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Adaptive Anti-synchronization of Identical and Non-identical 4-D Chaotic Systems

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Abstract - In this paper, we investigate the chaos anti-synchronization between two identical and different chaotic systems with fully unknown parameters via adaptive control. Based on the Lyapunov stability theory, an adaptive control law and a parameter update rule for unknown parameters are designed such that the two different chaotic systems can be anti-synchronized asymptotically. Theoretical analysis and numerical simulations are shown to verify the results.

Keywords: Chaotic LS system, Chaotic Qi system, Anti-synchronization, Adaptive control, Unknown parameters.

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1. INTRODUCTION

Since Pecora and Carroll [1] showed that it is possible to synchronize chaotic systems through a simple coupling, Chaos synchronization has attracted a great deal of attention and has been extensively studied in many areas such as chemical reactions, power converters, biological systems, information processing, secure communications, ecological systems, system identification, and so on [2,3].

Different types of synchronization phenomena have been observed in a variety of chaotic systems such as complete synchronization (CS), generalized synchronization (GS), phase synchronization (PS), lag synchronization (LS), anti-phase synchronization (APS) and anti-synchronization (AS) [4–11]. Anti-synchronization is a phenomenon that the state vectors of the synchronized systems have the same amplitude but opposite signs as those of the driving system. Therefore, the sum of two signals are expected to converge to zero when anti-synchronization appears.

The aim of this paper is to further develop the state observer method for constructing anti-synchronized slave system. The rest of the paper is organized as follows: Section 2 we present a novel adaptive anti-synchronization scheme with a parameter update law, anti-synchronization behavior of two identical Qi systems are studied in Sections 3. Section 4, deals with non-identical anti-synchronization between LS and Qi systems. In Section 5, we conclude the paper.

2. MATHEMATICAL MODELS

Consider the drive chaotic system in the form of

(2.1)

where is the state vector, is the unknown parameter vector of the system, is an matrix, is an matrix, the elements in matrix satisfies for . On the other hand, the response system is assumed by

(2.2)

where is the state vector, is the unknown parameter vector of the system, is an matrix, is an matrix, is control input vector, the elements in matrix satisfies for .

Let is the anti-synchronization error vector. Our goal is to design controller such that the trajectory of the response system (2.2) with initial condition can asymptotically approaches the drive system (2.1) with initial condition and finally implement the anti-synchronization, in the sense that,

(2.3)

where is the Euclidean norm.

2.1. Adaptive anti-synchronization controller design

Theorem 1. If the nonlinear control is selected as

$$u = -f(x) - F(x)\tilde{\alpha} - g(y) - G(y)\tilde{\beta} - ke \qquad (2.4)$$

and adaptive laws of parameters are taken as

$$\dot{\tilde{\alpha}} = [F(x)]^T e$$

$$\dot{\tilde{\beta}} = [G(y)]^T e \qquad (2.5)$$

then the response system (2.2) can anti-synchronize the drive system (2.1) globally and asymptotically, where $k > 0$ is a constant, $\tilde{\alpha}$ and $\tilde{\beta}$ are, respectively, estimations of the unknown parameters α and β.

Proof. From Eqs. (2.1) and (2.2), we get the error dynamical system as follows:

$$\dot{e} = F(x)(\alpha - \tilde{\alpha}) + G(y)(\beta - \tilde{\beta}) - ke \qquad (2.6)$$

Let $\hat{\alpha} = \alpha - \tilde{\alpha}$, $\hat{\beta} = \beta - \tilde{\beta}$, if a Lyapunov function candidate is chosen as

$$V(e, \hat{\alpha}, \hat{\beta}) = \frac{1}{2}\left[e^T e + (\alpha - \tilde{\alpha})^T(\alpha - \tilde{\alpha}) + (\beta - \tilde{\beta})^T(\beta - \tilde{\beta})\right] \qquad (2.7)$$

the time derivative of V along the trajectory of the error dynamical system (2.6) is as follows:

$$\dot{V}(e, \hat{\alpha}, \hat{\beta}) = \dot{e}^T e + (\alpha - \tilde{\alpha})^T \dot{\tilde{\alpha}} + (\beta - \tilde{\beta})^T \dot{\tilde{\beta}}$$

$$= \left[F(x)(\alpha - \tilde{\alpha}) + G(y)(\beta - \tilde{\beta}) - ke\right]^T e - (\alpha - \tilde{\alpha})^T [F(x)]^T e - (\beta - \tilde{\beta})^T [G(y)]^T e = -ke^T e \leq 0 \qquad (2.8)$$

Since V is positive definite, and $\dot{V}$ is negative semi-definite, it follows that $e, \tilde{\alpha} - \alpha, \tilde{\beta} - \beta \in L_\infty$. From the fact that $\int_0^t \|e\|^2 dt = \frac{1}{2}[V(0) - V(t)] \leq \frac{V(0)}{k}$, we can easily know that $e \in L_2$. From Eq.(2.6) we have $\dot{e} \in L_\infty$ Thus, by Barbalat's lemma, we have $\lim_{t\to\infty} e$, namely, $\lim_{t\to\infty}\|e\| = \lim_{t\to\infty}\|y(t, y_0) + x(t, x_0)\| = 0$. Thus the response system (2.2) can anti-synchronize the drive system (2.1) globally and asymptotically. This completes the proof.

Remark. If system (2.1) and system (2.2) satisfies $f(.) = g(.)$ and $F(.) = G(.)$, then the structure of systems (2.1) and (2.2) is identical. Therefore, Theorem 1 is also applicable to the adaptive anti-synchronization of two identical chaotic systems with unknown parameters.

2.2. systems description

The Lorenz-Stenflo system is described by the following nonlinear equations

$$\dot{x} = a(y - x) + cw$$

$$\dot{y} = x(d - z) - y$$

$$\dot{z} = xy - bz$$

$$\dot{w} = -x - aw \qquad (2.9)$$

which were formulated by Stenflo [12] from a low-frequency short-wavelength gravity wave equation. In (3.1), the dots denotes time derivatives, $a > 0, b > 0, c > 0, d > 0$ are respectively the Rayleigh number, Prandtl number, rotation number and geometric parameter. With the following parameters: $a = 1$, $b = 0.7$, $c = 1.5$, $d = 26$, the LS system exhibits the chaotic attractor.

The Qi system is described by the following nonlinear equations

$$\dot{x} = \alpha(y - x) + yzw$$

$$\dot{y} = \beta(x + y) - xzw$$

$$\dot{z} = -\gamma z + xyw$$

$$\dot{w} = -\lambda w + xyz \qquad (2.10)$$

where x, y, z and w are the state variables of the system and α, β, γ and λ are all positive real constant parameters. System (4.1) was recently introduced by Qi et al. [17] and it has been shown to exhibit complex dynamical behavior including the familiar period-doubling route to chaos as well as hope bifurcations [17]. With the following parameters: $\alpha = 30, \beta = 10, \gamma = 1, \lambda = 10$, the Qi system exhibits the chaotic attractor.

3. ADAPTIVE ANTI-SYNCHRONIZATION OF TWO IDENTICAL QI SYSTEMS

3.1. Formulation

Let us consider a Qi system given by

$$\dot{x}_1 = \alpha(y_1 - x_1) + y_1 z_1 w_1$$
$$\dot{y}_1 = \beta(x_1 + y_1) - x_1 z_1 w_1$$
$$\dot{z}_1 = -\gamma z_1 + x_1 y_1 w_1$$
$$\dot{w}_1 = -\lambda w_1 + x_1 y_1 z_1 \quad (3.2)$$

which drives a similar Qi system given as

$$\dot{x}_2 = \alpha(y_2 - x_2) + y_2 z_2 w_2 + u_1$$
$$\dot{y}_2 = \beta(x_2 + y_2) - x_2 z_2 w_2 + u_2$$
$$\dot{z}_2 = -\gamma z_2 + x_2 y_2 w_2 + u_3$$
$$\dot{w}_2 = -\lambda w_2 + x_2 y_2 z_2 + u_4 \quad (3.3)$$

where u_1, u_2, u_3, u_4 are four control functions to be designed, in order to determine the control functions to realize the anti-synchronization between systems equations (3.2) and (3.3), we add Eq. (3.3) to Eq. (3.2) and get

$$\dot{e}_1 = \alpha(e_2 - e_1) + y_2 z_2 w_2 + y_1 z_1 w_1 + u_1$$
$$\dot{e}_2 = \beta(e_2 + e_1) - x_2 z_2 w_2 - x_1 z_1 w_1 + u_2$$
$$\dot{e}_3 = -\gamma e_3 + x_2 y_2 w_2 + x_1 y_1 w_1 + u_3$$
$$\dot{e}_4 = -\lambda e_4 + x_2 y_2 z_2 + x_1 y_1 z_1 + u_4 \quad (3.4)$$

where $e_1 = x_1 + x_2, e_2 = y_1 + y_2, e_3 = z_1 + z_2, e_4 = w_1 + w_2$, our goal is to find proper control functions $u_i\ (i = 1,2,3,4)$ and parameter update rule, such that system equation (3.2) globally anti-synchronizes system equation (3.1) asymptotically, i.e. $\lim_{t\to\infty}\|e\| = 0$, where $e = [e_1, e_2, e_3, e_4]^T$. For two systems equations (3.1) and (3.2) without controls $(u_i = 0, i = 1,2,3,4)$, if the initial condition $(x_1(0), y_1(0), z_1(0), w_1(0)) \neq (x_2(0), y_2(0), z_2(0), w_2(0))$, then the trajectories of two systems will quickly separate each other and become irrelevant. However, when controls are applied, the two systems will approach anti-synchronization for any initial conditions by appropriate control functions. For this end, we propose the following adaptive control law for system equation (3.2):

$$u_1 = -\hat{\alpha}(e_2 - e_1) - y_2 z_2 w_2 - y_1 z_1 w_1 - e_1$$
$$u_2 = -\hat{\beta}(e_2 + e_1) + x_2 z_2 w_2 + x_1 z_1 w_1 - e_2$$
$$u_3 = (\hat{\gamma} - 1)e_3 - x_2 y_2 w_2 - x_1 y_1 w_1$$
$$u_4 = (\hat{\lambda} - 1)e_4 - x_2 y_2 z_2 - x_1 y_1 z_1 \quad (3.5)$$

and parameter update rule

$$\dot{\hat{\alpha}} = e_1 e_2 - e_1^2$$
$$\dot{\hat{\beta}} = e_1 e_2 + e_2^2$$
$$\dot{\hat{\gamma}} = -e_3^2$$
$$\dot{\hat{\lambda}} = -e_4^2 \quad (3.6)$$

Where $\hat{\alpha}, \hat{\beta}, \hat{\gamma}, \hat{\lambda}$ are estimates of $\alpha, \beta, \gamma, \lambda$, respectively.

Theorem 2. For any initial conditions, the two systems equations (3.2) and (3.3) are globally asymptotically anti-synchronized by adaptive control law equation (3.5) and parameter update rule equation (3.6).

Proof. Applying control law equation (3.5) to Eq. (3.4) yields the resulting error dynamics as follows:

$$\dot{e}_1 = \tilde{\alpha}(e_2 - e_1) - e_1$$
$$\dot{e}_2 = \tilde{\beta}(e_1 + e_2) - e_2$$
$$\dot{e}_3 = (\tilde{\gamma} - 1)e_3$$
$$\dot{e}_4 = (\tilde{\lambda} - 1)e_4 \quad (3.7)$$

where $\tilde{\alpha} = \alpha - \hat{\alpha}, \tilde{\beta} = \beta - \hat{\beta}, \tilde{\gamma} = \gamma - \hat{\gamma}, \tilde{\lambda} = \lambda - \hat{\lambda}$. Consider the following Lyapunov function:

$$V = \frac{1}{2}\left(e^T e + \tilde{\alpha}^2 + \tilde{\beta}^2 + \tilde{\gamma}^2 + \tilde{\lambda}^2\right) \quad (3.8)$$

then the time derivative of V along the solution of error dynamical system equation (3.7) gives that

$$\dot{V} = e^T\dot{e} + \tilde{\alpha}\dot{\tilde{\alpha}} + \tilde{\beta}\dot{\tilde{\beta}} + \tilde{\gamma}\dot{\tilde{\gamma}} + \tilde{\lambda}\dot{\tilde{\lambda}}$$
$$= e_1(\tilde{\alpha}(e_2 - e_1) - e_1) + e_2(\tilde{\beta}(e_1 + e_2) - e_2) + e_3((\tilde{\gamma} - 1)e_3) + e_4\left((\tilde{\lambda} - 1)e_4\right) + \tilde{\alpha}(e_1 e_2 - e_1^2)$$
$$= \tilde{\beta}(e_1 e_2 + e_2^2) + \tilde{\gamma}(-e_3^2) + \tilde{\lambda}(-e_4^2)$$
$$= e_1^2 - e_2^2 - e_3^2 - e_4^2 \leq 0 \quad (3.9)$$

Since V is positive definite and $\dot{V}$ is negative semi-definite in the neighborhood of zero solution of system equation (3.4), it follows that $e_1, e_2, e_3, e_4 \in$

$L_\infty, \hat{\alpha}, \hat{\beta}, \hat{\gamma}, \hat{\lambda} \in L_\infty$, from Eq. (3.7), we have $\dot{e}_1, \dot{e}_2, \dot{e}_3, \dot{e}_4 \in L_\infty$ since $\dot{V} = -e^T e$ then we obtain

$$\int_0^t \|e\|^2\, dt \le \int_0^t e^T e\, dt = \int_0^t -\dot{V}\, dt = V(0) - V(t) \le V(0) \quad \ldots.. \quad (3.10)$$

Thus, $\dot{e}_1, \dot{e}_2, \dot{e}_3, \dot{e}_4 \in L_2$, by Barbalat's lemma, we have $\lim_{t\to\infty}\|e(t)\| = 0$. Therefore, response system equation (3.3) can globally anti-synchronize drive system equation (3.2) asymptotically. This completes the proof.

3.2. Numerical simulations

To verify and demonstrate the effectiveness of the proposed method, we discuss the simulation result for the anti-synchronization between identical chaotic Qi systems. Assume that the initial condition $x_1(0) = 1.1$, $y_1(0) = -2.2$, $z_1(0) = 0.3$, $w_1(0) = 5.8$ and $x_2(0) = 1.15$, $y_2(0) = -2.3$, $z_2(0) = 0.4$, $w_2(0) = 5.5$, is employed. Hence the error system has the initial values $e_1(0) = 2.25$, $e_2(0) = -4.5$, $e_3(0) = 0.7$, $e_4(0) = 11.3$. The unknown parameters are chosen as $\alpha = 30, \beta = 10, \gamma = 1, \lambda = 0$, in simulations so that the both systems exhibit a chaotic behavior. Anti-synchronization of the systems (3.2) and (3.3) via adaptive control laws (3.5) and (3.6) with the initial estimated parameters $\hat{\alpha} = 0.1, \hat{\beta} = 0.1, \hat{\gamma} = 0.1, \hat{\lambda} = 0.1$ are shown in Figs.1-2. Fig.1 display the state response and Fig.2 shows the anti-synchronization errors of systems (3.2) and (3.3),

4. ADAPTIVE ANTI-SYNCHRONIZATION OF TWO DIFFERENT CHAOTIC SYSTEMS

In order to observe the anti-synchronization behavior between chaotic LS system (2.9) and chaotic Qi system (2.10), we assume that chaotic LS system is the drive system and chaotic Qi system is the response system. The drive and response systems are defined as follows:

$$\begin{aligned}
\dot{x}_1 &= a(y_1 - x_1) + cw_1 \\
\dot{y}_1 &= x_1(d - z_1) - y_1 \\
\dot{z}_1 &= x_1 y_1 - bz_1 \\
\dot{w}_1 &= -x_1 - aw_1
\end{aligned} \quad (4.1)$$

And
$$\begin{aligned}
\dot{x}_2 &= \alpha(y_2 - x_2) + y_2 z_2 w_2 + u_1 \\
\dot{y}_2 &= \beta(x_2 + y_2) - x_2 z_2 w_2 + u_2 \\
\dot{z}_2 &= -\gamma z_2 + x_2 y_2 w_2 + u_3 \\
\dot{w}_2 &= -\lambda w_2 + x_2 y_2 z_2 + u_4
\end{aligned} \quad (4.2)$$

where u_1, u_2, u_3, u_4 are four control functions to be designed, in order to determine the control functions to realize the anti-synchronization between systems equations (4.1) and (4.2), we add Eq. (4.2) to Eq. (4.1) and get

$$\begin{aligned}
\dot{e}_1 &= a(y_1 - x_1) + cw_1 + \alpha(y_2 - x_2) + y_2 z_2 w_2 + u_1 \\
\dot{e}_2 &= x_1(d - z_1) - y_1 + \beta(x_2 + y_2) - x_2 z_2 w_2 + u_2 \\
\dot{e}_3 &= x_1 y_1 - bz_1 - \gamma z_2 + x_2 y_2 w_2 + u_3 \\
\dot{e}_4 &= -x_1 - aw_1 - \lambda w_2 + x_2 y_2 z_2 + u_4
\end{aligned} \quad (4.3)$$

where $e_1 = x_1 + x_2, e_2 = y_1 + y_2, e_3 = z_1 + z_2, e_4 = w_1 + w_2$, our goal is to find proper control functions $u_i\ (i = 1,2,3,4)$ and parameter update rule, such that system equation (4.2) globally anti-synchronizes system equation (4.1) asymptotically, i.e. $\lim_{t\to\infty}\|e\| = 0$ where $e = [e_1, e_2, e_3, e_4]^T$. For two systems equations (4.1) and (4.2) without controls $(u_i = 0, i = 1,2,3,4)$, if the initial condition $(x_1(0), y_1(0), z_1(0), w_1(0)) \ne (x_2(0), y_2(0), z_2(0), w_2(0))$, then the trajectories of two systems will quickly separate each other and become irrelevant. However, when controls are applied, the two systems will approach anti-synchronization for any initial conditions by appropriate control functions. For this end, we propose the following adaptive control law for system equation (4.2):

$$\begin{aligned}
u_1 &= -\hat{a}(y_1 - x_1) - \hat{c}w_1 - \hat{\alpha}(y_2 - x_2) - y_2 z_2 w_2 - k_1 e_1 \\
u_2 &= -x_1(\hat{d} - z_1) + y_1 - \hat{\beta}(x_2 + y_2) + x_2 z_2 w_2 - k_2 e_2 \\
u_3 &= -x_1 y_1 + \hat{b} z_1 + \hat{\gamma} z_2 - x_2 y_2 w_2 - k_3 e_3 \\
u_4 &= x_1 + \hat{a} w_1 + \hat{\lambda} w_2 - x_2 y_2 z_2 - k_4 e_4
\end{aligned} \quad (4.4)$$

and parameter update rule

$$\begin{aligned}
\dot{\hat{a}} &= (y_1 - x_1)e_1 - w_1 e_4 \\
\dot{\hat{b}} &= -z_1 e_3 \\
\dot{\hat{c}} &= w_1 e_1
\end{aligned}$$

$$\dot{\hat{d}} = x_1 e_2$$
$$\dot{\hat{\alpha}} = (y_2 - x_2) e_1$$
$$\dot{\hat{\beta}} = (y_2 + x_2) e_2$$
$$\dot{\hat{\gamma}} = -z_2 e_3$$
$$\dot{\hat{\lambda}} = w_2 e_4 \quad (4.5)$$

where k_1, k_2, k_3, k_4 are four positive control coefficients, with which we can control the convergence speed of the scheme, $\hat{a}, \hat{b}, \hat{c}, \hat{d}, \hat{\alpha}, \hat{\beta}, \hat{\gamma}, \hat{\lambda}$, are estimates of $a, b, c, d, \alpha, \beta, \gamma, \lambda$, respectively.

Using theorem2, it can be seen that response system equation (4.2) globally anti-synchronize drive system equation (4.1) asymptotically.

4.1. Numerical simulations

To verify and demonstrate the effectiveness of the proposed method, we discuss the simulation result for the anti-synchronization between chaotic LS system and chaotic Qi system. We assume that the initial condition $x_1(0) = 0.028$, $y_1(0) = 0.02$, $z_1(0) = 0.03$, $w_1(0) = 0.04$ and $x_2(0) = 1.1$, $y_2(0) = -2.2$, $z_2(0) = 0.3$, $w_2(0) = 5.8$, is employed. Hence the error system has the initial values $e_1(0) = 1.128$, $e_2(0) = -1.18$, $e_3(0) = 0.33$, $e_4(0) = 5.84$. The unknown parameters are chosen as $a = 1, b = 0.7, c = 1.5, d = 26, \alpha = 30, \beta = 10, \gamma = 1, \lambda = 10$ in simulations so that the both systems exhibit a chaotic behavior. Anti-synchronization of the systems (4.1) and (4.2) via adaptive control laws (4.4) and (4.5) with the initial estimated parameters $\hat{a} = 0.1$, $\hat{b} = 0.1$, $\hat{c} = 0.1$, $\hat{d} = 0.1$, $\hat{\alpha} = 0.1$, $\hat{\beta} = 0.1$, $\hat{\gamma} = 0.1$, $\hat{\lambda} = 0.1$ are shown in Figs. 3-4. Fig. 3 displays the state response and Fig. 4 shows the anti-synchronization errors of systems (4.1) and (4.2).

5. CONCLUSION

This paper has examined the anti-synchronization of identical Qi system, and non-identical system consisting of the Lorenz–Stenflo system (as drive) and the Qi system (as response) using the technique of adaptive control. The designed adaptive controller ensures a stable anti-synchronization between the drive-response systems. Numerical simulations were also employed to illustrate the effectiveness of the approach.

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