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Abstract - Orthogonal Frequency division multiplexing (OFDM) is a very attractive technique in wireless communications which provides robustness to channel fading and immunity to impulse interference. Despite of its advantages, one of the major drawbacks of OFDM system is very high peak-to-average power ratio (PAPR). Among the various PAPR reduction techniques, companding appears attractive for its simplicity and effectiveness. In this paper novel Low density parity check (LDPC) encoded new companding technique is proposed which offers improved bit error rate, minimizes out-of-band interference and reduce PAPR effectively. Simulation results illustrates the performance of the system under Additive White Gaussian Noise (AWGN) and further evaluation is done for comparing the proposed companding technique with previous techniques.

Keywords - *Companding, Orthogonal frequency division multiplexing, Peak-to-average power ratio, Additive White Gaussian Noise, Low density parity check, Bit Error Rate.*

1. INTRODUCTION

Orthogonal Frequency Division Multiplexing (OFDM) has significant ability to support high data rates for wide area coverage, robustness to multipath fading, immunity to impulse interference [1,2]. However one of the major drawbacks of OFDM signal is its large envelope fluctuation, likely resulting in large peak-to average power ratio (PAPR), which distorts the signal if the transmitter contains the non-linear components such as power amplifiers and these may causes deficiencies such as intermodulation, spectral spreading and change in signal constellation. Minimizing PAPR allows higher average power to be transmitted for a fixed peak power and improving the overall signal to noise ratio at the receiver.

Some of the methods proposed in literature to reduce the PAPR of OFDM signals include several techniques such as amplitude clipping ,tone reservation(TR), active constellation extension(ACE) and coding [1,2][13], selective mapping [3], partial transmitting [4]. In [5], optimal companding coefficient is determined to enlarge small OFDM signals along with PAPR reduction. In [6], non-linear companding scheme is described by a single valued function which allows to be transformed before amplification. In exponential companding OFDM signals are transformed into uniformly distributed signals (with a specific degree) which are described in [7].

The idea behind these methods is that by clipping the peaks [8] of OFDM signal which is the simplest technique but it causes additional clipping noise and out-of-band interference (OBI) which degrades the system performance. But in μ-law companding, the PAPR is reduced at the expense of increasing the average power. In order to overcome the problem of increase of average power and to have efficient PAPR reduction, a non-linear companding technique namely exponential companding has been developed. In this paper, a simple but effective novel new companding technique which uses the special airy function to reduce the peak-to-average power ratio of OFDM signal is proposed. Furthermore, we extend the work to improve the performance of OFDM system by using some linear coding techniques, such as Low density parity coding (LDPC). In this respect we present a design of LDPC encoder to work in conjunction with new companding technique.

The paper is organized as follows: the PAPR problem in OFDM is briefly reviewed in section II. Section III, presents proposed algorithm using Low Density Parity-Check encoder to reduce the PAPR .In Section IV, the performance of proposed algorithm is compared with existing techniques. In Section V, we conclude.

II. PAPR IN OFDM

Let *X(0),X(1), · · ·, (−1)* represent the data sequence to be transmitted in an OFDM symbol with subcarriers. The baseband representation of the OFDM symbol is given by:

$$x(t)=\frac{1}{\sqrt{N}}\sum_{n=0}^{N-1}X(n)e^{j2\pi nt/N},0\le t\le T \qquad (1)$$

Where x(t) is OFDM symbol at time t, is the duration of the OFDM symbol. The input information symbols are assumed to be statistically independent and identically distributed. According to the central limit theorem, when is large, both the real and imaginary parts of ( ) becomes Gaussian distribution, each with zero mean and a variance of $E[|x(t)|^2]/2$, The amplitude, or modulus, of OFDM signal is given by

$$x_t=\sqrt{\mathrm{Re}^2\{x_t\}+\mathrm{Im}^2\{x_t\}} \qquad (2)$$

The amplitude has a Rayleigh distribution with the Cumulative Distribution Function (CDF) as follows.

$$F_{|x_t|(x)}=1-\exp(-\frac{x^2}{\sigma^2}),x\ge 0 \qquad (3)$$

The power of OFDM signal can be calculated as

$$|x_t|^2=\frac{1}{\sqrt{N}}\sum_{m=0}^{N-1}\sum_{k=0}^{N-1}X_mX_k\frac{\exp(j2\pi(m-k)t}{N} \qquad \text{….. (4)}$$

Where m=0,1,….N-1,k=0,1…N-1.Consequently it is possible that the maximum amplitude of OFDM signal may well exceed its average amplitude. Practical hardware (e.g. A/D and D/A converters, power amplifiers) has finite dynamic range; therefore the peak amplitude of OFDM signal must be limited.

The PAPR of the over sampled OFDM signal is mathematically defined as:

$$PAPR=10\log_{10}\left\{\frac{p_{\max}}{p_{avg}}\right\}=10\log_{10}\frac{\max\left[|x(t)|^2\right]}{\frac{1}{NT}\int_0^T|x(t)|^2dt}(dB) \qquad \text{…….. (5)}$$

The average power of the OFDM symbol in (4) can be written as :

$$p_{avg}=\frac{1}{T}\int_0^T\left|\sum_{v=0}^{N-1}c_ve^{j2\pi\frac{v}{NT}t_v}\right|^2dt \qquad (6)$$

$$p_{avg}=\frac{1}{T}\int_0^T(\sum_{v=0}^{N-1}c_v\cos(2\pi\frac{v}{NT}t_v))^2+(\sum_{v=0}^{N-1}c_v\sin(2\pi\frac{v}{NT}t_v))^2dt \qquad (7)$$

$$p_{avg}=\frac{1}{T}\int_0^T(\sum_{v=0}^{N-1}c^2{}_v)dt \qquad (8)$$

Where c_v is the magnitude of the modulated data.

The peak power occurs when modulated symbols are added with the same phase. The effectiveness of a PAPR reduction technique is measured by the complementary cumulative distribution function (CCDF), which is the probability that PAPR exceeds some threshold [11,12], i.e.

$$\text{CCDF} = \text{Probability}(\text{PAPR} > \text{PAPR}_0) \qquad (9)$$

Where $PAPR_0$ is the threshold level.

The next section describes linear coded LDPC and new companding techniques to reduce the effect of PAPR in independent multicarrier OFDM systems.

III. PAPR REDUCTION TECHNIQUES

A. Low-Density Parity Check Codes

LDPC codes (also known as Gallager codes) have recently received much attention from the communications industry because of their excellent error-correcting performance as well as having highly parallelizable decoding algorithm [15]. Fig. 1, depicts a OFDM system model with both LDPC encoder/decoder and companding implementation.

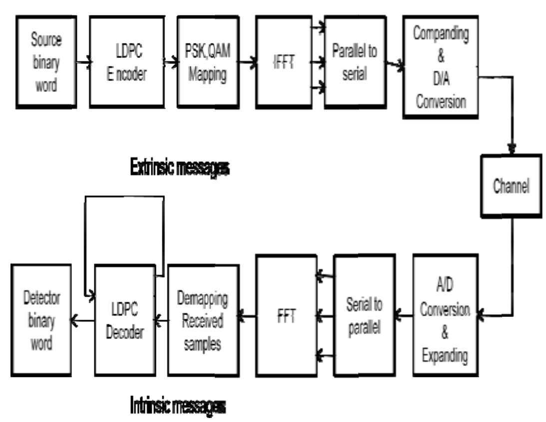

Fig. 1**:** Block diagram of LDPC-coded OFDM system with companding.

In designing the LDPC code the following design properties should be observed in order to obtain good performance; first the code should be long enough ,as the performance improves with code length, however a few cycles in the code bipartite graph may seriously degrade the error correcting performance if too many of them are used.

The size of each block matrix is b X b; the size of parity check matrix is $p_1 X p_2$, where p_1=mb and p_2=nb (where m and n define the size of the parity check matrix) and g=γb, where γ is the total number of blocks in the g submatrix. The row and column weight distributions are $\{w_{r_1}, w_{r_2}, ...w_{r_n}\}$ and $\{w_{c_1}, w_{c_2}, ...w_{c_n}\}$,where w_{r_i} and w_{c_j} represent the weight of i-th block rows and j-th columns respectively. The output from these parameters will provide the components of the $p_1 X p_2$ parity check matrix, **H**. This matrix will be either a right cyclic shift of an identity matrix or a zero matrix.

According the weight distribution of the matrix columns and rows, two different sets of weight distributions have been generated:

$$\{a_1, a_2, ..a_n\}, where\, a_j = w_{c_j}, if\, 1 \le j \le (n - m + \gamma)$$
$$= w_{c_j} - 1, if\,(n - m + \gamma \le j \le n) \quad .... (10)$$

$$\{b_1, b_2, ..b_m\}, where\, a_i = w_{r_i} - 1, if\, 1 \le i \le (m - \gamma)$$
$$= w_{r_i}, if\,(m - \gamma + 1 \le i \le m) \quad .... (11)$$

Starting with j=1, the a_j null blocks on the j-th block column will be replaced by a_j right cyclic shifted identity matrix. This is attained by:

STEP 1. Replacing $\mathbf{H}_{i,j}$ with a right cyclic shift of bXb identity matrix with a randomly generated shift value (i represents number of iterations and $b_i > 0$).

STEP 2. If the minimum cycle degree is less than the initial cycle degree, the replacement will be rejected and step 1 will be repeated (bearing in mind that for all variable nodes on a cycle, the sum of degrees1 is defined as the cycle degree of this cycle. It is, therefore, intuitively desirable to make the cycle degree as large as possible for those unavoidable small cycles).

STEP 3. Let $b_i = b_i - 1$.

STEP 4. If $d < d_{\min}$, terminate and restart the procedure where d and $d_{\min}$ are the calculated node degree distribution and the minimum node distribution threshold.

The remaining null blocks should then be replaced with zero matrices resulting in the output matrix. For more clarity, a flowchart of the encoding procedure is shown in Fig 2. The encoder design is accomplished by exploiting the structural property of the code parity check matrix.

B. New Companding Algorithm

In OFDM system, the ideal case is to reduce the PAPR to make the amplitude of the complex base-band signal constant. Quantification of the OBI caused by companding requires the knowledge of the power spectral density (PSD) of the companded signal. Unfortunately analytical expression of the PSD is in general mathematically intractable, because of the nonlinear companding transform involved. Here we take an alternative approach to estimate the OBI. Let f ( ) be a nonlinear companding function, and x( ) = sin( ) be the input to the compander. The companded signal y( ) is:

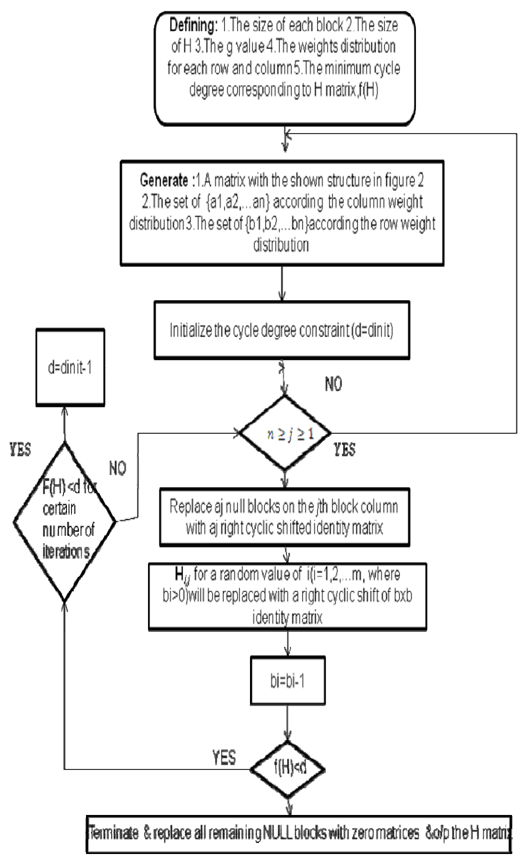

Fig. 2 : The flowchart of generating the H matrix

$$y(t) = f[x(t)] = f[\sin(wt)] \quad (12)$$

We now propose a new companding technique using a smooth function, namely the airy special function. The *companding* function is as follows:

$$f(x) = \beta . sign(x) . [airy(0) - airy(\alpha . |x|)] \quad (13)$$

Where airy(·) is the airy function of the first kind. α is the parameter that controls the degree of companding.β is the factor adjusting the average output power of the compander to the same level as the average input power:

$$\beta = \sqrt{\frac{E\left[|x|^2\right]}{E\left[|airy(0) - airy(\alpha.|x|)|^2\right]}} \quad (14)$$

Where $E[\cdot]$ denotes the expectation.

The decompanding function is the inverse of f(x):

$$f^{-1}(x) = \frac{1}{\alpha} . sign(x) . airy^{-1}\left[airy(0) - \frac{|x|}{\beta}\right] \quad (15)$$

Next we examine the BER performance of the proposed algorithm with linear LDPC. Let y(t) denote the output signal of the compander, w(t) the white Gaussian noise. The received signal can be expressed as:

$$z(t) = y(t) + w(t) \quad (16)$$

The decompanded signal $\bar{x}(t)$ simply is:

$$\bar{x}(t) = f^{-1}[z(t)] = f^{-1}[y(t) + w(t)] \quad (17)$$

It is worth to mention that BER and PAPR affect each other adversely and therefore there is a tradeoff.

IV. NUMERICAL RESULTS

The performance of the proposed OFDM system architecture (Fig.1) is evaluated with Complimentary Cumulative Distribution Function (CCDF) under both linear LDPC and new companding technique. Some of the simulation parameters are listed in Table 1.

TABLE 1: LIST OF OFDM PARAMETERS

Number of Transmit antenna	**1**
Number of Receive antenna	1
Number of Data streams	1
FFT Size	256
Number of Data Subcarriers	172
Channel model	Additive White Gaussian Noise
Modulation	BPSK
Number of Pilot Subcarriers	84
Companding Input Power	3dbm
Control parameter	30

The spectrums of the uncompressed and compressed OFDM signals by the proposed scheme are illustrated in Fig.3.From the simulation results; it is observed that the proposed algorithm produces OBI almost 3dB lower than the exponential algorithm, 10dB lower than the μ-law. Fig.4 depicts the CCDF of the three companding schemes. The new algorithm is roughly 1.5dB inferior to the exponential, but surpasses the μ-law by 6dB. The BER vs. SNR is plotted in Fig.5 proposed algorithm has improved bit error rate compared with exponential and µ-law algorithms. The amount of improvement increases as SNR becomes more. One more observation from the simulation is that unlike the exponential companding whose performance is found almost unchanged under different degrees of companding, the new algorithm is flexible in adjusting its specifications simply by changing the value of α in the companding function.

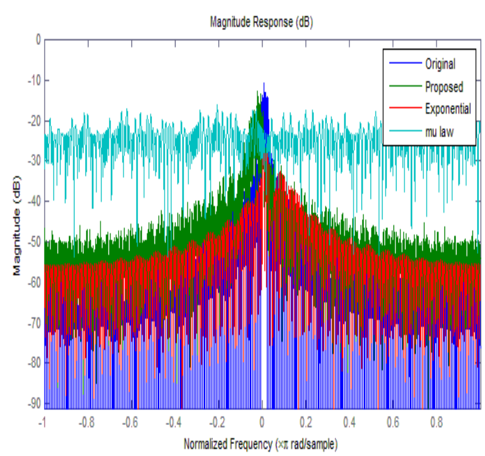

Fig. 3 : Power spectral density of original and companded signals (companded i/p power = 3dBm, α = 30)

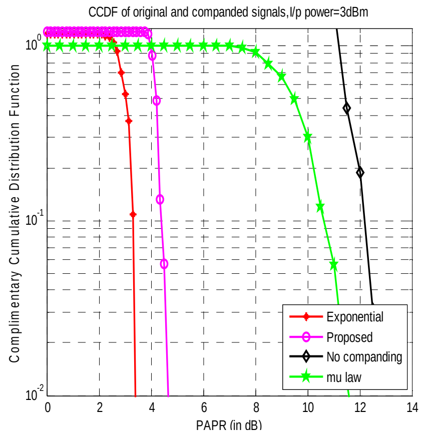

Fig. 4 : Complimentary Cumulative Distribution function of original and companded signals (companded i/p power = 3dBm, α = 30)

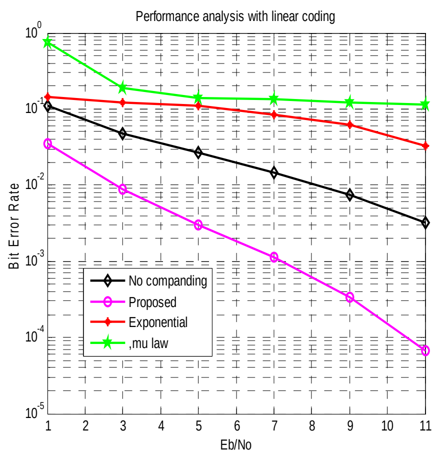

Fig. 5 : BER Vs SNR for original and companded signals in AWGN channel and Linear LDPC coding (companded i/p power = 3dBm, α = 30)

V. CONCLUSION

A novel companding algorithm is proposed to effectively reduce PAPR problem in Orthogonal Frequency Division Multiplexing (OFDM) with Low-Density-Parity check (LDPC) encoder. By careful selection of the control parameter α explained in the paper, the PAPR reduction can be achieved in a better way and the BER performance can be improved. Simulation results show, that the proposed algorithm offers improved performance in terms of BER and OBI while reducing PAPR effectively compared with exponential and μ-law companding schemes.

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