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A Novel Approach for the Optimal PMU Placement using Binary Integer Programming Technique

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Abstract - Phasor measurement units (PMUs) are considered as a promising tool for future monitoring, protection and control of power systems. In this paper, a novel approach is proposed in order to determine the optimal number and locations of PMUs to make the power system observable. The PMU placement problem is formulated as a binary integer linear programming, in which the binary decision variables (0, 1) determine where to install a PMU, preserving the system set observability, such as full or partial. The proposed approach integrates the impacts of both existing conventional power injection/flow measurements (if any) and the possible failure of single or multiple PMU / communication line into the decision strategy of the optimal PMU allocation. The network topology and hence network connectivity matrix remains unaltered for the inclusion of conventional measurements. Program is developed using Matlab software and is tested on sample seven-bus system and IEEE 14 bus system.

Keywords : Binary integer linear programming , optimal PMU placement, maximum redundancy , failure of PMU / Communication line

I. INTRODUCTION

Synchronized Phase Measurement Unit (PMU) is a monitoring device, which was first introduced in mid-1980s. Phasor measurement units (PMU) are devices, which use synchronization signals from the global positioning system (GPS) satellites and provide the phasors of voltage and currents measured at a given substation. PMU is considered as the most promising measurement technology for monitoring the power systems. This unit is composed of a number of phasors that capture measurements of analog voltage, current waveform and the line frequency. As the PMUs become more and more affordable, their utilization will increase not only for substation applications but also at the control centers for the EMS applications. The Phasor Data Concentrator (PDC), which is located in the control center, gathers all the phasor data received from geographically distributed PMUs via the communication network. The communication network is one of the components that can be investigated and upgraded, to provide more accurate real-time information between the PMU and the control center. In this way, the observance of the power balance can be facilitated.

Wide Area Monitoring and Control (WAMC) systems based on Phasor Measurement Unit (PMU)

provides dynamic coverage of the network. Applications of the phase measuring units includes measuring frequency and magnitude of phasors, state estimation, instability prediction, adaptive relaying, and improved control. There are three basic approaches to conduct network observability analysis; namely numerical, topological, and hybrid approaches. The numerical observability approach is based on the fact that a unique solution for the state vector can be estimated if the gain matrix is nonsingular or equivalently if the measurement Jacobian matrix has a full column rank and well conditioned. Conventionally, numerical methods involve huge matrix manipulations and are computationally expensive. Also, in case of large power systems, the measurement matrix may become ill-conditioned. Many optimal PMU placement techniques, based on this concept, have been devised e.g. Simulated Annealing, particle swarm optimization [7,8], Tabu Search , Genetic Algorithm [2,10] based methods. However, all these techniques, being iterative in nature, require longer convergence time and their convergence depend upon the initial guess. The topological observability approach is based on the fact that a network is fully observable if the set of measurements can form at least one measurement spanning tree of full rank. Topological observability based approaches utilize the graph theoretic concept to find the optimal locations

and, thus, to make the system topologically observable. Few methods, based on this concept, are Depth First Search, Spanning Tree based method [6], and Integer Linear Programming based methods [4,5] etc. Due to the high cost of having a PMU at each node, some of the researcher focused on PMU placement and pseudo measurements for complete or partial observability of the power system. The placement of a minimal set of phasor measurement units at a strategic location to make the power system observable is the main objective of this paper. This paper has proposed a modified topological observability based PMU placement formulation to ensure complete system observability with and without conventional power flow and power injection measurements with one depth of unobservability and maximum redundancy. Further the formulation is extended for failure of PMU or communication line.

II. PROBLEM FORMULATION

PMUs provide two types of measurements: bus voltage phasors and branch current phasors. In this paper, it is assumed that each PMU has enough channels to record the bus voltage phasor at its associated bus and current phasors along all branches that are incident to this bus. Also the cost of all installed PMU is same.

The objective of the PMU placement problem is to render an observable system by using a minimum number of PMUs.

A. Without Conventional Measurements for Complete Observability

The optimal placement of PMU is formulated as follows

$$\text{Min } \sum x_i \sum_{i=1}^n x_i$$

Subject to the constraints

$$C X \geq b$$

Where, C is a connectivity matrix of the system, the entry of which can be defined as

$$C_{i,j} = 1 \text{ if } i=j$$

$$= 1 \text{ if } i \text{ and } j \text{ are connected}$$

$$= 0 \text{ otherwise}$$

$$X = [x_1 \ x_2 \ \dots \ x_n]$$

x_i is the PMU placement variable

b is a vector of length n (no. of nodes)

An example for a sample 7-bus system is as shown in Fig. 1.

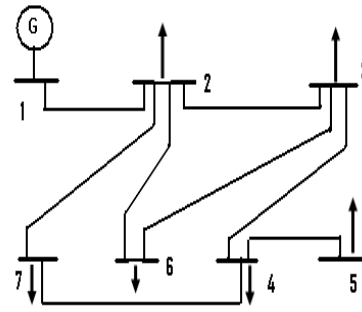


Fig. 1 Sample 7-bus system

For 7-bus sample system,

$$C = \begin{pmatrix} 1 & 1 & 0 & 0 & 0 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 & 1 & 1 \\ 0 & 1 & 1 & 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 1 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 & 0 & 0 & 1 \end{pmatrix}$$

B. Without Conventional Measurements for Incomplete Observability (Depth of one unobservability)

The optimal placement of PMU is formulated as follows

$$\text{Min } \sum x_i \sum_{i=1}^n x_i$$

Subject to the constraints

$$A C X \geq b_1$$

Where C is a connectivity matrix of the system A is branch to node incident matrix

$X = [x_1 \ x_2 \ \dots \ x_n]$; x_i is the PMU placement variable. b_1 is a vector of length m (no. of branches)

For 7-bus sample system,

$$A = \begin{pmatrix} 1 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 1 \end{pmatrix}$$

C. With Conventional Measurements for Complete Observability
The optimal placement of PMU is formulated as follows

$$\text{Min } \sum x_i \sum_{i=1}^n x_i$$

Subject to the constraints

$$T P C X \geq b_2$$

Where, C is a connectivity matrix of the system

P is a permutation matrix

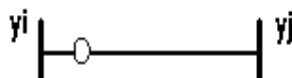
$$T = \begin{bmatrix} I_{M \times M} & 0 \\ 0 & T_{\text{meas}} \end{bmatrix}$$

Where M is the no. of buses not associated to conventional measurements

$X = [x_1 \ x_2 \ \dots \ x_n]$; x_i is the PMU placement variable b_2 is a vector defined according to cases

For Conventional measurement, following three cases needs to be analyzed in detail :

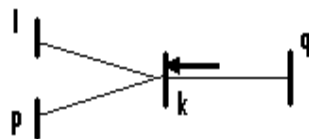
If a power flow measurement is on line ij



then the following needs to be held : $y_i + y_j \geq 1$

which means one bus voltage can be solved from this measurement and the other needs to be covered by PMU

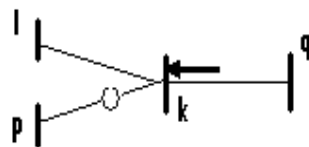
Suppose that an injection measurement is at bus k as follows



then the following inequality needs to be held :

$$y_i + y_p + y_k + y_q \geq 3$$

The power flow measurements and the injection measurements are associated



According to the approaches introduced in above mentioned case (1) and (2), the following two inequalities are as follows :

$$y_p + y_k \geq 1 \quad \text{and} \quad y_i + y_p + y_k + y_q \geq 3$$

In order to satisfy $y_p + y_k \geq 1$, the first inequality needs to be subscribed from the second inequality corresponding to the injection measurement and consider that its right-hand side needs to be reduced by one due to the injection I_k as shown below :

$$(y_i + y_p + y_k + y_q - y_p - y_k) \geq 3 - 1$$

The second inequality becomes $y_i + y_q \geq 1$

If bus s is not associated to any conventional measurement, then the corresponding constraint of the minimization problem is still kept $y_s \geq 1$

For 7-bus sample system,

$$T = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 & 1 \end{bmatrix}$$

$$P = \begin{bmatrix} 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

D. With Conventional Measurements for Incomplete Observability (Depth of one unobservability)

The optimal placement of PMU is formulated as follows

$$\text{Min } \sum x_i \sum_{i=1}^n x_i$$

Subject to the constraints

$$P_1 A C X \geq P_1 b_1$$

Where, C is a connectivity matrix of the system

A is branch to node incident matrix

P_1 is a matrix that keeps the branches that are associated with zero injection measurement and removes the branches that are not associated with zero injection measurement.

$X = [x_1 \ x_2 \ \dots \ x_n]$; x_i is the PMU placement variable

b_1 is a vector of length m (no. of branches)

For 7-bus sample system,

$$P1 = \begin{pmatrix} 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

In a sample 7-bus system, branch flow measurement is on line 2-3 and an injection measurement at bus 2 whereas in IEEE 14 bus system, a zero injection measurement is at bus 7 and no other power flow measurements.

Table I gives the system detail and size of matrix for the considered cases as follows. Fig. 2 shows single line diagram for IEEE 14- bus system

System	No. of variables	X	C	A	b	b1
7-bus	7	7	7 × 7	8 × 7	7 × 1	8 × 1
14-bus	14	14	14 × 14	20 × 14	14 × 7	14 × 1

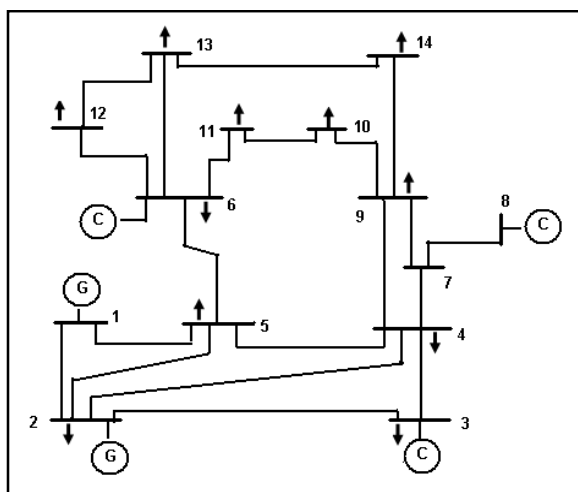


Fig. 2 IEEE 14 bus System

III. RESULT

Program is developed in MATLAB software using binary integer programming for determining optimum no. of PMU's and their location considering 4 cases for 2 different power system.

Program is further extended for the Placement of minimal no. of PMU considering maximum redundancy as well as failure of PMU / Communication line for the above mentioned 4 cases.

The proposed method attempts to provide a local PMU redundancy to allow for the loss of PMUs while preserving the global network observability. Entry for b

is set to 1 in order to guarantee the observability of each bus via at least one PMU. If it is changed to 2, then for that bus to be observable, at least two PMUs must be installed in the set of buses formed by all buses incident to bus , including bus itself. When the concept is extended to all buses, the network global observability will be directly maintained, with possible single or multiple PMU / Communication loss. It should be admitted that when the problem is solved in this way, a higher number of PMUs will be expected for the network observability. The method can also be made adaptive. Different numbers can be assigned to the right-hand side vector to account for different levels of PMU loss. These numbers can be assigned according to the heaviness of connectivity of each bus in the system. For IEEE 14 bus system, considering all the constraints, the optimum no. of PMU required for meeting objective function is found to be 9 with different level and different combination of number, only the placement of PMU changes. The cost of all PMUs are assumed to be equal at 1 p.u. Costs of PMU differ and increases as the number of channels. Unequal cost of the PMUs in the formulation of the problem is not considered in this paper as the maximum no. of measurand at any bus is not more than 7 for any mentioned cases for IEEE 14 bus system. Thus PMU is assumed to have sufficient no. of channels. The PMU placement at zero-injection buses will help find the optimal solution. PMUs at zero-injection buses measure current phasors of corresponding lines; thus, the KCL at zero-injection bus provides no additional information. The removal of PMUs from zero-injection buses will reduce the search space which could enhance the solution speed. Thus with zero injection bus in system reduces no. of PMU. Same is tested for sample 7-bus system and IEEE 14-bus system and is presented in table II.

This paper proposes a novel binary integer linear programming formulation for optimal PMU placement under different cases considering maximum redundancy as well as failure of PMU / Communication line. This formulation, considering the situations with and without zero injection measurements, shows that the problem of optimal PMU placement can be modeled linearly and can be solved by binary integer linear programming. Further Formulation can be extended to any depth of unobservability, even though result is presented and restricted for one depth only from power system reliability point of view. It is concluded that 9 no. of PMU's located at bus no. 2,4,5,6,7,8,9,11&13 will give maximum measurement redundancy for all cases including failure of device / communication for IEEE 14 bus system or PMU can be placed in 2-stage, first on bus no. 2,6,7 and 9 and in second stage on bus no. 4,5,8,11 and 13 as suggested in paper[10]. Proposed

formulation / algorithm is computationally simple, fast efficient and can be used in practice.

Table II gives the optimum no. of PMU and its location for the considered cases as follows

System	Observability / Constraint	Optimum no. of PMU		PMU Location		Remark
		Without zero injection	With zero injection	Without zero injection	With zero injection	
7-bus	Complete	2	2	2,4	2,5	No. of PMU's required is same, but location changes when zero injection is considered
	Incomplete (Depth of one unobservability)	1	1	3	4	
	Completeobservability with max. redundancy as well as failure of PMU / Comm. line	4	2	1,2,4, &5	2 & 4	No. of PMU's required reduces with zero injection consideration for the same constraint and objective function
	Incompleteobservability (Depth of one unobservability)with max. redundancy as well as failure of PMU / Comm. line	3	2	2,4 & 5	2 & 4	
System	Observability / Constraint	Optimum no. of PMU		PMU Location		Remark
		Without zero injection	With zero injection	Without zero injection	With zero injection	
14-bus	Complete	4	3	2,6,7,9	2,6,9	No. of PMU's required reduces, when zero injection is considered
	Incomplete (Depth of one unobservability)	2	1	4,6	6	

	Complete observability with max. redundancy as well as failure of PMU / Comm. line	9	4	2, 4, 5,6, 7, 8, 9,11 & 13	4, 5 ,6 & 9	No. of PMU's required reduces with zero injection consideration for the same constraint and objective function
	Incomplete observability (Depth of one unobservability)with max. redundancy as well as failure of PMU / Comm. line	7	4	2 ,4 ,5,6, 9,10 & 13	3 ,4 , 5 & 8	

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