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Rashmi Ranjan Ota
Siksha O Anusandhan Deemed to be University, Bhubaneswar, rashmiranjanota@soa.ac.in

Swetarani Tripathy
Siksha O Anusandhan Deemed to be University, Bhubaneswar, swetaranitripathy123@gmail.com

Dr Mitali Madhusmita Nayak
Siksha O Anusandhan Deemed to University, mitalinayak7@gmail.com

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A Case StudyonSolutions of Linear FractionalProgrammingProblems

RashmiRanjan Ota*, SwetaraniTripathy**, MitaliMadhusmitaNayak***
I.T.E.R, S'O'A (Deemed To Be University)
Bhubaneswar,Odisha
*rashmiranjanota@soa.ac.in,
**swetaranitripathy123@gmail.com,
***mitalinayak@soa.ac.in

Abstract

In some decision-making problems,the objective function can be defined as the ratio of two linear functionalsubjects to given constraints. These types of problems are known as linear fractional programming problems. The importance of linear fractional programming problems comes from the fact that many real-life problems can be expressed as the ratio of physical or economical values represented by linear functions, for example, traffic planning, game theory, and production planning, etc. In this article,a suitable attempthas been made to form the mathematical model of a production planning problem using fractional programming and to solve it, variousfractional programmingtechniques have been used. Finally the result is compared with the solution obtained by the graphical method. To illustrate the efficiency of the stated method a numerical example has been given.

Keywords: *-fractional programming; linear programming;efficiency;production planning.*

1. Introduction:-

Fractionalprogramming in which the relationship among the variables is linear, the objective function to be optimized is a ratio of two linear functionsand the given constraints arein the linear form known as a linear fractional programming problem. Many researchers in the past have been attracted by the fascinating character of fractional programming. The main reason for their attraction towards fractional programming comes from the verity that the programming models could be suitablefor real-life problems if we consider optimization problems as the ratio between the physical quantities. Initially, the concept of fractional programming was developed by the Hungarian mathematician B.Matros[10] in the year 1960. Nowadays,this method is frequently used in different fields.In many practical applications like optimal chains for Markov chains, blending problems, stock problems and sensitivity of linear programming problems, optimization of ratios of objective function give more insight than optimization of a single objective function. Fractional programming is of various types. Some of them deal with the theoretical part [1], some of them related to solution methods [4] and some are deal with applications [9].

Several methods have been developed to solve linear fractional programming (LFP) problems. In the year 1962, Charnes and Cooper [5] have proposed how the linear fractional programming problems(LFP) converted to their equivalent linear programming problems. But Bitran and Novaes[2] have solved the linear fractional programming problems by computing the local gradient of objective functions. WhereasSwarup[15] has extended the concept of the simplex method of Dantzig[6,7] in finding solutions ofLFP problems. For solving LFPs Tantway[17,18] bring forwardtwo different methods namely; the feasible direction approach and the duality approach method. In a similar mannerMojtaba et .al[3] have solved linear fractional programming problems with integer coefficients in objective functions which are based on Charnes and Cooper criterion. Odior[11] has solved linear fractional programming problems by using an algebraic approach based upon the duality and the partial fraction. In the year 1981, Schaible discussed some applications and algorithms of fractional programming. In 1997 Stancu-minasian[16] studied the theory, methods, and applications of fractional programming.In 2012 Guzel, et .al[8] have studied fractional transportation problems with interval coefficients. In the same year, Sohraband Morteza[14] introduced interval-valued linear fractional programming problems. Most of the available methods solved linear fractional programming problems by using the simplex type method. But F.A.Simi and M. S. Talukder[13], in their recent paper, have shown how to solve linear fractional programming problems considering the objective function as a linear fractional function where constraints are in the form of inequalities. They have transformed the linear fractional programming into linear programming and solved the problem algebraically using the concept of duality. In recent years, S. K. Saha et.al[12] have shown a new approach to solve linear fractional programming problems using any type of fractional programming techniques converting them in to linear programming form. They have developed a FORTRAN program to solve such problems.The novelty of the paper is to develop mathematical model of a real-life problem using fractional programming and to solve it by different optimization techniques.

This article is organized as follows; followed by the introduction,the concept of linear programming andlinearfractional programming problems have given in sec.2 and sec.3 respectively. The solution procedure for linear fractional programming by Charnes and Cooper and the denominator objective restriction method has been discussed in sec.4 and sec.5 respectivelywhereasthe solution by the graphical method is given in sec.6. An illustrative example has been given in sec.7 to establish the validity of our discussions and finally, some conclusionsdrawn are presented in sec.8.

2. Linear programming problem:-

Linear programming problem is concerned with finding the optimal value of a linear function of several variables subject to the given conditions.

Linear programming can be used to express a wide variety of different kinds of problems. We can use the algorithms of linear programming to solve the max-flow problem, solve the min-cost max-flow problem, find mini-max-optimal strategies in games, and many other things.

Mathematically,A linear programming problem can be represented in canonical form as

$$Maximize\ Z = c^T X$$

$$subject\ to \quad AX \leq b$$
$$X \geq 0$$

where $'X'$ represents the vector of decision variables to be determined, $'c'$ vectors of cost coefficients, $'b'$ is the price vector,and $'A'$ is a matrix of coefficients.

The Linear programming problem can be expressed in standard form as

$$Max\ Z = cx$$

$$subject\ to \quad Ax = b$$
$$x \geq 0$$

Where $x = (x_1, x_{2,} \dots \dots .. x_n)^T$, $c = (c_1, c_2, \dots, c_n)$, $b^T = (b_1, b_2, \dots, b_m)$ *and*

$$A = (a_{ij}), \qquad i = (1,2,\dots,m), \quad j = (1,2,\dots,n)$$

3. Linear Fractional Programming problem:-

In mathematical optimization,linear fractional programming is a generalization of linear programming. The objective function in linear fractional programming is a ratio of two linear functions. A linear program can be regarded as a special case of a linear fractional programming problem in which the denominator is the constant function.

Mathematically, linear fractional programming can be stated as

Max/Min$\frac{c^T x+\alpha}{d^T x+\beta}$

Subject to $Ax \le b$

$$x \ge 0$$

Where $x \in R^n$ represents the vectors of the variable to be determined c, d $\in R^n$ and b $\epsilon\, R^m$ are vectors of known coefficients and α, β are the scalars.

The constraints have to restrict the feasible region to $\{x | d^T x + \beta > 0\}$ i.e the region on which the denominator is positive.

Linear programming provides the best outcome such as maximum profit or least cost. In contrast, linear fractional programming is used to achieve the highest ratio of outcome to the cost that is the ratio representing the highest efficiency.

4. Charnes and Cooper method:-

Charnes&Cooper method is a simple technique for solving linear fractional programming problem was developed in 1967. In this method, the linear fractional programming problem is transformed into a linear programming problem by using some substitution. Then the linear programming problem is solved by the simplex method.

The general form of a classical linear fractional programming problem by Charnes and Cooper method can be stated as follows:

$$max \frac{c^T x + \alpha}{d^T x + \beta}$$

$$x \in X = \{x \in R^n : Ax(\le or = or \ge)b, x \ge 0, b \in R^m$$

Where c, d ϵR;α, βϵ R, x is nonempty and bounded.

<u>Procedures ofCharnes and Cooper transformation</u>**:-**

Let the following function to be optimized

Max $Z = \frac{ax+b}{cx+d}$

Subject to $Ax = b$

$$x \ge 0$$

By substitution method, Let consider

$$y = tx \qquad or \qquad x = \frac{y}{t}$$

Now the problem can be modified as

Max $\quad Z = \frac{\frac{ay}{t}+b}{\frac{cy}{t}+d} = \frac{ay+bt}{cy+dt}$

Subject to $\frac{Ay}{t} = b \Rightarrow Ay - bt = 0$ $, y, t \geq 0$

Assume $cy + dt = 1$

$$\text{Max } Z = ay + bt$$
$$subject\ to\ \ cy + dt = 1$$
$$Ay - bt = 0$$
$$y, t \geq 0$$

Now, we can solve the above problem using simplex method

5.Denominator objective restriction method:-

Let P bethe linear fractional programming problem whose objective function with numerator N and denominator D. To find the solution concerning to given constraints,we have tofollow two theorems given below.

Theorem 1:

Let N be the problem with optimal solutionX_0. If the problem D having$\{X_n\}$ is a sequence of a basic feasible solutions, with $Z(X_k) \leq Z(X_{k+1})$ for all k= 0, 1, 2…..n-1 and $Z(X_n) \geq Z(X_{k+1})$ then X_n is an optimal solution to the problem P by the simplex method considering X_0 as an initial feasible solution. Theorem 2:Let the problem N having optimal solution X.IfD is the problem witha sequence of basic feasible solution$\{X_n\}$with $Z(X_k) \leq Z(X_{k+1})$for k= 0, 1, 2…..n-1and $X_{n+1}$is an optimal solution to the problem D by simplex method considering X_0as an initial feasible solution, then X_{n+1} is an optimal solution to the problem P.

The denominator objective restriction method for finding the optimal solution to the linear fractional programming problem P can be stated as follows:-

Step -1:-With respect to the given problem P, construct two single objective linear programming problemsconsidering numerator N as one objective function andthe other as the denominator D using stated constraints.

Step-2:- Using the simplex method compute the optimal solution to the problem N. Let X_0 be the optimal solution to the problemNand $Z(X_0) = Z_0$.

Step-3:-To find a sequence of improved basic feasible solutions $\{X_n\}$to the problem D, use the optimal table of problem N as an initial simplex table to the problem D and using the simplex method find the value of Z at each of the improved basic feasible solutions.

Step-4(a):-Stop the computation process, if $Z(X_k) \leq Z(X_{k+1})$ for all k=0,1,2,….,n-1 and $Z(X_n) \geq Z(X_{n+1})$ for some 'n' and then go to step 5.

Step-4(b):-Stop the computation process, if $Z(X_k) \leq Z(X_{k+1})$ for all k=0, 1, 2,… n and X_{n+1} will be an optimal solution to the problem D for some 'n' and then go to step 6.

Step-5:-$X_n$will be an optimal solution to the problem Pand Max$(X) = Z(X_n)$.

Step -6:-X_{n+1} is an optimal solution to the problem P and Max$(X) = Z(X_{n+1})$.

6. Graphical Method:-

Alinear fractional programming problem (LFPP) can be solved by using the graphical method. To solve LFPP,the problem will be first transferred into its corresponding single linear programming problem, and then it will be solved. In a graphical method, anLFPPcan be solved by finding the highest or lowest point of intersection between the focus point of the objective function and the feasible region on the graph.

Algorithm:-

Step1:-Let the LFPP problem given by

$$Optimize\ \ F(x) = \frac{N(x)}{D(x)}$$
$$= \frac{n_1x_1 + n_2x_2 + n_0}{d_1x_1 + d_2x_2 + d_0}$$
$$subject\ to$$
$$a_{i1}x_1 + a_{i2}x_2 \le b_i, i = 1,2, \ldots .., m$$
$$x_1, x_2 \ge 0$$

Where X_1, X_2 are the nonnegative unknown variables.

Step 2:-Convert the LFP into standard form

$$Optimize\ \ F(x) = \frac{N(x)}{D(x)}$$
$$= \frac{n_1x_1 + n_2x_2 + n_0}{d_1x_1 + d_2x_2 + d_0}$$
$$subject\ to$$
$$a_{i1}x_1 + a_{i2}x_2 = b_i, i = 1,2, \ldots .., m$$
$$x_1, x_2 \ge 0$$

Step 3:- From the standard form of LFPP, we have

$N(x) = n_1x_1 + n_2x_2 - n_0$=0

$$D(x) = d_1x_1 + d_2x_2 - d_0 = 0$$

Step 4:- Solve $N(x)$ and $D(x)$ separately and find the focus point (P).

Step 5:- Solve the constraints and find the extreme points

$$A(x_1, x_2), B(x_1, x_2), C(x_1, x_2), E(x_1, x_2)$$

Step 6:-a) Plot the focus point by using $N(x)$and $D(x)$. If $N(x)$ and $D(x)$intersects each other then the point is the focus point andit has a unique solution. If $N(x)$ and $D(x)$ are not intersecting, then it hasnosolution.

b) Plot the extreme points

$A(x_1, x_2), B(x_1, x_2), C(x_1, x_2), E(x_1, x_2)$.

c) Find the feasible region.

If a constraint has $\le$ sign then the shaded region will be below the line.

If the constraint has $\ge$ sign then the shaded region will be above the line.

Step 7:-By putting the extreme points in $F(x)$we get the values of

$$F(A), F(B), F(C), F(E)$$

Step 8:- From the value of $F(A), F(B), F(C), F(E)$, we determine the objective function $F(x)$. For the minimumvalue of the problem, we have to check the maximum value among $F(A)$, $F(B)$, $F(C)$, $F(E)$.For the maximum value, we check the minimum among $F(A)$, $F(B)$, $F(C)$, $F(E)$.

Step 9:-Join the focus points to the maximal point $F(x) = F_{max}$ and minimum point $F(x) = F_{min}$ and draw the arbitrary line from the focus point

$$P(x) = k.$$

Step 10:-If there is a closed common region, then find the bounded feasible region otherwise we find the unbounded feasible region.

7. Numerical Example:-

For the illustration of the above methods given in sec.4 and sec.5, we have considered the following test problem.

A Company manufactures two types of sandal soaps A & B with a profitof aroundRs.4&Rs.3 per unit respectively. However, the cost of each unit of the product is around Rs.3&Rs.2 respectively. A fixed cost of around Rs.1 is added to the cost function through the process of production. Raw materials for the products A & B areavailable around 3 units per pound & 5 units per pound respectively. The supply of raw material is restricted to 15 pounds. Man-hours per unit for product A & B is about 5 hrs & 2 hrs respectively. Daily total man hours available are 10 hrs. Determine the no of units of products A &B be manufactured to maximize the total profit.

Mathematical Model formulation:-

Let a company produce x_1 unit of A andx_2 unit of B

Per unit A's profit = Rs.4

Per unit B's profit = Rs.3

Total profit = $4x_1 + 3x_2$

Per one unit A's cost = Rs.3

Per one unit B's cost = Rs.2

Rs.1 is added extra in cost through the process ofmanufacturing.

Total cost =$3x_1 + 2x_2 + 1$

For A, raw material needed = 3 units per pound

For B, raw material needed = 5 units per pound

Raw material restrict = 15 pound

Total= $3x_1 + 5x_2 \leq 15$

Considering 1 unit of A is 1 pound, 1 unit of B is 1 pound.

Manhrs.Per unit for A = about 5 hrs.

Man hrs.Per unit for B = about 2hrs

Total man hrs.availableis about 10hrs.dailyi.e.$5x_1 + 2x_2 \leq 10$

Aim of the company is to maximize the total profit w.r.to total cost is given by

$$Max \quad f(x) = \frac{4x_1 + 3x_2}{3x_1 + 2x_2 + 1}$$

$$\begin{gathered} Subject\ to \\ 3x_1 + 5x_2 \leq 15 \\ 5x_1 + 2x_2 \leq 10, x_1, x_2 \geq 0 \end{gathered}$$

Solution by Denominator objective restriction method:-

Maximize $N(x) = 4x_1 + 3x_2$

$$\begin{gathered} subject\ to \\ 3x_1 + 5x_2 \leq 15 \\ 5x_1 + 2x_2 \leq 10 \\ x_1, x_2 \geq 0 \end{gathered}$$

Maximize $N(x) = 4x_1 + 3x_2 + 0s_1 + 0s_2$

$$\begin{gathered} subject\ to \\ 3x_1 + 5x_2 + s_1 = 15 \\ 5x_1 + 2x_2 \qquad + s_2 = 10 \\ x_1, x_2, s_1, s_2 \geq 0 \end{gathered}$$

Minimize $D(x) = 3x_1 + 2x_2 + 1$

$$\begin{gathered} subject\ to \\ 3x_1 + 5x_2 \leq 15 \\ 5x_1 + 2x_2 \leq 10 \\ x_1, x_2 \geq 0 \end{gathered}$$

Maximize $D(x) = -3x_1 - 2x_2 - 1 + 0.s1 + 0.s_2$

$$\begin{gathered} subject\ to \\ 3x_1 + 5x_2 + s_1 = 15 \\ 5x_1 + 2x_2 \qquad + s_2 = 10 \\ x_1, x_2, s_1, s_2 \geq 0 \end{gathered}$$

Now, we applysimplex method to solve it as follows:

Table-1:-

c_j 4 3 0 0

c_B	x_B	x_1	x_2	s_1	s_2	b	θ=min+ve ratio
0	s_1	3	5	1	0	15	5
0	s_2	5	2	0	1	10	2

z_j 0 0 0 0

$z_j - c_j$-4 -3 0 0

Continuing as above, the final simplex table will be

Table -2:-

c_j 4 3 0 0

c_B	x_B	x_1	x_2	s_1	s_2	b	θ
3	x_2	0	1	5/19	-3/19	45/19	-
4	x_1	1	0	-2/19	5/19	20/19	-

z_j 4 3 7/19 11/19

$z_j - c_j$0 0 7/19 11/19

$$x_1 = \frac{20}{19}, \qquad x_2 = \frac{45}{19}$$

$z = \frac{4x_1+3x_2}{3x_1+2x_2+1} = \frac{215}{169}$

Using above table, now we solve objective function in denominator

Table -3:-

c_j-3 -2 0 0

c_B	x_B	x_1	x_2	s_1	s_2	b	θ
-2	x_2	0	1	5/19	-3/19	45/19	-15
-3	x_1	1	0	-2/19	5/19	20/19	4

z_j -3 -2 -4/19 -9/19

$z_j - c_j$ 0 0 -4/19-9/19

Table-4:-

c_j -3 -2 0 0

c_B	x_B	x_1	x_2	s_1	s_2	b	θ
-2	x_2	3/5	1	1/5	0	57/19	-
0	s_2	19/5	0	-2/5	1	4	-

z_j-6/5 -2 -2/5 0

$z_j - c_j$9/5 0 -2/5 0

$$x_1 = 0, x_2 = \frac{57}{19}$$

$$z = \frac{4x_1 + 3x_2}{3x_1 + 2x_2 + 1} = \frac{171}{133} = 1.28$$

Table-5:-

c_j -3 -2 0 0

c_B	x_B	x_1	x_2	s_1	s_2	b	θ
0	s_1	3	5	1	0	15	-
0	s_2	5	2	0	1	190/19	-

z_j 0 0 0 0

$z_j - c_j$ 3 2 0 0

$$x_1 = 0, x_2 = 0, z = 0$$

Hence $x_1 = 0$, $x_2 = \frac{57}{19}$, $z = 1.28$

<u>Solution byCharnes and Cooper method</u>**:-**

Let $y_1 = tx_1, \; y_2 = tx_2$

$$Maximize\ f(x) = \frac{\frac{4y_1}{t} + \frac{3y_2}{t}}{\frac{3y_1}{t} + \frac{2y_2}{t} + 1}$$

$$subject\ to$$

$$\frac{3y_1}{t} + \frac{5y_2}{t} \le 15$$
$$\frac{5y_1}{t} + \frac{2y_2}{t} \le 10$$
$$y_1, y_2, t \ge 0$$
$$=> Maximize f(x) = \frac{4y_1 + 3y_2}{3y_1 + 2y_2 + t}$$

$$subject\ to$$
$$3y_1 + 5y_2 - 15t \le 0$$
$$5y_1 + 2y_2 - 10t \le 0$$
$$y_1, y_2, t \ge 0$$

$$=> Maximize\ f(x) = 4y_1 + 3y_2$$
$$subject\ to$$
$$3y_1 + 2y_2 + t + A_1 = 1$$
$$3y_1 + 5y_2 - 15t + s_1 = 0$$
$$5y_1 + 2y_2 - 10t + s_2 = 0$$
$$y_1, y_2, t, s_1, s_2 \ge 0$$

Applying simplex two phase method, we have the following result

Table-1:-

c_j 0 0 0 -1 0 0

c_B	y_B	x_B	y_1	y_2	t	A_1	s_1	s_2
-1	A_1	1	3	2	1	1	0	0
0	s_1	0	3	5	-15	0	1	0
0	s_2	0	5	2	-10	0	0	1

z_j -3 -2 -1 -1 0 0

$z_j - c_j$ -3 -2 -1 0 0 0

Continuing in the same manner as the previous tables, we obtain the final simplex table will be as follows:

Table-4:-

c_j 4 3 0 0 0

c_B	y_B	x_B	y_1	y_2	t	s_1	s_2
0	t	133/1183	0	0	1	-4/169	-315/5915
3	y_2	45/169	0	1	0	35/169	-48/169
4	y_1	140/1183	1	0	0	-22/169	-887/5915

z_j 4 3 0 17/169 1492/5915

$z_j - c_j$ 0 0 0 17/169 1492/5915

$$y_1 = \frac{140}{1183}, y_2 = \frac{45}{169}, t = \frac{133}{1183}$$

$z = 4y_1 + 3y_2 = \frac{15}{11}$=1.27

Solution by graphical method:-

Standard form of the given problem is as follows

$$Max\ \ f(x) = \frac{4x_1 + 3x_2}{3x_1 + 2x_2 + 1}$$

$$Subject\ to$$
$$3x_1 + 5x_2 = 15$$
$$5x_1 + 2x_2 = 10, x_1, x_2 \ge 0$$

To find the focus point solve the numerator and denominator separately

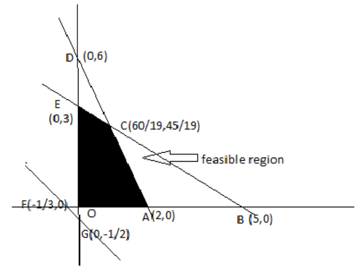
$$N(x) = 4x_1 + 3x_2$$
$$D(x) = 3x_1 + 2x_2 + 1$$

For $N(x)$ points are (0,0) and for $D(x)$ points are (0,-1/2) and (-1/3,0)

For calculating extreme point we solve the constraints

$3x_1 + 5x_2 = 15$, Points are (0, 3) and (5,0)

$5x_1 + 2x_2 = 10$, Points are (0, 5) and (2,0)



From above figure, feasible region is OACEO with O (0, 0),A(2,0),C(60/19,45/19)

E (0, 3) and max f =1.29 at C.

Result Analysis:-

Global solutions to the test problems can be obtained by all the discussedmethods. But Charnes& Cooper method is better as compared to another method from point of calculation, as it converted anLFPPinto a linear programming problem. The graphical method is a nearly approach method. It can also give a better approximation solution.

8. Conclusion:-

As we know, linear fractional programming problems have global solutions because of convexity property and can be solved by different methods. In this paper, we have used Charnes and Cooper and the denominator objective restriction method to solve the model related to the productivity planning problem.Finally, we have compared the result obtained by both the method with the solution obtained by graphical approach method for importance and efficiency of the methods. All the methods discussed are useful in the solution of the economic problems as well as many real-life problems in which different activities utilize fixed resources in proportion to their level of their values.

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