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Efficient Power Allocation for LDPC-Coded MIMO Systems

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Abstract **-** In this papers an efficient ordering scheme for an ordered successive interference cancellation detector is determined under the bit error rate minimization criterion for multiple-input multiple-output(MIMO) communication systems using transmission power control. From the convexity of the Q-function, we evaluate the choice of suitable quantization characteristics for both the decoder messages and the received samples in Low Density Parity Check (LDPC)-coded systems using M-QAM schemes. We derive the ordering strategy that makes the channel gains converge to their geometric mean. Based on this approach, the fixed ordering algorithm is first designed, for which the geometric mean is used for a constant threshold using correlation among ordering results.

Key words **-** *Multiple-Input Multiple-Output (MIMO), QR-decomposition, Detection Ordering, Low Density Parity Check (LDPC), Bit Error Rate(BER).*

I. INTRODUCTION

The multiple-input multiple-output (MIMO) system has been an active area of research as well as practical transceiver implementations for their great potential of enhancing the system's performance [1][2]. The V-BLAST architecture proposed in [3] and [4], also referred to as the BLAST-ordered successive interference cancellation (B-OSIC) detector, is regarded as an attractive solution that exploits this potential. In a B-OSIC receiver, the data stream with the strongest signal-to-interference-noise ratio (SINR) is selected first and subtracted from the received signal, and the procedure is successively performed for the remaining multiple data streams. For equal power allocation (PA)[5][6] across the transmit antenna array, it is optimal in terms of bit error rate (BER).

Low Density Parity Check (LDPC) codes are state-of-art error correcting codes [7] and [8], included in several standards for broadcast transmissions. Iterative soft-decision decoding algorithms for LDPC codes reach excellent error correction capability. Great attention has been paid, in recent literature, to the topic of quantization for LDPC decoders, but mostly focusing on binary modulations and analyzing finite precision of the receiver. The current scenario of error correcting codes is dominated by schemes using Soft-Input Soft-Output (SISO) decoding. Among them, an important role is played by Low- Density Parity-Check (LDPC) codes, which permit to approach the theoretical Shannon limit, while ensuring reduced complexity. For the name reason, these codes have been included in some recent telecommunication standards.

We provide the BER minimization condition, derived from the convexity of the Q -function in the PA scheme. It is demonstrated that the ordering strategy, which makes the channel gains converge to their geometric average, achieves the improved error performance. By using this observation, we develop the two ordering algorithms, which are identical except for the threshold adaptation. The basic algorithm determines the detection-order using the geometric mean as a constant threshold, whereas the modified ordering scheme for robust convergence adaptively updates the threshold by taking into account the previous ordering results. The comparison of the cumulative distribution is conducted to confirm the superiority of the adaptive design. There is no closed form solution for the BER or FER of coded systems except for some trivial cases that include un-coded systems and orthogonal coding [9,10]. Upper bounds are the conventional method for performance analysis of coded systems. The upper bounds of coded systems are obtained using the union bound technique.

The remainder of the paper is organized as follows: In section II, we propose system model of MIMO. In section III, proposed ordering scheme for power allocation and decoders are implemented. In section IV, SIC Receiver based on QR decomposition is presented. In section V, numerical analysis and Simulation results are briefly discussed. Finally, we concludes in Section VI.

II. SYSTEM MODEL

Fig.1, depicts a MIMO system with transmit antennas and receive antennas. The flat-fading MIMO channel is expressed by the matrix H with the element h_{ji} representing the channel gain from i_{th} transmit antenna to j_{th} receive antenna. The received signal vector $r = [r_1 \ldots\ldots r_{Nr}]^T$ is written as

$$r = \sqrt{\frac{E_s}{N_t}} Hpx + n \quad (1)$$

Where $x=[x_1,x_2,\ldots..x_{Nt}]^T$ denotes the x1 transmitted signal vector, and $n=[n_1,n_2,\ldots\ldots n_{Nt}]^T$ is the Nr dimensional noise vector with elements following complex zero mean Gaussian distribution with variance of σ_n^2 .E_s is the total transmitted signal energy on N_t transmit antenna. To express the signal model for the MMSE-QR detector, an $(N_r+N_t)\times N_t$ augmented channel matrix $\hat{H}$, an $(N_r+N_t)\times 1$ extended receive vector $\hat{r}$ and an $N_t \times 1$zero matrix O N_t,1 can be written as

$$H = \begin{bmatrix} H \\ \sigma_n I_{N_t} \end{bmatrix} \xrightarrow{\text{ordering}} \overline{Q}\overline{R} \text{ and } \hat{r} = \begin{bmatrix} r \\ 0_{N_T,1} \end{bmatrix} \quad (2)$$

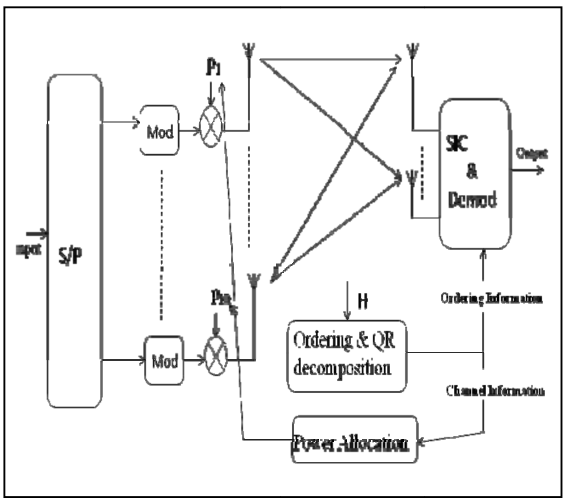

Fig. 1 : MIMO transmission model with QR-OSIC detector.

The upper triangular matrix $\bar{R}$ is differently defined by the detection-order; The QR-decomposition based OSIC detection for BER-minimized PA transmission can be performed using proposed architecture. Based on the feedback information of the diagonal elements, transmission power P_k is assigned to each data stream. The independently encoded symbols are processed through a diagonal PA matrix and then transmitted from N_t data streams. The QR-OSIC receiver detects the transmit symbols sequentially in accordance with the designated detection-order.

$$\rho_k = \frac{E_s}{\sigma_n^2} p_k^2 \overline{R}_{k,k}^{2} - 1, \quad k=1,\ldots..,N_t. \quad (3)$$

The analysis we have developed is quite general and can be applied, to any value of M. However, for better evidence, in the following, we will mainly refer to the specific case of a 16-QAM constellation. For any equal to an even power of 2, a Gray labeling can be adopted to match every sequence of encoded bits to each symbol.

The LDPC encoder maps each M–bit word produced by the source into an M-bit LDPC codeword. Each codeword is then passed to the mapper and modulator block, for M-QAM constellation. The modulated signal is then transmitted over an Additive White Gaussian Noise (AWGN) channel. At the receiver side, the demapper block is a maximum a posteriori (MAP) symbol-to-bit metric calculator, that is able to produce an initial likelihood value for each received bit (such values are denoted as *intrinsic* or channel messages). These messages serve as input for the Sum-Product Algorithm (SPA) that starts iterating and, at each iteration, produces updated versions of the *extrinsic* and the *a posteriori* messages. Gray labeling for 16-QAM are used as input for the subsequent iteration, while the latter represent the decoder output, and serve to obtain an estimated codeword that is subject to the hard decision and the parity-check test.

III. PROPOSED ORDERING SCHEME FOR POWER ALLOCATION AND DECODERS

A. *Power allocation schemes in OFDM*

A.1 Q-function in PA

Power allocated to the nulls of the frequency response is likely to be wasted. The performance of a coded OFDM system can be represented by upper bounds. For simple analysis, we assume that the channel code is linear and the modulation is BPSK or QPSK. Here we will present an efficient power allocation algorithm for a coded OFDM system to minimize the target BER under a constant total power constraint. Using the union bound technique, the upper bounds of BER give by

$$p_{BER} \le \frac{1}{N_b} \sum_i b_i.Z_i \quad (4)$$

Where N_b is the total number of information bits in a frame and *bi* is the Hamming distance of the $c_o \oplus c_i$. The pair wise error probability Z_i is a *Q* function,

$$Z_i = Q\left(\sqrt{2\sum_j |\rho_j|^2 . d_{i,j}^2 . P_j / N_0}\right) \quad (5)$$

Where $Q(x)=\sqrt{\frac{1}{2\Pi}}\int_x^{\infty} e^{-(t2/2)} dt$

Where $|\rho j|^2$ is the channel gain of the j^{th} symbol, P_j is the allocated power at the j^{th} symbol, d_{ij} is the Euclidean distance between codeword $c0$ and $c0 \oplus ci$ at the j^{th} symbol location with unit symbol power and N_0 is the noise spectral density of the AWGN.

The objective of power allocation is to minimize the target BER or FER under power constraints given by

$$\sum_i p_i \le p_T \quad (6)$$

$$p_i \ge 0 \quad \text{for } i=1,\ldots\ldots, N_8$$

Where Pi is the transmission power of the *i*-th subcarrier, P_T is the total transmission power and *Ns* is the number of modulation symbols in a frame. The sum of exponential functions and that of *Q*-functions are convex functions. A detailed proof of this statement will be given in the next section. Therefore, power optimization to minimize the FER or BER bound under a constant power constraint is a convex optimization problem.

$$P_{BER} \le \sum_i \frac{b_i}{N_b} Q\left(\sqrt{2\sum_j |p_j|^2 . d^2_{i,j} . p_j / N_o}\right)$$
$$\le \sum_i \frac{b_i}{2N_b} \exp\left(\sqrt{-\sum_j |p_j|^2 . d^2_{i,j} . p_j / N_o}\right) \quad (7)$$

Sums of exponential functions and *Q*-functions are log-convex functions. It is difficult to compute the summations of the BER bounds for all possible code words, especially when there are many code words in the codebook. As the number of codewords increases, the complexity of the optimization increases. In this case, it is possible to approximate the BER bounds by using the N_t terms which are nearest to the transmitted data sequence c_0.The above BER bounds are obtained assuming linearity of the coding and modulation. The general expression for non-linear coding needs the average for the transmitted code words which increases the complexity of the BER computation.

A.2. LDPC

Consider a transmitter with *Nt* antennas and a receiver with *Nr* antennas in Figure 1. The data bits from *L* layers are individually encoded by LDPC encoders. The coded bits are bit-wise interleaved, prior to Gray mapping to M_l-QAM symbols with unity average power at the l^{th} layer. The resulting symbols are grouped by the *K* symbols and fed into an OSTBC encoder producing an $M \times T$ code matrix. An OSTBC matrix X_l with the symbols $x1, x2, \ldots\ldots xK$ is constructed as

$$X_l = \sum_{k=1}^{K}\left(x_k^R U_k + j x_k^I V_k\right) \quad (8)$$

Where x_k^R and $xI\ k$ are the real part and imaginary part of xk, respectively. U_k and V_k are dispersion matrices of size $M \times T$. which satisfy the following conditions:

$$U_n U_n^H = I_M, V_n V_n^H = I_M, \quad 1 \le n \le K$$

$$U_n U_m^H = -U_m U_n^H, \quad 1 \le n \ne m \le K$$

$$V_n V_m^H = -V_m V_n^H, \quad 1 \le n \ne m \le K$$

$$U_n V_m^H = V_m U_n^H, \quad 1 \le n, m \le K$$

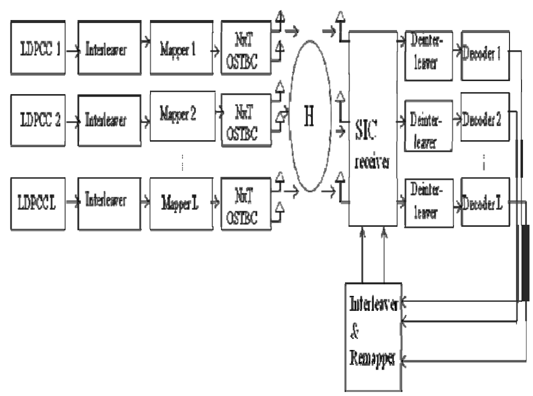

Fig. 2 **:** Group layered multi-antenna architecture using LDPC-coded transmission and detection based on SIC.

Where I_M denotes the identity matrix of size $M \times M$. The OSTBC matrices from the *L* layers are transmitted in parallel over N_t antennas. That is, the transmission matrix is given by

$$X = [X_1^t, X_2^t, \ldots\ldots X_L^t]^t \quad (9)$$

and $N_t = ML$, where A^t denotes the transpose of a matrix A. The $N_r \times T$ received matrix Y is given by

$$Y = HX + N, \quad (10)$$

Where H is an $N_r \times N_t$ channel matrix whose entries are independent and identically distributed (i.i.d.) complex

Gaussian random variables with zero mean and unit variance, and N is the noise matrix whose entries are statistically independent zero-mean complex Gaussian random variables with variance $2\sigma^2$. We assume that H is constant over a block of T symbols and perfectly known to the receiver.

IV. SIC RECEIVER BASED ON QR DECOMPOSITION

The receiver for our system performs the QR decomposition of the channel matrix, i.e., H = QR, where Q is a unitary matrix with size $N_r \times N_t$ and R is an upper triangular matrix with size $N_t \times N_t$. By multiplying Q† to the received matrix Y, we get

$$\tilde{Y} = Q^{\dagger} Y = RX + \tilde{N} \tag{11}$$

Where Q† is the Hermitian of Q and $\tilde{N}$ = Q†N has i.i.d. zero-mean complex Gaussian entries with variance $2\sigma^2$. The matrix R can be written as,

$$R = \begin{bmatrix} R_{1,1} & R_{1,2} & \ldots & R_{1,L} \\ 0 & R_{2,2} & \ldots & R_{2,L} \\ . & . & .. & . \\ 0 & 0 & \ldots & R_{L,L} \end{bmatrix}$$

Where $R_{l,k}$ is an $M \times M$ sub matrix of R, given by

$$R_{l,k} = \begin{bmatrix} r_{M(l-1)+,M(k-1)+1} & \ldots & r_{M(l-1)+1,Mk} \\ . & & . \\ . & & . \\ r_{Ml,M(k-1)+1} & \ldots & r_{Ml,Mk} \end{bmatrix} \tag{12}$$

and $r_{u,v}$ is the (u, v) entry of R and is zero, whenever $u > v$. The statistical properties of $r_{i,j}$, $1 \le i < j \le N_t$, may be directly deduced by applying the following theorem to the case that $m = N_r$ and $n = N_t$.

Theorem 1: Let H be an $m \times n$ complex Gaussian matrix with $m \ge n$, whose entries are i.i.d. complex Gaussian random variables with zero-mean and unit variance. Denote its QR decomposition by H = QR. The matrix R is upper triangular with real valued diagonal. The entries of R are independent of each other. Moreover, the square of the ith diagonal element of R, $r_{i,j}^2$, is of CHI-SQUARE distribution with the degree of freedom $2(m-i+1)$. The off-diagonal elements $r_{i,j}$, for $1 \le i < j \le n$, are zero-mean complex Gaussian with unit variance. The l^{th} $M \times T$ sub block in Ỹ for decoding of the l^{th} layer may be written as

$$\bar{Y}_l = R_{l,l} X_l + \sum_{k=l+1}^{L} R_{l,k} X_k + \bar{N}_l \tag{13}$$

Where $\tilde{N}_l$ is the l^{th} M ×T sub block of Ñ. The decoding process is successively done from the l^{th} layer to the first layer by using the information from the previously decoded layers. That is, the decision matrix Z_l for decoding at the l^{th} layer is given by

$$Z_l = \tilde{Y}_l - \sum_{k=l+1}^{L} R_{l,k} \hat{X}_k \tag{14}$$

Where $\hat{X}_k$ is the OSTBC matrix recovered by re-mapping the codeword decoded in the k^{th} layer to Mk-QAM symbols and re-encoding the symbols to the OSTBCs. We assume that there are no errors in decoding of the previous layers so that error propagation does not occur. Then Z_l in can be written as

$$Z_l = R_{l,l} X_l + \tilde{N}_l \tag{15}$$

By denoting the m^{th} rows of Z_l, $R_{i,i}$, and $\tilde{N}_l$ by Z_m, r_m, and n_m, respectively, and applying X_l, we get the following vector equation

$$\tilde{z}_l = \tilde{X}_l \tilde{R}_l + \tilde{n}_l \tag{16}$$

where,

$\tilde{z}_l = [(z_1^R, z_1^I, \ldots, (z_M^R, z_M^I)]$, $\tilde{x}_l = [(x_1^R, x_1^I, \ldots, (x_k^R, x_k^I)]$,

$\tilde{n}_l = [(n_1^R, n_1^I, \ldots, (n_M^R, n_M^I)]$,

and the superscripts R and I denote the real part and imaginary part, respectively. The equivalent real channel matrix $\bar{R}_1$.

$$\tilde{R}_l = \begin{bmatrix} \tilde{r}_1 U_1 & \ldots & \tilde{r}_M U_1 \\ \tilde{r}_1 V_1 & \ldots & \tilde{r}_M V_1 \\ & & \\ \tilde{r}_1 U_1 & \ldots & \tilde{r}_M U_1 \\ \tilde{r}_1 V_1 & \ldots & \tilde{r}_M V_1 \end{bmatrix} \tag{17}$$

where $U_k = \begin{bmatrix} U_k^R & U_k^I \\ -U_k^I & U_k^R \end{bmatrix}, V_k = \begin{bmatrix} -V_k^I & V_k^R \\ -V_k^R & -V_k^I \end{bmatrix}$, and

$\bar{r}_m = \left[r_m^R, r_m^I \right]$ From the properties of the dispersion matrices in it may be easily checked that Ř

mutually orthogonal rows, i.e., $\overline{R}_l \overline{R}_l^H = \left\| R_{l,l} \right\|^2 I_{2k}$ where $\left\| . \right\|$ denotes the Frobenius norm of a matrix. Thus, we get the following decision statistic:

$$\hat{z}_l = \frac{\overline{z}_l \overline{R}_l^H}{\left\| R_{i,i} \right\|^2} = \overline{x}_l + \hat{n}_l \qquad (18)$$

Where the components of ň$_l$ are i.i.d. zero-mean real Gaussian random variables with variance $\sigma 2 / \left\| R_{l,l} \right\|^2$. Note that Ž$_l$ in (10) is equivalent to the received vector for QAM symbols transmitted over K parallel Gaussian channels having $\left\| R_{l,l,} \right\|$ as a channel gain. In addition, it is easily shown by Theorem 1 that $\left\| R_{l,l} \right\|^2$ is of chi-square distribution with the degree of freedom $2M(N_r - M(l - 1))$. The kth element ž$_{l,k}$ in ž$_l$ has the following conditional probability density function (pdf):

$$p\left(\hat{z}_{l,k} \middle| \hat{x}_{l,k,p}\right) = \sqrt{\frac{\rho}{\Pi}} \exp\left\{-\rho(z_{l,k} - \overline{x}_{l,k})^2\right\} \qquad (19)$$

A The BER Performance

As in the derivation of the post-detection SINR, the error rate is also affected by the channel gains and the transmission power. A Power Allocation scheme for the average BER minimization under the assumption of the QR-decomposition of the channel matrix and no error propagation in successive cancellation of the data streams has been proposed. The PA scheme for M-QAM modulation can be expressed as the average BER of the PA can be approximated with a constellation-specific constant the average BER as well as the post-detection SINR is determined by the allocated power and the channel gain. Because of the convexity property of the Q-function, the resulting BER is minimized by the detection ordering of the QR-OSIC receiver such that all diagonal elements of the matrix are equal to their geometrical average , and alternatively the PA scheme at the transmitter which makes the product of two variables and identical for all data streams. As the real MIMO channel is characterized by several spatial-temporal properties, the condition (i) Is not practical in spite of its optimality. (ii), Different detection-order leads to different transmission power and hence power gain should be also differently assigned. This indicates that an appropriate detection ordering strategy incorporates with the PA scheme can achieve the improved BER performance.

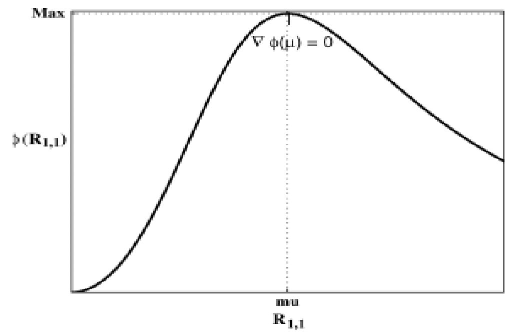

Fig. 3 : Graph ofΦ (Ř$_{1,1}$)

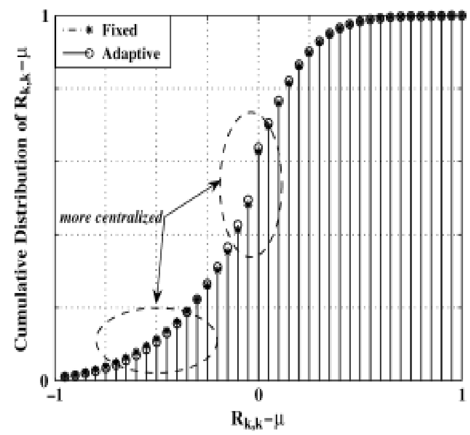

Fig. 4 : Comparision of cumulative destribution Ř$_{k,k}$ -μ

V. NUMERICAL RESULTS

For each of the MIMO systems and for a specific value of SNR, a quasi-static channel is assumed for the performance evaluation, for which the channel gain is constant over a frame and changed independently from frame to frame. To concentrate our point on comparing ordering algorithms, we postulate the perfect channel estimation at the receiver and error-free PA information at the transmitter. Fig. 4 shows the average BER performance comparison for MIMO systems with three transmit/receive antennas and the simulation results of four transmit/receive antennas are depicted in Fig. 5. Here, the dashed line indicates a system with the BER-minimized PA scheme, whereas the solid line represents a system without the PA. Implementation of QR receiver with PA and without ordering in denoted as QR-SIC. The QR receiver with the PA but no ordering, denoted as QR-SIC w/ PA, has similar performance to the open-loop OSIC systems without the PA.

This demonstrates the importance of the detection-order for successive detection. As expected, without the PA, the B-OSIC outperforms the QR-OSIC receiver. Despite the reduced complexity, however, power controlled MIMO systems employing the proposed ordering strategy achieve the improved error performance compared to those with the B-OSIC algorithm. It is sufficient to confirm the superiority of the proposed design because the ordering algorithms of previous studies comply with the strategy of the B-OSIC. A further performance improvement in the high SNR region can be explained in terms of the error propagation, since the PA scheme as well as the proposed QR-OSIC receiver is designed under the assumption of the error-free decision in previous detection stages.

SIMULATION RESULTS

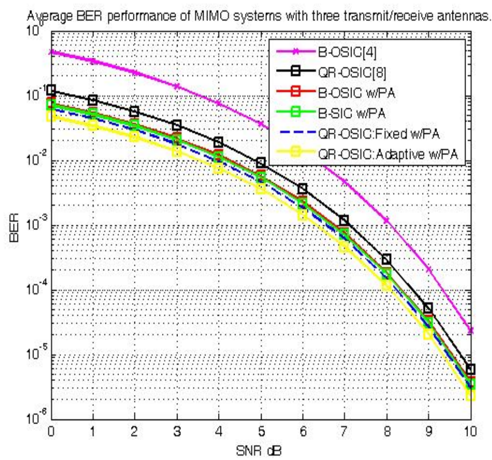

Fig. 5: BER performance of MIMO system with three transmit/receive antennas

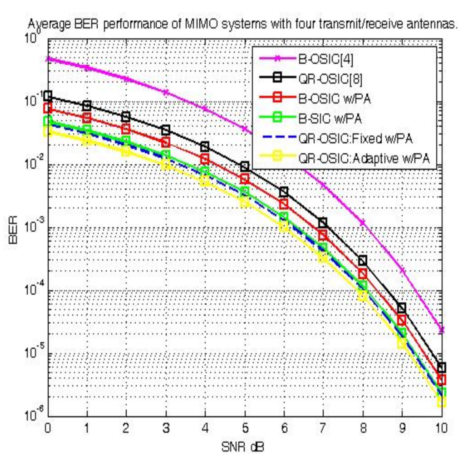

Fig. 6 : BER performance of MIMO system with four transmit/receive antennas

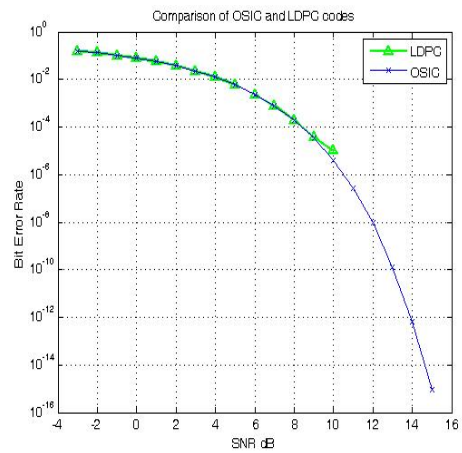

Fig. 7: Comparison of OSIC and LDPC codes

VI. CONCLUSIONS

In this study, we investigated the QR-OSIC receiver design for the transmitter-side power allocated MIMO system. Based on the properties of the Q-function and ordering results, we develop the efficient power allocation for LDPC in digital system. We evaluate the choice of suitable quantization characteristics for both the decoder messages and the received samples in Low Density Parity Check (LDPC)-coded systems using M-QAM schemes. Because of the post-detection SINR increment, the coded systems with the derived approach can also be expected to achieve the performance improvement.

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