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Quality-Adaptive sharpness enhancement and noise removal of a colour images based on the bilateral filtering

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ABSTRACT

In this paper, we present the Adaptive Bilateral Filter (ABF) for sharpness enhancement and noise removal of a colour images. The ABF sharpens an image by increasing the slope of the edges without producing overshoot or undershoot. It is an approach to sharpness enhancement that is fundamentally different from the unsharp mask (USM). This new approach to slope restoration also differs significantly from previous slope restoration algorithms. Compared with an USM based sharpening method, the optimal unsharp mask (OUM), In terms of noise removal, ABF will outperform the bilateral filter and the OUM. ABF works well for both gray images and color images. Due to operation of sharpening of colour images along the edge slope tend to poseterize the image using ABF by pulling up or pulling down the colour images. The proposed method is effective at removing signal noise while enhancing the experimental results in perceptual quality both quantatively and qualitatively.

Keywords:Bilateral filters,Colour images,noise removal,sharpness enhancement, slope restoration.

1. INTRODUCTION

The bilateral filter[1] is essentially a smoothing filter; it does not restore the sharpness of a degraded image. Bilateral filtering produces no phantom colours along edges in colour images, and reduces phantom colours where they appear in the original image. The Aleksic *et al.* modified bilateral filter to perform both noise removal and sharpening by adding a high-pass filter to the conventional bilateral filter[2].

In this we present the adaptive bilateral filter (ABF) for sharpness enhancement and noise removal[4]. ABF works well for both gray images and colour images. The Adaptive Bilateral Filter is able to smooth the noise[5][6], while enhancing edges and textures in the image. The bilateral

restored images are significantly less sharper than the Adaptive bilateral filter[15].

The ABF is useful tool for the following applications: Imagedenoising[7],Computergraphics[8],Videoprocessing[9],Imageinterpolation[10],illuminationestimation[11],Relight ingandtexturemanipulation[12][13],Dynamicrangecompression,Image restoration, Digital half toning, Color correction, Exposure Correction, Flash photography enhancement, Photo printing applications, Astronomical image restoration.

There is a Training-based approaches have been used to develop imaging algorithms for a variety of applications, including image interpolation[14],image restoration[8], digital half toning[9][13], descreening [14], and colour correction[12].The contents that Training-based approaches have in common when used for development of imaging algorithms are:

- 1) A set of training pairs each consisting of an input image and a desired output image.
- 2) An architecture for the algorithm consisting of free parameters.
- 3) A cost function under which those free parameters may be optimized.

The two most common forms of degradation an image suffers are loss of sharpness or blur, and noise. The degradation model we use consists of a linear, shift-invariant blur followed by additive noise[8]. First to develop a sharpening method that is fundamentally different from the unsharp mask filter (USM)[6], which sharpens an image by enhancing the high-frequency cocomponents of the image. In the spatial domain, the boosted higfrerequencfrequency components lead to overshoot and undershoot around edges, which causes objectionable ringing or halo artifacts in general.

Our goal is to develop a sharpening algorithm that increases the slope of edges without producing overshoot and undershoot, which renders clean, crisp, and artifact free

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edges, thereby improving the overall appearance of the image.

In terms of noise removal, conventional linear filters work well for removing additive Gaussian noise, but they also significantly blur the edge structures of an image. Therefore, a great deal of research has been done on edge-preserving noise reduction. Hybrid schemes combining both rank order filtering and linear filtering have been proposed in order to take advantage of both approaches.

2. BILATERAL FILTER

The bilateral filter proposed by Tomasi and Manduchi[1]. It is a smoothing filter. Removes noise, retains the edges but details are blurred. It uses low pass filters, the parameters are fixed. It does not restore the sharpness of a degraded image.

Bilateral filtering smoothes images while preserving edges[1], by means of a nonlinear combination of nearby image values. The method is non-iterative, local, and simple. It combines gray levels or colors based on both their geometric closeness and their photometric similarity, and prefers near values to distant values in both domain and range. In contrast with filters that operate on the three bands of a color image separately, a bilateral filter can enforce the perceptual metric underlying the CIE-Lab color space[34], and smooth colors and preserve edges in a way that is tuned to human perception.

We then combine range and domain filtering, and show that the combination is much more interesting. We denote the combined filtering as Bilateral Filtering.

In particular, bilateral filters can be applied to color images just as easily as they are applied to black-and white images. the bilateral filter acts essentially as a standard domain filter, and averages away the small, weakly correlated differences between pixel values caused by noise. Consider now a sharp boundary between a dark and a bright region When the bilateral filter is centered, say, on a pixel on the bright side of the boundary, the similarity function assumes values close to one for pixels on the same side, and close to zero for pixels on the dark side.

The similarity function is shown in figure 1 (b) for a filter support centered two pixels to the right of the step in figure 1 (a). As a result, the filter replaces the bright pixel at the center by an average of the bright pixels in its vicinity, and essentially ignores the dark pixels.

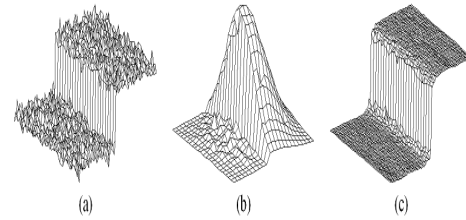


Figure 1: (a) A 100-gray-level step perturbed by Gaussian noise with $\sigma = 10$ gray levels. (b) Combined similarity weights $c(x)$ for a 23×23 neighborhood centered two pixels to the right of the step in (a). The range component effectively suppresses the pixels on the dark side. (c) The step in (a) after bilateral filtering with $\sigma_r = 50$ gray levels and $\sigma_d = 5$ pixels.

Conversely, when the filter is centered on a dark pixel, the bright pixels are ignored instead. Thus, as shown in figure

3. BILATERAL FILTER PROPERTIES:

The bilateral filter proposed by Tomasi and Manduchi in 1998 is a nonlinear filter that smoothes the noise while preserving edge structures[1]. The shift-variant filtering operation of the bilateral filter is given by,

$$f[m, n] = \sum_k \sum_l h[m, n, k, l] g[k, l]$$

$f[m, n]$ is the restored image.

$h[m, n, k, l]$ is the response at $[m, n]$ to an impulse $[k, l]$ and $g[m, n]$ is the degraded image.

where $[m_0, n_0]$ is the center pixel of the window.

σ_d and σ_r are the standard deviations of the domain and range Gaussian filters, respectively.

$$r_{m_0, n_0} = \sum_{m=m_0-N}^{m_0+N} \sum_{n=n_0-N}^{n_0+N} \exp(-\frac{(m-m_0)^2}{2\sigma_d^2} - \frac{(n-n_0)^2}{2\sigma_d^2}) \times \exp(-\frac{(g[m, n] - g[m_0, n_0])^2}{2\sigma_r^2})$$

is a normalization factor that assures that the filter preserves average gray value in constant areas of the image.

This ensures that averaging is done mostly along the edge and is greatly reduced in the gradient direction. This is the reason why the bilateral filter can smooth the noise while preserving edge structures.

Fig. 1(b) shows that a bilateral filter with $\sigma_d = 2$ and $\sigma_r = 20$ removes much of the noise that appears in the degraded image shown in Fig. 1(a) and preserves the edge structures. In Fig. 1(c), where the spatial domain Gaussian with $\sigma_d = 2$ is applied alone, the edges are significantly blurred. The range filter with $\sigma_r = 20$ at the edge pixel A is shown in Fig. 1(e).

4.1 ADAPTIVE BILATERAL FILTER (ABF) FOR SHARPENING AND DE-NOISING:

The Adaptive Bilateral Filter is a new training based approach to image restoration .it also consists of two filters 1.Domain Filter 2.Range Filter.

The domain filter for noise removal. Domain filter gives higher weight to pixels that are spatially close to the center pixel. Range filter acts as a derivative filter that processes the histogram of the image for sharpness. The range filter gives higher weight to pixels that are similar to the center pixel in gray value.

In this section, we present a new sharpening and smoothing algorithm: the adaptive bilateral filter (ABF). The response at [mo,no] of the proposed shift-variant ABF to an impulse at [m,n] is given by (4), where [mo,no] and $\Omega m_o, n_o$ are defined as before, and the normalization factor is given by (5),

$$h[m_o, n_o; m, n] = \begin{cases} \frac{1}{\sigma_d \sigma_r} \exp(-(m - m_o)^2 / 2\sigma_d^2) \exp(-(n - n_o)^2 / 2\sigma_r^2) \exp(-(g[m, n] - g[m_o, n_o])^2 / 2\sigma_g^2), & [m, n] \in \Omega_1 \\ 0 & \text{else} \end{cases} \tag{5}$$

$$r_{m_o, n_o} = \sum_{m=m_o-N}^{m_o+N} \sum_{n=n_o-N}^{n_o+N} \exp(-(m - m_o)^2 / 2\sigma_d^2) \exp(-(n - n_o)^2 / 2\sigma_r^2) \exp(-(g[m, n] - g[m_o, n_o])^2 / 2\sigma_g^2)$$

$$m_o, n_o = \sum_{m=m_o-N}^{m_o+N} \sum_{n=n_o-N}^{n_o+N} \exp(-(m - m_o)^2 / 2\sigma_d^2) \exp(-(n - n_o)^2 / 2\sigma_r^2)$$

The ABF retains the general form of a bilateral filter, but contains two important modifications. First, an offset ζ is introduced to the range filter in the ABF. Second, both ζ and the width of the range filter in the ABF are locally adaptive. If $\zeta = 0$ and w is fixed, the ABF will degenerate into a conventional bilateral filter. For the domain filter, a fixed low-pass Gaussian filter with $w=1.0$ is adopted in the ABF.

The combination of a locally adaptive ζ and w transforms the bilateral filter into a much more powerful filter that is capable of both smoothing and sharpening. To understand how the ABF works, we need to understand the role of ζ and 'w' in the ABF.

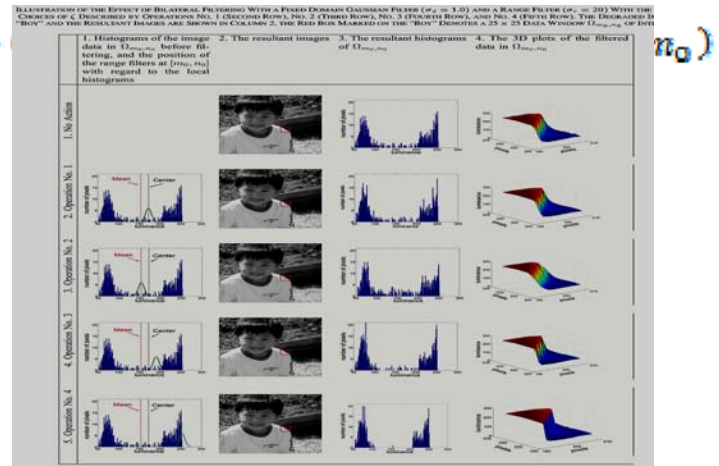
4.2. ROLE OF OFFSET ζ IN THE ABF:

The range filter can be interpreted as a 1-D filter that processes the histogram of the image. We will illustrate this

viewpoint for the window of data enclosed in the red box in the boy portrait images in Table I. We index the images in the table by their [row, column] coordinates. The original degraded image with the red data box is shown in [1, 2], for which the histogram is shown in [1, 3]. For the conventional bilateral filter, the range filter is located on the histogram at the gray value of the current pixel and rolls off as the pixel values fall farther away from the center pixel value as shown in [2, 1]. By adding an offset ζ to the range filter, we are now able to shift the range filter on the histogram, as shown in [3, 1], [4, 1], and [5, 1]. As before, let $\Omega m_o, n_o$ denote the set of pixels in the window of pixels $(2N + 1) \times (2N + 1)$ centered at [mo,no]. Let MIN, MAX, and MEAN denote the operations of taking the minimum, maximum, and average value of the data in $\Omega m_o, n_o$, respectively. Let

$\Delta m_o, n_o = g[m_o, n_o] - \text{MEAN}(\Omega m_o, n_o)$ we will demonstrate the effect of bilateral filtering with a fixed domain Gaussian filter $\sigma_d = 1.0$ and a range filter $\sigma_r = 20$ shifted by the following choices for ζ .

- 1) No offset (conventional bilateral filter): $\zeta[m_o, n_o] = 0$
 - 2) Shifting towards the MEAN: $\zeta[m_o, n_o] = -\Delta m_o, n_o$
 - 3) Shifting away from the MEAN: $\zeta[m_o, n_o] = \Delta m_o, n_o$
 - 4) Shifting away from the MEAN, to the MIN/MAX
- $$\zeta[m_o, n_o] = \begin{cases} \text{MAX}(\Omega m_o, n_o) - g[m_o, n_o], & \text{if } \Delta m_o, n_o > 0 \\ \text{MIN}(\Omega m_o, n_o) - g[m_o, n_o], & \text{if } \Delta m_o, n_o < 0 \\ 0, & \text{if } \Delta m_o, n_o = 0 \end{cases}$$



The locations of the resultant range filters with regard to the histogram of the data in $\Omega m_o, n_o$ are illustrated in Table I, rows two to five. Here $N=12$, As we can see from Table, shifting the range filter towards $\text{MEAN}(\Omega m_o, n_o)$ will blur the image [3, 2]. Shifting the range filter away from $\text{MEAN}(\Omega m_o, n_o)$

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will sharpen the image [4, 2]).

In the extreme case, if for every pixel above $MEAN(\Omega_{m_0, n_0})$, we shift the range filter to $MAX(\Omega_{m_0, n_0})$, and for every pixel below $MEAN(\Omega_{m_0, n_0})$, we shift the range filter to $MIN(\Omega_{m_0, n_0})$, we will see a drastic sharpening effect and the image will appear over-sharpened ([5, 2]). The reason behind these observations is the transformation of the histogram of the input image by the range filter.

In our case, the data window marked by the red box in [1, 2] contains an edge. Therefore, the histogram of the data in Ω_{m_0, n_0} has two peaks, which correspond to the darker and brighter sides of the edge, respectively ([1, 3]). Any pixels located between the two peaks appear on the slope of the edge. A 3-D plot of the image data in Ω_{m_0, n_0} is shown in [1, 4]. The conventional bilateral filter (no shift to the range filter) does not significantly alter the histogram of the data ([2, 3]) and, consequently, does not change the slope of the edge ([2, 4]).

Shifting the range filter to $MEAN(\Omega_{m_0, n_0})$ at each pixel will redistribute the pixels towards the center of the histogram ([3, 3]). Hence, the slope is reduced ([3, 4]). On the other hand, if we shift the range filter further away from $MEAN(\Omega_{m_0, n_0})$ pixels will be compressed against the two peaks ([4, 3]). The slope will then be increased. In the case of operation No. 4,

4.3 OPTIMIZATION OF THE ABF PARAMETERS:

The parameter optimization is formulated as a minimum mean squared error (MMSE) estimation problem. We classify the pixels into T classes, and during the training process estimate the optimal ζ and σ_r for each class that minimizes the overall MSE between the original and restored images. Let P be the total number of training image sets. The k th set ($k = 1, 2, \dots, P$) consists of an original image $f_k[m, n]$, a degraded image $g_k[m, n]$, the class index image $L_k[m, n]$, and the restored image $\hat{f}_k[m, n]$. All four of these images have dimensions $M_k \times N_k$. Let

$S^{(k)} = \{[m, n], [m, n] \in [0, M_k - 1] \times [0, N_k - 1]\}$ be the set of indices for the pixels in these images. Also let S_i be the set of indices for the pixels belonging to the class i in image k. Given the P training image sets as described .

$$S_i^{(k)} = \{[m, n], L_k[m, n] = i \text{ and } [m, n] \in S^{(k)}\}$$

$$i = 1, 2, \dots, T$$

above, the optimal parameters $\vec{\zeta}$ and $\vec{\sigma}_r$ satisfy

$$\{\vec{\zeta}, \vec{\sigma}_r\} = \arg \min_{\{\zeta, \sigma_r\}} \sum_{k=1}^P \sum_{[m, n] \in S_i^{(k)}} \| \hat{f}_k[m, n] - f_k[m, n] \|^2$$

Where $\|A\|_2$ denotes the L-2 norm of the $\vec{\zeta} = \{\zeta_i | i = 1, 2, \dots, T\}$ array A over the index set B, and $\vec{\sigma}_r = \{\sigma_{r,i} | i = 1, 2, \dots, T\}$

Since the classes are independent and non-overlapping, we can separately estimate the optimal $\vec{\zeta}$ and $\vec{\sigma}_r$ for each class

$$\{\zeta_i^*, \sigma_{r,i}^*\} = \arg \min_{\{\zeta_i, \sigma_{r,i}\}} \sum_{k=1}^P \sum_{[m, n] \in S_i^{(k)}} \| \hat{f}_k[m, n] - f_k[m, n] \|^2$$

$$i = 1, \dots, T$$

To find the pair of parameters which minimizes the MSE for each class, we perform an exhaustive search in the parameter space

$$\Phi = \{(\zeta, \sigma_r) | \zeta \in \Phi_\zeta \text{ and } \sigma_r \in \Phi_{\sigma_r}\}$$

The range and the step size of the parameters are chosen empirically such that they can yield adequate sharpening and smoothing for all types of image structures with a balance between accuracy and computational cost. The offset in our case is found to be more effective if the offset $(\vec{\zeta}) = \text{delta}(\Delta_{m_0, n_0})/5$. Adding and subtracting the offset from the pixel values makes the image more dynamic in appearance. This also modifies histogram averaging of the low pass Gaussian filter.

Estimated Optimal $\vec{\zeta}$ and $\vec{\sigma}_r$

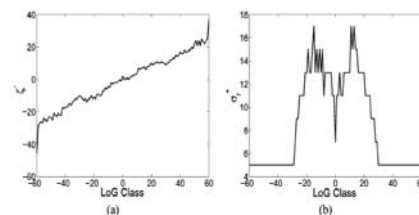
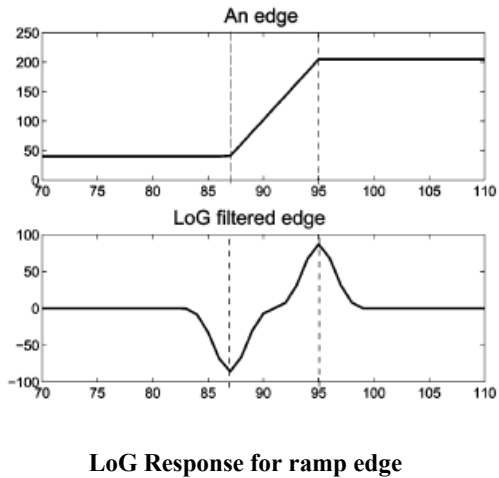
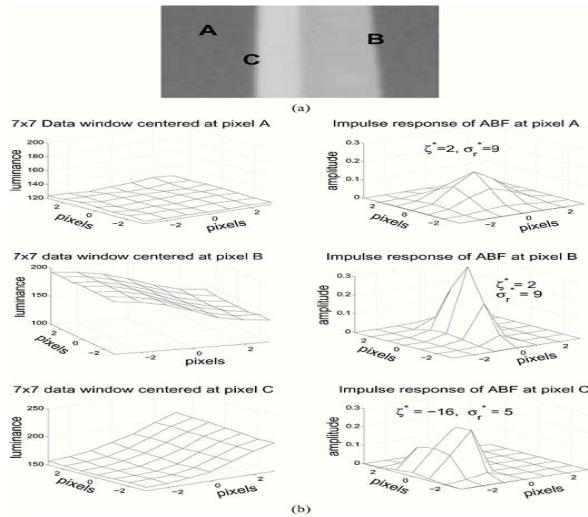


Fig. 6. Estimated optimal parameters for the ABF. (a) Optimal offset ζ , for each class. (b) Optimal σ_r , for each class.

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4.4 FEATURE DESIGN:

The feature for pixel classification plays an important role in the success of the training. we described how shifting the range filter according to Δ , the difference between the center pixel value and the mean of the local data window will impact the output image.

general guidelines for choosing the feature(s) are:

- 1) be able to reflect the strength of edges,
- 2) it can distinguish the regions, to process differently, mainly, the regions for smoothing and sharpening,
- 3) it have some robustness to noise.

4.5 LAPLACIAN OF GUASSIAN(LoG) OPERATOR:

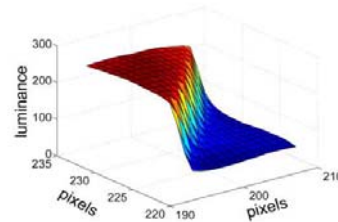
The laplacian of Gaussian is a feature used for pixel classification. It is used to determine whether a pixel belongs to bright or dark side of an edge. It calculates Laplacian. It is a high pass filter. Computes the second order derivative of input image.

LoG FILTER RESPONSE:

The magnitude of response near edges is high. In smooth regions, and The magnitude of response is low. On the center of edge, response is zero.

REASONS FOR SELECTING LoG OPERATOR:

Magnitude of LoG strength reflects the local edge structure. More robust to noise compared to other operators. The sign of response gives the location of a pixel With respect to edge.



Edge structure

ALGORITHM:

- STEP1: Load color images into MATLAB environment.
- STEP2: Convert into Double precision in the interval [0,1]
- STEP3: Apply Blur Point Spread Function.
- STEP4: Introduce pseudo-random noise into images
- STEP5: Display histograms of both images
- STEP6: Set bilateral filter width
- STEP7: Set Domain and range filter standard deviations
- STEP8: Apply Adaptive Bilateral Filter to each image
- STEP9: Verify that the input image exists and is valid
- STEP10: Verify bilateral filter window size
- STEP11: Verify bilateral filter standard deviations
- STEP12: Select appropriate filter function depending on color image
- STEP13: Apply color bilateral filtering
- STEP14: Convert input sRGB image to Lab (YIQ) color space
- STEP15: Calculate Laplacian of Gaussian response
- STEP16: Calculate Gaussian Domain filter weights
- STEP17: Show Domain filter response
- STEP18: Create wait bar.
- STEP19: Extract local region.

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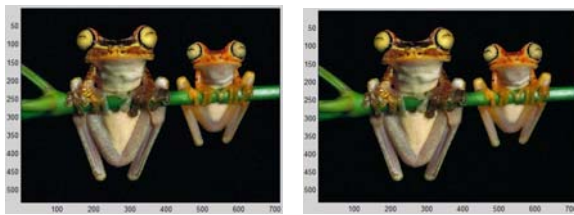
- STEP20: Calculate local mean, minimum, maximum values
- STEP21: Increase sharpness using mean, minimum, maximum values
- STEP22: Make sigma, width adaptive with Laplacian.
- STEP23: Compute range Gaussian intensity weights
- STEP24: Calculate bilateral filter response
- STEP25: Close wait bar
- STEP26: Display color input image and filtered output
- STEP27: Display Histograms after filtering the images
- STEP28: Calculate MSE between original and filtered image.
- STEP29: Display Mean Squared Error
- STEP30: THE END.

5. RESULTS AND DISCUSSIONS:

The bilateral filter has been widely used since it was first proposed. The ABF is an extension of the traditional bilateral filter. With its capability of adaptive sharpening and smoothing, it offers the potential for wider application and better image quality. Although the ABF is developed in the framework of image restoration, it can be used as an image enhancing algorithm, as we have demonstrated with the test images. In particular, our main motivation for developing the ABF is to enhance the quality of digital images that consumers want to view and/or print.

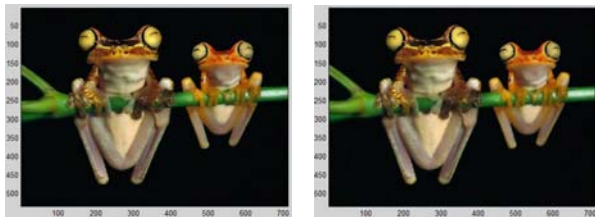
5.1 EFFECT OF ABF ON COLOR IMAGE:

Variance (σ) of Adaptive Bilateral Filter: $\sigma=14$



$\sigma=1$

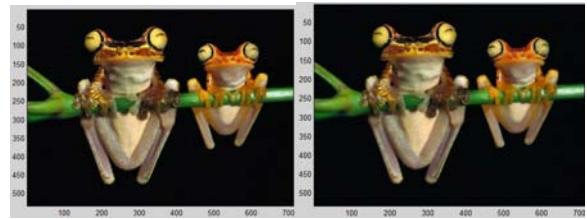
$\sigma=4$



$\sigma=14$

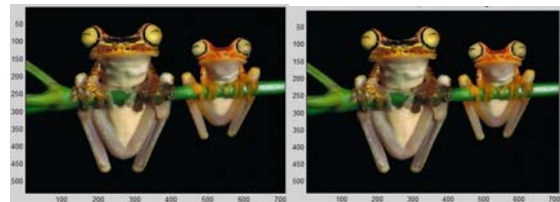
$\sigma=20$

ABF FILTERED IMAGES: w=5



w=2

w=4



w=5

w=9



w=12

From the above tables $\sigma=14$, $w=4$ are the optimum parameters for the color image. The values of pixels vary in the range of [0,1] for the B&W image, but for the color image values are ranged upto 100. So, the filter parameters are also adjusted accordingly

The best parameters are those which produce minimum MSE and smooth images with sharp edges. I feel that visual inspection of the test results provides a better indication of the quality of ABF restored images.

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Width(w)	MSE	Visual Observations
W=2	0.0014	Noise is not removed
W=4	0.0010	Noise is reduced. edges are sharp ,image appeared smooth.
W=5	0.0012	Noise is removed, smooth image.
W=7	0.0014	Slight Blurring is observed.
W=9	0.0016	Blurring occurred
W=12	0.0019	Not effective on image, noise is not removed

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