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Abstract **-** In this paper, simple approach is proposed to determine reduced order model of a unstable open-loop position control system. This approach is based on Krishnamurthy's approach on Routh criterion on reduced order modelling. The results are simulated in Matlab environment.

Keywords **-** *Reduced order modelling, dc motor, Routh criterion.*

I. INTRODUCTION

Reduced order modelling is an important issue in control systems. In literature, various research papers have been published for reducing order modelling of linear systems[1-11]. It is observed that reduced order modelling using Routh criterion is a simple and effective method for reduced order modelling. The first paper on reduced order modelling using Routh criterion is appeared in 1975[2]. This paper improves the drawbacks of Pade approximation method. A simple approach on Routh criterion is proposed by V. Krishnamurthy and V. Seshadri [3]. It is found that in this method, system can be unstable if it has both numerator and denominator coefficients. In [6], a mixed method is proposed for reduced order modelling of linear systems. It is observed that all these method are applicable considering only stable systems. It is observed that most of the practical systems are open-loop unstable systems such as d.c. motor, inverted pendulum etc [12, 13]. To the best of our knowledge, these existing approaches can not be easily applicable to open-loop unstable system. Therefore, in this paper , simple approach based on Krishnamurthy's results on Routh criterion [2] is presented for reducing order modelling of an open-loop unstable system. In order to show the applications of the approach, open-loop unstable transfer function of position control d.c. motor is considered. The order of this position control system is three. It has been reduced to second-order. In order to verify whether the original position control system matches with the reduced order position control system, step responses of the unity feedback closed loop-original system and reduced order system are checked. It is shown that the responses are exactly matching. This paper is presented as follows:

This paper is presented as follows:

Section- II: Includes a method for obtaining reduced order model, Section-III: Includes determination of reduced order model of position control system, Section-IV: Includes simulation results, Section –V: Conclusions.

II. REDUCED ORDER MODEL OF LINEAR SYSTEM BASED ON V. KRISHNA MURTHY'S AND SESHADRI RESULT

In [2], a method is presented for reduced order modelling of a linear systems using Routh criterion. Based on this approach, a simple approach is proposed for reduced order modelling of an open- loop unstable system.

Suppose, an open-loop transfer function of the system is represented as

$$G(s) = \frac{K}{s^n + a_1 s^{n-1} + a_2 s^{n-2} + ... + a_{n-1} s} \qquad (1)$$

The open-loop transfer function of above system is unstable since there is one pole at origin. But, the stability of the closed-loop system depends on the gain K. Therefore, the following steps are suggested to determine reduced order model.

Step 1: Determine closed-loop characteristics equation. Suppose, it is denoted as $C_r(s)$

$$C_r(s) = a_0 s^n + a_1 s^{n-1} + a_2 s^{n-2} + ... + a_{n-1} s + K. \qquad (2)$$

Step 2: Applying Routh criterion to $C_r(s)$.

Step 3: Using Krishnamurthy approach based on Routh criterion, reduced order characteristic equation will be obtained. Suppose, it is dented by $C_{rd}(s)$.

Step 4: Suppose $C_{rd}(s)$ can be written as

$$C_{rd}(s) = a_{r0}s^m + a_{r1}s^{m-1} + a_{r2}s^{m-2} + ... + a_{rm}, m < n. \tag{3}$$

Step 5: From above eq. (3), open- loop reduced order model of the system will be

$$G_r(s) = \frac{a_{rm}}{a_{r0}s^m + a_{r1}s^{m-1} + a_{r2}s^{m-2} + ... + a_{r(m-1)}s}. \tag{4}$$

III. REDUCED ORDER MODEL OF POSITION CONTROL SYSTEM

The open-loop transfer function of a position control system is given as [12, 13]

$$G(s) = \frac{\theta(s)}{E_f(s)} = \frac{K_T}{s(L_f s + R_f)(Js + B)} = \frac{K_T}{L_f Js^3 + (BL_f + JR_f)s^2 + BR_f} \tag{5}$$

Where,

K_T =Motor torque constant

L_f = Inductance of a field winding

R_f = Resistance of a field winding

J = Moment of inertia of the motor

B = Viscous-friction coefficient of the motor

Using steps as explained in section-II, reduced order model can be determined as follows:

Step 1: The closed-loop characteristics equation of position control d.c. servomotor as

$$C(s) = 1 + G(s) = L_f Js^3 + (BL_f + JR_f)s^2 + BR_f + K_T. \tag{6}$$

Step 2: Applying Routh criterion to above characteristic equation, we get

$$\begin{array}{lll} s^3 & L_f J & BR_f \\ s^2 & (BL_f + JR_f) & K_T \\ s^1 & z = \dfrac{(BL_f + JR_f)BR_f - L_f JK_T}{(BL_f + JR_f)} & \\ s^0 & K_T & \end{array} \tag{7}$$

Step 3: Using Krishnamurthy's approach, reduced order closed-loop characteristic equation is

$$C_r(s) = (BL_f + JR_f)\ s^2 + zs + K_T. \tag{8}$$

Where,

$$z = \frac{(BL_f + JR_f)BR_f - L_f JK_T}{(BL_f + JR_f)} \tag{9}$$

Step 4: From above equation, reduced-order open-loop transfer function model can be written as

$$G_r(s) = \frac{K_T}{(BL_f + JR_f)^2\ s^2 + zs}. \tag{10}$$

IV. SIMULATION RESULTS

Suppose, parameters of dc servomotor are as given below:

$$J = 2.2\times10^{-3}, B = 3.025\times10^{-3}, L_f = 0.5\times10^{-2}, K = 25\times10^{-3}, Rf = 0.9. \tag{11}$$

Using eq. (5), original transfer function model and reduced order model of the system are as given below:

Original model:

$$\frac{0.025}{1.1\times10^{-5}s^3 + 0.001995s^2 + 0.002723s} \tag{12}$$

Reduced-order model:

$$\frac{0.025}{0.001995s^2 + 0.002585\ s} \tag{13}$$

The step responses of unity feedback closed-loop original model and reduced order model are shown in fig. 1.

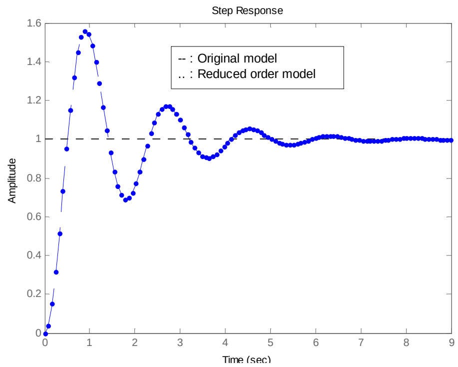

Fig.1 : Step response of original model and reduced order model of position control system

From above responses, it is shown that closed-loop original system and reduced-order system have exactly the same response.

V. CONCLUSION

In this note, it shown that the reduced order model can be determined for open-loop unstable system. Here, this approach is applied to unstable position control system. In future, this approach can be applied for other type of electrical and mechanical systems.

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