta_p \triangleq \frac{\sum P_i^{(2)}}{P_i^{(1)}} - 1 \quad (18)$$

Where  $P_i^{(1)}$  is a reference potential calculated, for example, from the data sampled before control tuning, and  $P_i^{(2)}$  is calculated from data sampled after the tuning.

However for calculating the final scalar measure ( $\eta_p$ ) must be constant. This alternative solutions method is checked for different delay and different spread times ( $\tau$ ), the plot reflects for a limited number of samples when  $\tau$  tends to infinity the value of  $\eta_p$  tends to be constant value, but for a large number of samples  $\eta_p$  tends to increase incrementally as shown

in Fig. 2. Here we assume  $\tau$  is constant subsequently and plot  $\eta_p$  as shown in second and third graph in Fig. 3.

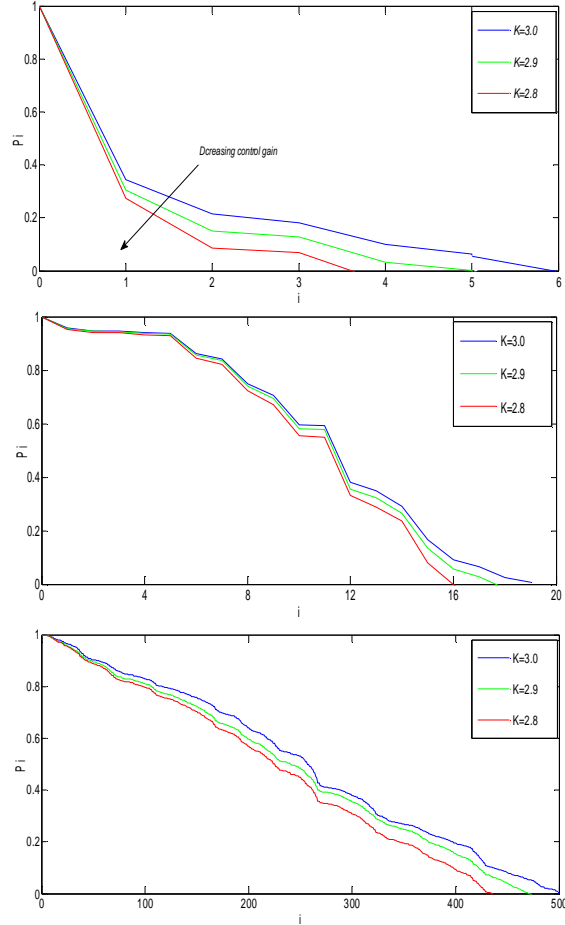


Fig. 3. Plot for first 6,20 and 500 samples

## VI. CONCLUSION

In this paper, the discussion of alternative and simple solutions to multi-variate feedback control performance assessment with prior knowledge and without any prior knowledge of the interactor matrices. The proposed an algorithm to obtained performance measure based on closed-loop potential and the solution is based on the multi-step optimal prediction error. This alternative method has been mentioned which is acceptable for limit of samples and it should improve to find proper control assessment index. The simulation examples have shown the features of the proposed algorithms.

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